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SOME THEOREMS IN A GRADE CONSISTENT MICROPOLAR THEORY OF THERMOELASTICITY

BY

D. VIERU

1. **Introduction.** B r u l i n and H j a l m a r s [1, 2] have derived a theory of micropolar continua, where the second-order displacement gradient is added to the classic set of independent constitutive variables. The grade consistent micropolar theory of thermoelasticity was studied in [2, 3].

The present paper is concerned with the linear theory of thermoelastic solids. The purpose of this paper is to prove some basic theorems concerning the uniqueness of solution, the reciprocity relations and the variational characterization of the solution.

2. **Basic equations.** We assume that the body occupies a bouded region B with Lipschitz boundary ∂B which consists of a finite number of smooth surfaces. Denote γ_α intersections of two adjoined smooth surfaces and $\Gamma=\bigcup_\alpha \gamma_\alpha$. We denote by u_i the components of the displacement vector and by φ_i the components of the microrotation vector.

The basic equations of the linear theory are (see [1], [2]),

— the equations of motion

$$\tau_{ji,j}-\mu_{kji,kj}+F_i=\rho\ddot{u}_i,\quad m_{ji,j}+\varepsilon_{irs}\tau_{rs}+M_i=I_{ij}\ddot{\varphi}_j, \tag{2.1}$$

— the equation of entropy

$$\rho T_0\dot{\eta}-q_{i,i}=S, \tag{2.2}$$

— the constitutive equations

$$\begin{gathered}\tau_{ij}=A_{ijrs}e_{rs}+B_{ijrs}\varkappa_{rs}+D_{ijpqr}\varkappa_{pqr}+a_{ij}\theta,\\ m_{ij}=B_{rsij}e_{rs}+C_{ijrs}\varkappa_{rs}+E_{ijpqr}\varkappa_{pqr}+c_{ij}\theta,\\ \mu_{ijk}=D_{rsijk}e_{rs}+E_{rsijk}\varkappa_{rs}+F_{ijkpqr}\varkappa_{pqr}+g_{ijk}\theta,\\ \rho\eta=d\theta-a_{ij}e_{ij}-c_{ij}\varkappa_{ij}-g_{ijk}\varkappa_{ijk},\\ q_i=k_{ij}\theta_{,j},\end{gathered} \tag{2.3}$$

— the geometrical equations

$$e_{ij}=u_{j,i}+\varepsilon_{jik}\varphi_k,\quad \varkappa_{ij}=\varphi_{j,i},\quad \varkappa_{ijk}=u_{k,ij}. \tag{2.4}$$

The coefficients in (2.3) have the following symmetries

$$A_{ijrs}=A_{rsij},\ C_{ijrs}=C_{rsij},\ D_{ijpqr}=D_{ijqpr},$$
$$E_{ijpqr}=E_{ijqpr},\ F_{ijkpqr}=F_{jikpqr}=F_{pqrijk}. \tag{2.5}$$

We introduce the notations

$$P_i=[\tau_{ji}-\mu_{sji,s}+(\mu_{pji}n_p n_r-\mu_{pri}n_p n_j),_r]\,n_j,$$
$$R_i=\mu_{rsi}n_r n_s,\ Q_i=<\mu_{pji}n_p\eta_j>,\ \eta_j=\varepsilon_{jrp}\tau_r n_p. \tag{2.6}$$

where n_i represents the components of the unit outward normal to ∂B, τ_i represents the unit tangent vector to γ_α and $<\cdot>$ denotes the difference of the limits of the two sides of γ_α. Let us consider subsets $\Sigma_s(s=1,\ldots,8)$ of ∂B so that $\Sigma_1\cup\Sigma_2\cup\Gamma=\Sigma_3\cup\Sigma_4\cup\Gamma=\Sigma_5\cup\Sigma_6\cup\Gamma=\Sigma_7\cup\Sigma_8\cup\Gamma,\ \Sigma_1\cap\Sigma_2=\Sigma_3\cap\Sigma_4=\Sigma_5\cap\Sigma_6=\Sigma_7\cap\Sigma_8=\varnothing$.

We consider the following boundary conditions

$$u_i=\tilde u_i \text{ on } \Sigma_1\times[0,\ t_0),\ P_i=\tilde P_i \text{ on } \Sigma_2\times[0,\ t_0),$$
$$\varphi_i=\tilde\varphi_i \text{ on } \Sigma_3\times[0,\ t_0),\ m_i=\tilde m_i \text{ on } \Sigma_4\times[0,\ t_0),$$
$$Du_i=g_i \text{ on } \Sigma_5\times[0,\ t_0),\ R_i=\tilde R_i \text{ on } \Sigma_6\times[0,\ t_0), \tag{2.7}$$
$$\theta=\tilde\theta \text{ on } \Sigma_7\times[0,\ t_0),\quad q=\tilde q \text{ on } \Sigma_8\times[0,\ t_0),$$
$$Q_i=\tilde Q_i \text{ on } \Gamma\times[0,\ t_0).$$

where $\tilde u_i,\ \tilde P_i,\ \tilde\varphi_i,\ \tilde m_i,\ g_i,\ \tilde R_i,\ \tilde\theta,\ \tilde q,\ \tilde Q_i$ are prescribed functions and t_0 is some time instant that may be infinite.

To the system of field equations we adjoin the boundary conditions (2.7) and the initial conditions

$$u_i(x,0)=a_i(x),\ \dot u_i(x,\ 0)=b_i(x),\ \varphi_i(x,0)=c_i(x),\ \dot\varphi_i(x,\ 0)=d_i(x),$$
$$\eta(x,\ 0)=\eta_0(x). \tag{2.8}$$

where $a_i, b_i, c_i, d_i, \eta_0$ are prescribed functions.

3. General theorems. The internal energy per unit volume in the reference configuration is

$$2W=A_{ijrs}e_{ij}e_{rs}+C_{ijrs}\varkappa_{ij}\varkappa_{rs}+F_{ijkpqr}\varkappa_{ijk}\varkappa_{pqr}+$$
$$+2B_{ijrs}e_{ij}\varkappa_{rs}+2D_{ijpqr}e_{ij}\varkappa_{pqr}+2E_{ijpqr}\varkappa_{ij}\varkappa_{pqr}. \tag{3.1}$$

Theorem 3.1. **(Uniqueness theorem).** *Let W be positive semi-definite, k_{ij} be a positive semi-definite tensor, I_{ij} be a positive definite tensor and $\rho>0$, $T_0>0$, $d>0$. Then the mixed problem has at most one solution.*

Proof. With the help of (2.3) we have

$$\tau_{ij}\dot e_{ij}+m_{ij}\dot\varkappa_{ij}+\mu_{ijk}\dot\varkappa_{ijk}+\rho\theta\dot\eta=\dot W+\theta d\theta. \tag{3.2}$$

On the other hand, from (2.1), (2.2), it follows that

$$\int_B(\tau_{ij}\dot e_{ij}+m_{ij}\dot\varkappa_{ij}+\mu_{ijk}\dot\varkappa_{ijk}+\rho\theta\dot\eta)\,dv=\int_B(F_i v_i+M_i\nu_i+(1/T_0)\,\theta S)\,dv$$
$$-\int_B(\rho\dot v_i v_i+I_{ij}\dot\nu_i\nu_j)\,dv-(1/T_0)\int_B q_i\,\theta_{,i}\,dv+\int_{\partial B}(P_i v_i+R_i Dv_i+m_i\nu_i+ \tag{3.3}$$
$$+(1/T_0)\,\theta q)\,da+\int_\Gamma Q_i v_i ds.$$

Let us introduce the function E on $[0,\ t_0)$ defined by

$$E=(1/2)\int_B(\rho v_i v_i+I_{ij}\nu_i\nu_j+b\theta^2)\,dv+\int_B W dv. \tag{3.4}$$

From (3.2)–(3.4) we obtain the identity

$$\dot E=\int_B(F_i v_i+M_i\nu_i+(1/T_0)\,\theta S)\,dv+\int_{\partial B}(P_i v_i+R_i Dv_i+m_i\nu_i+$$
$$+(1/T_0)\,\theta q)\,da+\int_\Gamma Q_i v_i ds-(1/T_0)\int_B q_i\,\theta_{,i}\,dv. \tag{3.5}$$

Let $\mathcal{S}^\circ=\{u_i^\circ,\ \varphi_i^\circ,\ \theta^\circ\}$ be the difference between two solutions. According to the linearity of the problem, $\mathcal{S}^\circ$ is a solution corresponding to null data. By using the initial conditions, the hypotheses and (3.4), (3.5) we obtain $u_i^\circ=0,\ \varphi_i^\circ=0,\ \theta^\circ=0$ on $\bar B\times[0,\ t_0)$.

In what follows we will use an alternative formulation of the problem.

Let u and v be functions defined on $\bar B\times[0,\ \infty)$, continuous on $[0,\ \infty)$ with respect to the time t for each $x\in\bar B$. We denote by $u*v$ the convolution of u and v

$$[u*v]\,(x,t)=\int_0^t u(x,\ t-\tau)\,v(\tau)\,d\tau. \tag{3.6}$$

Let us introduce the notations

$$g(t)=t,\ f_i=g*F_i+\rho(tb_i+a_i),\ l_i=g*M_i+I_{ij}(td_j+c_j),\ l(t)=1,$$
$$w=(1/T_0)l*S+\rho\eta_0. \tag{3.7}$$

Proceeding as in [6, 7] one can prove the following theorem.

Theorem 3.2. *The functions $u_i,\ \varphi_i,\ \eta,\ \tau_{ij},\ m_{ij},\ \mu_{ijk}$, satisfy the equations (2.1), (2.2) and the initial conditions (2.8), if and only if*

$$\mathcal{L}_i\equiv g*(\tau_{ji,\,j}-\mu_{sji,\,sj})+f_i=\rho u_i,$$
$$\mathcal{M}_i\equiv g*(m_{ji,\,j}+\varepsilon_{irs}\tau_{rs})+l_i=I_{ij}\varphi_j, \tag{3.8}$$
$$\rho\eta=(1/T_0)\,l*q_{i,\,i}+w.$$

Thus $\mathcal{S}$ is a solution of the boundary-initial-value problem if and only if it satisfies the equations (2.3), (2.4), the boundary conditions (2.7) and the equations (3.8).

Let us consider the body subjected to two different systems of loadings $L^{(\alpha)}=\{F_i^{(\alpha)},\ M_i^{(\alpha)},\ S^{(\alpha)},\ \tilde u_i^{(\alpha)},\ \ldots,\ d_i^{(\alpha)},\ \tilde\theta^{(\alpha)},\tilde q^{(\alpha)},\ \eta_0^{(\alpha)}\}$ and the two corresponding states $\mathcal{S}^{(\alpha)}$, $\alpha=1,\ 2$.

Theorem 3.3. (Reciprocal theorem). *If a thermoelastic solid with symmetric conductivity tensor is subjected to two systems of loadings $L^{(\alpha)}$ $(\alpha=1, 2)$ — then between the corresponding states $\mathcal{S}^{(\alpha)}$ there is the following relation*

$$\begin{aligned}&\int_B [f_i^{(1)} * u_i^{2} + l_i^{(1)} * \varphi_i^{(2)} - g * w^{(1)} * \theta^{(2)}]\,dv + \int_{\partial B} g * [P_i^{(1)} * u_i^{(2)} + \\ &+ m_i^{(1)} * \varphi_i^{(2)} + R_i^{(1)} * Du_i^{(2)} - (1/T_0)\, l * q^{(1)} * \theta^{(2)}]\,da + \\ &+ \int_\Gamma g * Q_i^{(1)} * u_i^{(2)} ds = \int_B [f_i^{(2)} * u_i^{(1)} + l_i^{(2)} * \varphi_i^{(1)} - g * w^{(2)} * \theta^{(1)}]\,dv + \\ &+ \int_{\partial B} g * [P_i^{(2)} * u_i^{(1)} + m_i^{(2)} * \varphi_i^{(1)} + R_i^{(2)} * Du_i^{(1)} - (1/T_0)\, l * q^{(2)} * \theta^{(1)}]\,da + \\ &+ \int_\Gamma g * Q_i^{(2)} * u_i^{(1)}\, ds.\end{aligned} \tag{3.9}$$

Proof. We will use method given in [5]. On the basis of the relations (2.5) we conclude from the constitutive equations that

$$\begin{aligned}&\tau_{ij}^{(1)} * e_{ij}^{(2)} + m_{ij}^{(1)} * \varkappa_{ij}^{(2)} + \mu_{ijk}^{(1)} * \varkappa_{ijk}^{(2)} - \rho\eta^{(1)} * \theta^{(2)} = \tau_{ij}^{(2)} * e_{ij}^{(1)} + m_{ij}^{(2)} * \varkappa_{ij}^{(1)} + \\ &\mu_{ijk}^{(2)} * \varkappa_{ijk}^{(1)} - \rho\eta^{(2)} * \theta^{(1)}.\end{aligned} \tag{3.10}$$

If we introduce notations

$$L_{\alpha\beta} = \int_B g * [\tau_{ij}^{(\alpha)} * e_{ij}^{(\beta)} + m_{ij}^{(\alpha)} * \varkappa_{ij}^{(\beta)} + \mu_{ijk}^{(\alpha)} * \varkappa_{ijk}^{(\beta)} - \rho\eta^{(\alpha)} * \theta^{(\beta)}]\,dv, \tag{3.11}$$

then from (3.10) we have

$$L_{12} = L_{21}. \tag{3.12}$$

Using the equations (2.4), (3.8), the divergence theorem and Stokes' theorem we get

$$\begin{aligned}L_{\alpha\beta} = &\int_B [f_i^{(\alpha)} * u_i^{(\beta)} + l_i^{(\alpha)} * \varphi_i^{(\beta)} - g * w^{(\alpha)} * \theta^{(\beta)}]\,dv - \int_B \rho u_i^{(\alpha)} * u_i^{(\beta)} dv - \\ &- \int_B I_{ij}\varphi_j^{(\alpha)} * \varphi_i^{(\beta)} dv + (1/T_0)\int_B g * l * q_i^{(\alpha)} * \theta_{,i}^{(\beta)} dv + \int_{\partial B} g * [P_i^{(\alpha)} * u_i^{(\beta)} + \\ &+ m_i^{(\alpha)} * \varphi_i^{(\beta)} + R_i^{(\alpha)} * Du_i^{(\beta)} - (1/T_0)\, l * q^{(\alpha)} * \theta^{(\beta)}]\,da + \int_\Gamma g * Q_i^{(\alpha)} * u_i^{(\beta)} ds.\end{aligned} \tag{3.13}$$

From (3.12) and (3.13) we obtain (3.10). The reciprocal theorem can be used to obtain variational characterizations of the solutions [10]. In what follows, we will present a variational theorem of G u r t i n type [11].

We introduce the notations

$$s_i = (1/T_0)\, q_i, \quad \theta_{,i} = T_0\lambda_{ij} s_j, \quad \Lambda = \rho\eta + a_{pq}e_{pq} + c_{pq}\varkappa_{pq} + g_{pqr}\varkappa_{pqr}.$$

Theorem 3.3. (Variational theorem). *Let $\mathcal{P}$ denote the set of all admissible processes,* [8], *and for each $t \in [0, t_0)$ define the functional $\mathcal{D}_t\{\cdot\}$ on $\mathcal{P}$ by*

$$\begin{aligned}\mathcal{D}_t\{\Pi\} = &\int_B \{(1/2)\, g * [A_{ijkl}e_{ij} * e_{kl} + 2B_{ijkl}e_{ij} * \varkappa_{kl} + 2D_{ijpqr}e_{ij} * \varkappa_{pqr} + \\ &+ C_{ijkl}\varkappa_{ij} * \varkappa_{kl} + 2E_{ijpqr}\varkappa_{ij} * \varkappa_{pqr} + F_{ijkpqr}\varkappa_{ijk} * \varkappa_{pqr} + (1/d)\Lambda * \Lambda +\end{aligned}$$

$$\begin{aligned}&+ \rho u_i * u_i + I_{ij}\varphi_i * \varphi_j + T_0\lambda_{ij} l * s_i * s_j - g * (\tau_{ij} * e_{ij} + \varkappa_{ij} * m_{ij} + \\ &+ \mu_{ijk} * \varkappa_{ijk} + \rho\eta * \theta + l * s_i * \theta_{,i}) - \mathcal{L}_i * u_i - \mathcal{M}_i * \varphi_i + g * \theta * w\}\,dv + \\ &+ \int_{\Sigma_1} g * P_i * \bar{u}_i da + \int_{\Sigma_2} g * (P_i - \bar{P}_i) * u_i da + \int_{\Sigma_3} g * m_i * \bar{\varphi}_i da + \\ &+ \int_{\Sigma_4} g * (m_i - \bar{m}_i) * \varphi_i da + \int_{\Sigma_5} g * R_i * g_i da + \int_{\Sigma_6} g * (R_i - \tilde{R}_i) * Du_i da + \\ &+ (1/T_0)\int_{\Sigma_7} g * l * (\theta - \tilde{\theta}) * q da + (1/T_0)\int_{\Sigma_8} g * l * \theta * \bar{q} da + \\ &+ \int_\Gamma g * (Q_i - \tilde{Q}_i) * u_i ds,\end{aligned} \tag{3.14}$$

for every $\Pi \in \mathcal{P}$. Then $\delta\mathcal{D}_t\{\Pi\} = 0, (0 \leq t < t_0)$ at an admissible process Π if and only if Π is a solution of the mixed problem.

Proof. We introduce the formal notation

$$\delta_{\Pi'}\mathcal{D}_t\{\Pi\} = \frac{d}{d\lambda}\mathcal{D}_t\{\Pi + \lambda\Pi'\}|_{\lambda=0}$$

A straightforward calculation yields

$$\begin{aligned}\delta_{\Pi'}\mathcal{D}_t\{\Pi\} = &\int_B \{g * [A_{ijkl}e_{kl} + B_{ijkl}\varkappa_{kl} + D_{ijpqr}\varkappa_{pqr} + (1/d)\, a_{ij}\Lambda - \\ &- \tau_{ij}] * e_{ij} + g * [B_{rsij}e_{rs} + C_{ijrs}\varkappa_{rs} + E_{ijpqr}\varkappa_{pqr} + (1/d)\, c_{ij}\Lambda - \\ &- m_{ij}] * \varkappa'_{ij} + g * [D_{rsijk}e_{rs} + E_{rsijk}\varkappa_{rs} + F_{ijkpqr}\varkappa_{pqr} + (1/d)\, g_{ijk}\Lambda - \\ &- \mu_{ijk}] * \varkappa'_{ijk} + g * [(1/d)\Lambda - \theta] * \rho\eta' - (\mathcal{L}_i - \rho u_i) * u'_i - (\mathcal{M}_i - \\ &- I_{ij}\varphi_j) * \varphi'_i + g * l * (T_0\lambda_{ij}s_j - \theta_{,i}) * s'_i - g * (\rho\eta - l * s_{i,i} - \\ &- w) * \theta' - g * (e_{ij} - u_{j,i} - \varepsilon_{jik}\varphi_k) * \tau'_{ij} - g * (\varkappa_{ij} - \varphi_{j,i}) * m'_{ij} - \\ &- g * (\varkappa_{ijk} - u_{k,ij}) * \mu'_{ijk}\}\,dv - \int_{\Sigma_1} g * (u_i - \tilde{u}_i) * P'_i da + \\ &\int_{\Sigma_2} g * (P_i - \tilde{P}_i) * u'_i da - \int_{\Sigma_3} g * (\varphi_i - \bar{\varphi}_i) * m'_i da + \int_{\Sigma_4} g * (m_i - \bar{m}_i) * \varphi'_i da - \\ &- \int_{\Sigma_5} g * (Du_i - g_i) * R'_i da + \int_{\Sigma_6} g * (R_i - \tilde{R}_i) * Du'_i da + \\ &+ (1/T_0)\int_{\Sigma_7} g * l * (\theta - \tilde{\theta}) * q' da - (1/T_0)\int_{\Sigma_8} g * l * (q - \bar{q}) * \theta' da + \\ &+ \int_\Gamma g * (Q_i - \tilde{Q}_i) * u'_i ds, \text{ for } 0 \leq t < t_0.\end{aligned} \tag{3.15}$$

If Π is a solution of the mixed problem, it follows from (2.3), (2.4), (2.7), (3.8) that $\delta_{\Pi'}\mathcal{D}_t\{\Pi\} = 0$, $(0 \leq t < t_0)$ and therefore $\delta\mathcal{D}_t\{\Pi\} = 0$, $0 \leq t < t$. The proof can be continued by using the procedure of [8, 11].

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EXISTENCE THEOREMS IN ELASTOSTATICS OF MATERIALS WITH VOIDS

BY

GH. RUSU

1. Introduction. In [1], J. W. Nunziato and S. C. Cowin gave a nonlinear theory of elastic materials with voids. The intended applications of this theory are to geological materials like rocks and soils and to manufactured porous materials. The linear theory of the elastic materials with voids was established by S. C. Cowin and J. W. Nunziato in [2]. Some basic theorems for statical and dynamical cases were obtained by D. Ieşan ([3]). In this paper we consider some fundamental three-dimensional boundary value problems of equilibrium for isotropic and homogeneous elastic materials with voids. In Section 3 the fundamental matrix is given and its properties are established. A. Somigliana's type representation of the solution is given in Section 4. In Sections 5 and 6, using the method of potentials ([4], [5]) we reduce the boundary value problems to two-dimensional singular integral equations for which Fredholm's basic theorems hold ([6]). In Section 7, existence theorems for interior and exterior problems are established.

2. Basic Equations. Throughout this paper a rectangular coordinate system $Ox_k(k=1, 2, 3)$ is employed. We shall use summation and differentiation conventions: Latin subscripts — unless otherwise specified — are understood to range over the integers (1, 2, 3), summation over repeated subscripts is implied and a subscript preceded by a comma denotes partial differentiation with respect to the corresponding cartesian coordinate.

Let us assume that the body occupies a finite region B bounded by a closed Liapunov surface Σ.

The basic equations of the linear theory of homogeneous and isotropic materials with voids are ([2])

— the equations of equilibrium

$$t_{ij,j}+f_i=0, \quad h_{i,i}+g+l=0, \tag{2.1}$$

— the constitutive equations

$$\begin{aligned} t_{ij}&=\lambda e_{rr}\delta_{ij}+2\mu e_{ij}+\beta\varphi\delta_{ij},\\ h_i&=\alpha\varphi_{,i}\\ g&=-\xi\varphi-\beta e_{rr}, \end{aligned} \tag{2.2}$$