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DYNAMIC PLANE PROBLEM OF INFINITE MICROPOLAR ELASTIC SOLID

BY

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1. Introduction. In a recent paper [1], the author obtained the fundamental solution of dynamic micropolar three dimensional equations for an infinite body under the action of impulsive concentrated body force.

The present paper is concerned with the dynamic plane strain problem of an infinite micropolar elastic solid. By using Fourier-Laplace transforms, we obtain the fundamental solutions of dynamic two dimensional equations for the infinite micropolar solid under the action of an impulsive, suddenly applied, and periodic body force acting at a point.

In the thermoelastic case the dynamic plane strain problem of infinite micropolar bodies were studied by Shanker and Dhaliwal [4].

2. Basic equations. For a homogeneous isotropic centrosymmetric micropolar solid, the linearized equations of motion are [2]:

$$\begin{aligned}(2.1)\qquad &(\mu+\alpha)\nabla^2 u_i+(\lambda+\mu-\alpha)(u_{j,j})_{,i}+2\alpha(\varepsilon_{ijk}\varphi_{j,k})+F_i=\rho\ddot{u}_i;\\ &[(\gamma+\varepsilon)\nabla^2-4\alpha]\varphi_i+(\beta+\gamma-\varepsilon)(\varphi_{j,j})_{,i}+2\alpha(\varepsilon_{ijk}u_{j,k})+M_i=J\ddot{\varphi}_i,\end{aligned}$$

where we have used the following notations: u_i — displacement field components, φ_i — rotation field components, F_i — body force components, M_i — body couple components, ρ — density, J — rotational inertia, ε_{ijk} — Ricci's permutation symbol, and the dots denote the time derivatives.

The symbols α, β, γ, ε, μ, λ denote the elastic constants of the micropolar material.

3. Dynamic plane problem. In the case of the plane state of strain, the body force F_i, the body couple M_i and the corresponding effects depend on two variables x_1, x_2 and t only.

We shall consider an infinite micropolar solid $-\infty < x_1, x_2, x_3 < +\infty$, $t \geqslant 0$. The system of equations (2.1) which govern the state of the body can be decomposed in two independent systems.

In the first system there appear the vectors $u=(u_1, u_2, 0)$, $\varphi=(0, 0, \varphi_3)$, while in the second one the vectors $u=(0,0,u_3)$, $\varphi=(\varphi_1, \varphi_2, 0)$.

The systems take the form.

$$(3.1)\qquad \begin{cases}(\mu+\alpha)\nabla^2 u_1+(\lambda+\mu-\alpha)(u_{1,11}+u_{2,21})+2\alpha\varphi_{3,2}+F_1=\rho\ddot{u}_1;\\ (\mu+\alpha)\nabla^2 u_2+(\lambda+\mu-\alpha)(u_{1,12}+u_{2,22})-2\alpha\varphi_{3,1}+F_2=\rho\ddot{u}_2;\\ [(\gamma+\varepsilon)\nabla^2-4\alpha]\varphi_3+2\alpha(u_{2,1}-u_{1,2})+M_3=J\ddot{\varphi}_3,\end{cases}$$

and

$$(3.2)\quad \begin{cases} (\mu+\alpha)\nabla^2 u_3+2\alpha(\varphi_{2,1}-\varphi_{1,2})+F_3=\rho\ddot{u}_3, \\ [(\gamma+\varepsilon)\nabla^2-4\alpha]\varphi_1+(\beta+\gamma-\varepsilon)(\varphi_{1,11}+\varphi_{2,21})+2\alpha u_{3,2}=J\ddot{\varphi}_1, \\ [(\gamma+\varepsilon)\nabla^2-4\alpha]\varphi_2+(\beta+\gamma-\varepsilon)(\varphi_{1,12}+\varphi_{2,22})+2\alpha u_{3,1}=J\ddot{\varphi}_2, \end{cases}$$

where $\nabla^2=\partial^2/\partial x_1^2+\partial^2/\partial x_2^2$.

Let us suppose the initial conditions have the form

$$(3.3)\quad u_i(x,0)=\dot{u}_i(x,0)=0\ \ (i=1,2),\ \ \varphi_3(x,0)=\dot{\varphi}_3(x,0)=0.$$

Here we turn our attention on the first system of equations (3.1) under the initial conditions (3.3). In order to integrate the system (3.1) it is convenient to apply the Fourier-Laplace transforms method. Let us introduce, the Laplace transform $f^{(1)}(x_1,x_2,p)$ of a function $f(x_1,x_2,t)$ as

$$(3.4)\quad f^{(1)}(x_1,x_2,p)=\int_0^\infty f(x_1,x_2,t)\,e^{-pt}\,dt$$

and the Fourier-Laplace transform $f^{(2)}(\xi_1,\xi_2,p)$ of $f(x_1,x_2,t)$ as

$$(3.5)\quad f^{(2)}(\xi_1,\xi_2,p)=\frac{1}{2\pi}\iint_{-\infty}^{+\infty} f^{(1)}(x_1,x_2,p)\,e^{i\xi_k x_k}\,dx,$$

where $dx=dx_1dx_2$; $\xi_k x_k=\xi_1x_1+\xi_2x_2$.

Taking the Fourier-Laplace transform of equations (3.1) we obtain the following algebraic system of equations in terms of $u_i^{(2)}$ $(i=1,2)$ and $\varphi_3^{(2)}$:

$$(3.6)\quad \begin{cases} (\gamma^{*2}\xi_1^2+\xi^2+\beta_2^2)u_1^{(2)}+\gamma^{*2}\xi_1\xi_2u_2^{(2)}+is\xi_2\varphi_3^{(2)}=mF_1^{(2)}; \\ \gamma^{*2}\xi_1\xi_2u_1^{(2)}+(\gamma^{*2}\xi_2^2+\xi^2+\beta_2^2)u_2^{(2)}-is\xi_1\varphi_3^{(2)}=mF_2^{(2)}; \\ -iq\xi_2u_1^{(2)}+iq\xi_1u_2^{(2)}+(\xi^2+\overline{\gamma}^2+\beta_4^2)\varphi_3^{(2)}=nM_3^{(2)}, \end{cases}$$

where

$$\gamma^{*2}=\frac{\lambda+\mu-\alpha}{\mu+\alpha};\quad s=\frac{2\alpha}{\mu+\alpha};\quad m=\frac{1}{\mu+\alpha};\quad q=\frac{2\alpha}{\gamma+\varepsilon};\quad n=\frac{1}{\gamma+\varepsilon};$$

$$\beta_i=\frac{p}{c_i}\ (i=2,4);\quad c_2^2=\frac{\mu+\alpha}{\rho};\quad c_4^2=\frac{\gamma+\varepsilon}{J}.$$

4. The effect of a concentrated body force. Let us consider the case of point body force of magnitude F applied at the origin in the Ox_2-direction. In this case we have

$$(4.1)\quad F_1=0,\ F_2=F\delta(x_1)\,\delta(x_2)\,f(t),\ M_3=0,$$

where $\delta(x)$ is the Dirac delta function.

The Fourier-Laplace transform of (4.1) is given by

$$(4.2)\quad F^{(2)}_{1(\xi,p)}=0,\quad F_2^{(2)}(\xi,p)=(F/2\pi)\,f^{(1)}_{(p)},\quad M^{(2)}_{3(\xi,p)}=0.$$

Substituting (4.2) into (3.6), we obtain the following algebraic equations:

$$(4.3)\quad \begin{aligned} &(\gamma^{*2}\xi_1^2+\xi^2+\beta_2^2)u_1^{(2)}+\gamma^{*2}\xi_1\xi_2u_2^{(2)}+is\xi_2\varphi_3^{(2)}=0; \\ &\gamma^{*2}\xi_1\xi_2u_1^{(2)}+(\gamma^{*2}\xi_2^2+\xi^2+\beta_2^2)u_2^{(2)}-is\xi_1\varphi_3^{(2)}=\frac{mF}{2\pi}f^{(1)}_{(p)}; \\ &-iq\xi_2u_1^{(2)}+iq\xi_1u_{(2)}^{(2)}+(\xi^2+\overline{\gamma}^2+\beta_4^2)\varphi_3^{(2)}=0. \end{aligned}$$

The solution of (4.3). is

$$(4.4)\quad \begin{aligned} &u_1^{(2)}(\xi,p)=-\frac{F}{2\pi}\cdot\frac{sq+\gamma^{*2}(\xi^2+\overline{\gamma}^2+\beta_4^2)}{\Delta}\cdot\xi_1\xi_2f^{(1)}_{(p)}; \\ &u_2^{(2)}(\xi,p)=\frac{F}{2\pi}\cdot\left[\frac{1}{\nu^{*2}\xi^2+\beta_2^2}+\frac{\gamma^{*2}(\xi^2+\overline{\gamma}^2+\beta_4^2)}{\Delta}\cdot\xi_1^2\right]\cdot f^{(1)}_{(p)}; \\ &\varphi_3^{(2)}(\xi,p)=-\frac{qF}{2\pi}\cdot\frac{1}{(\xi^2+\beta_2^2)(\xi^2+\overline{\gamma}^2+\beta_4^2)-sq\xi^2}\cdot i\xi_1f^{(1)}_{(p)}, \end{aligned}$$

where

$$\Delta=(\nu^{*2}\xi^2+\beta_2^2)\,[(\xi^2+\beta_2^2)(\xi^2+\overline{\gamma}^2+\beta_4^2)-sq\xi^2],\quad \nu^{*2}=1+\gamma^{*2}=\frac{c_1^2}{c_2^2},\quad c_1^2=\frac{\lambda+2\mu}{\rho}.$$

5. **The effect of impulsive body forces.** Let us assume now that the body force is applied impulsively at the origin, and acts in the Ox_2-direction. Such a body force may be written as (4.1) where $f(t)=\delta(t)$.

The Laplace transform of $f(t)=\delta(t)$ is $f^{(1)}(p)=1$. Substituting $f^{(1)}(p)$ into (4.4) we get

$$(5.1)\quad \begin{aligned} &u_1^{(2)}(\xi,p)=-\frac{F}{2\pi}\cdot\frac{sq+\gamma^{*2}(\xi^2+\overline{\gamma}^2+\beta_4^2)}{\Delta}\cdot\xi_1\xi_2; \\ &u_2^{(2)}(\xi,p)=\frac{F}{2\pi}\cdot\left[\frac{1}{\nu^{*2}\xi^2+\beta_2^2}+\frac{sq+\gamma^2(\xi^2+\overline{\gamma}^2+\beta_4^2)}{\Delta}\,\xi_1^2\right]; \\ &\varphi_3^{(2)}(\varepsilon p)=-\frac{qF}{2\pi}\cdot\frac{1}{(\xi^2+\beta_2^2)(\xi^2+\overline{\gamma}^2+\beta_4^2)-sq\xi^2}\cdot i\xi_1. \end{aligned}$$

By inverting the Fourier transform with respect to ξ_1,ξ_2 we obtain:

$$(5.2)\quad \begin{aligned} &u_1^{(1)}(x,p)=\frac{F}{(2\pi)^2}\cdot\frac{\partial^2 I_1}{\partial x_1\partial x_2}; \\ &u_2^{(1)}(x,p)=\frac{F}{(2\pi)^2}\cdot\left[I_2-\frac{\partial^2 I_1}{\partial x_1^2}\right];\quad \varphi_3^{(1)}(x,p)=\frac{qF}{(2\pi)^2}\cdot\frac{\partial I_3}{\partial x_1} \end{aligned}$$

where

$$
(5.3)\qquad \begin{aligned}
I_1 &= \int_{R^1} \frac{sq+\gamma^{*2}(\xi^2+\bar{r}^2+\beta_4^2)}{\Delta}\cdot e^{-ix_k\xi_k}\,d\xi\,; \qquad I_2 = \int_{R^1} \frac{1}{\nu^{*2}\xi^2+\beta_2^2}\cdot e^{-ix_k\xi_k}\,d\xi\,;\\
I_3 &= \int_{R^1} \frac{1}{(\xi^2+\beta_2^2)(\xi^2+\bar{r}^2+\beta_4^2)-sq\xi^2}\cdot e^{-ix_k\xi_k}\,d\xi.
\end{aligned}
$$

If we estimate the integrals I_1, I_2, I_3 by adopting the same procedure like in [4], and replace them in (5.2), we obtain

$$
(5.4)\qquad \left\{\begin{aligned}
u_1^{(1)}(x,p) &= \frac{F}{2\pi}\cdot\frac{c_2^2}{c_1^2}\cdot\frac{x_1x_2}{r^2}\left[\frac{sq+\gamma^{*2}(\beta_4^2+\bar{r}^2-\beta_1^2)}{(\beta_1^2+\lambda_1^2)(\beta_1^2+\lambda_2^2)}\cdot\beta_1^2K_2(\beta_1 r) - \right.\\
&\left.\frac{sq+\gamma^{*2}(\beta_4^2+\bar{r}^2+\lambda_1^2)}{(\beta_1^2+\lambda_1^2)(\lambda_1^2-\lambda_2^2)}\cdot\lambda_1^2K_2(i\lambda_1 r) + \frac{sq+\gamma^{*2}(\beta_4^2+\bar{r}^2+\lambda_2^2)}{(\beta_1^2+\lambda_1^2)(\lambda_1^2-\lambda_2^2)}\cdot\lambda_2^2K_2(i\lambda_2 r)\right];\\
u_2^{(1)}(x,p) &= \frac{F}{2\pi}\cdot\frac{c_2^2}{c_1^2}\cdot K_0(\beta_1 r) - \frac{x_1}{x_2}u_1^{(1)}(x_1,p)\,;\\
\varphi_3^{(1)}(x,p) &= -\frac{iqF}{2\pi}\cdot\frac{1}{\lambda_1^2-\lambda_2^2}\left[\lambda_1K_1(i\lambda_1 r)-\lambda_2K_1(i\lambda_2 r)\right]
\end{aligned}\right.
$$

where K_0, K_1, K_2 are the modified Bessel functions of the third kind, $r=\sqrt{x_1^2+x_2^2}$ and λ_1^2, λ_2^2 are the roots of the equation:

$$
(\xi^2+\beta_2^2)(\xi^2+\bar{r}^2+\beta_4^2)-sq\xi^2=0.
$$

It is a difficult task to obtain the inverse Laplace transforms of the above functions and this is the reason for which we shall confine only to the case of small time approximations. If we calculate the roots λ_1, λ_2 by approximation with respect to p, we obtain

$$
\lambda_1 \cong -i\frac{p}{c_4}\,, \qquad \lambda_2 \cong -i\frac{p}{c_2}\,.
$$

Expanding the right hand terms of (5.4) into inverse powers of p, and retaining only the terms up to the order of $p^3p^{1/2}$, we obtain

$$
\begin{aligned}
u_1^{(1)}(x_1 p) &= \frac{F}{2\pi}\sqrt{\frac{\pi}{2}}\cdot\frac{c_2^2}{c_1^2}\cdot\frac{x_1x_2}{r^2\sqrt{r}}\left\{e^{-\frac{p}{c_1}r}\left[-\frac{\gamma^{*2}}{a^2-b^2}\frac{1}{c_1\sqrt{c_1}}\frac{1}{p^{1/2}} - \frac{15\gamma^{*2}}{8(a^2-b^2)\sqrt{c_1}r}\cdot\frac{1}{pp^{1/2}} + \right.\right.\\
&+\left(\frac{sq+\gamma^{*2}r^{-2}}{a^2(a^2-b^2)c_1\sqrt{c_1}} - \frac{105\gamma^{*2}\sqrt{c_1}}{128r^2(a^2-b^2)}\right)\cdot\frac{1}{p^2p^{1/2}} + \left.\frac{15(sq+\gamma^{*2}r^{-2})}{8a^2(a^2-b^2)\sqrt{c_1}r}\cdot\frac{1}{p^3p^{1/2}}\right]+\\
&+e^{-\frac{p}{c_4}r}\frac{sq+\gamma^{*2}r^{-2}}{a^2b^2c_4\sqrt{c_4}}\left[\frac{1}{p^2p^{1/2}}+\frac{15c_4}{8r}\cdot\frac{1}{p^3p^{1/2}}\right]+e^{-\frac{p}{c_2}r}\left[\frac{\gamma^{*2}}{(a^2-b^2)c_2\sqrt{c_2}}\cdot\frac{1}{p^{1/2}}+\right.\\
&+\frac{15\gamma^{2}}{8(a^2-b^2)\sqrt{c_2}r}\cdot\frac{1}{pp^{1/2}}+\left(\frac{105\gamma^{*2}}{128(a^2-b^2)\sqrt{c_2}r^2}-\frac{sq+\gamma^{*2}r^{-2}}{b^2(a^2-b^2)c_2\sqrt{c_2}}\right)\frac{1}{p^2p^{1/2}} -
\end{aligned}
$$

$$
(5.5)\qquad \begin{aligned}
&\left.\left.-\frac{15(sq+\gamma^{*2}r^{-2})}{8b^2(a^2-b^2)\sqrt{c_2}r}\cdot\frac{1}{p^3p^{1/2}}\right]\right\} + O\left(\frac{1}{p^4p^{1/2}}\right);\\
u_2^{(1)}(x,p) &= \frac{F}{2\pi}\sqrt{\frac{\pi}{2}}\cdot\frac{c_2^2}{c_1\sqrt{c_1}r}e^{-\frac{p}{c_1}r}\cdot\left[\frac{1}{p^{1/2}}-\frac{8c_1}{r}\cdot\frac{1}{pp^{1/2}}+\frac{9c_1^2}{128r^2}\cdot\frac{1}{p^2p^{1/2}}\right]-\\
&-\frac{x_1}{x_2}u_1^{(1)}(x,p)+O\left(\frac{1}{p^4p^{1/2}}\right);\\
\varphi_3^{(1)}(x_1 p) &= \frac{qF}{2\pi}\sqrt{\frac{\pi}{2}}\frac{1}{b^2}\left[\frac{1}{\sqrt{c_2}}e^{-\frac{p}{c_2}r}\left(\frac{1}{pp^{1/2}}+\frac{3c_2}{r}\cdot\frac{1}{p^2p^{1/2}}+\frac{3c_2^2}{128r^2}\frac{1}{p^3p^{1/2}}\right)-\right.\\
&\left.-\frac{1}{\sqrt{c_4}}e^{-\frac{p}{c_4}r}\left(\frac{1}{pp^{1/2}}+\frac{3c_4}{r}\cdot\frac{1}{p^3p^{1/2}}+\frac{3c_4^2}{128r^2}\cdot\frac{1}{p^3p^{1/2}}\right)\right]+O\left(\frac{1}{p^4p^{1/2}}\right),
\end{aligned}
$$

where $a^2=1/c_1^2-1/c_4^2$, $b^2=1/c_2^2-1/c_4^2$.

The expression of the right hand side of (5.5) are now convenient for the calculation of Laplace inversion; using special tables of integral transforms [6], we obtain the displacements and rotation field:

$$
(5.6)\qquad \begin{aligned}
u_1(x,t) &= \frac{F}{2\pi}\sqrt{\frac{\pi}{2}}\cdot\frac{c_2^2}{c_1^2}\cdot\frac{x_1x_2}{r^2\sqrt{r}}\left\{-\frac{\gamma^{*2}}{a^2-b^2}\frac{1}{c_1\sqrt{c_1}}L_1(t)-\frac{15\gamma^{*2}}{8(a^2-b^2)}\cdot\frac{1}{\sqrt{c_2}r}M_1(t)+\right.\\
&+\left[\frac{sq+\gamma^{*2}r^{-2}}{a^2(a^2-b^2)}\frac{1}{c_1\sqrt{c_1}}-\frac{105\gamma^{*2}}{128(a^2-b^2)}\cdot\frac{\sqrt{c_1}}{r^2}\right]\cdot N_1(t)+\frac{15(sq+\gamma^{*2}\gamma^{-2})}{8a^2(a^2-b^2)\sqrt{c_1}r}\cdot P_1(t)+\\
&+\frac{sq+\gamma^{*2}r^{-2}}{a^2b^2c_4\sqrt{c_4}}\left[N_4(t)+\frac{15c_4}{8r}P_4(t)\right]+\frac{1}{\sqrt{c_2}(a^2-b^2)}\left[\frac{\gamma^{*2}}{c_2}L_2(t)+\frac{15\gamma^{*2}}{8r}M_2(t)+\right.\\
&\left.\left.+\left(\frac{105\gamma^{*2}}{128r^2}-\frac{sq+\gamma^{*2}r^{-2}}{b^2c_2}\right)N_2(t)-\frac{15(sq+\gamma^{*2}r^{-2})}{8b^2r}P_2(t)\right]\right\};\\
u_2(x,t) &= \frac{F}{2\pi}\sqrt{\frac{\pi}{2}}\cdot\frac{c_2^2}{c_1\sqrt{c_1}r}\left[L_1(t)-\frac{8c_1}{r}M_1(t)+\frac{9c_1^2}{128r^2}N_1(t)\right]-\frac{x_1}{x_2}u_1(x,t)\,;\\
\varphi_3(x,t) &= \frac{qF}{2\pi}\sqrt{\frac{\pi}{2}}\frac{1}{b^2\sqrt{r}}\left\{\frac{1}{\sqrt{c_2}}\left[M_2(t)+\frac{3c_2}{r}N_2(t)+\frac{3c_2^2}{128r^2}P_2(t)\right]-\right.\\
&\left.-\frac{1}{\sqrt{c_4}}\left[M_4(t)+\frac{3c_4}{r}N_4(t)+\frac{3c_4^2}{128r^2}P_4(t)\right]\right\}
\end{aligned}
$$

where by $L_i(t)$, $M_i(t)$, $N_i(t)$, $P_i(t)$ we have denoted

$$
L_i(t)=\begin{cases}0, & 0<t<\dfrac{r}{c_i}\\[2mm] \left(t-\dfrac{r}{c_i}\right)^{-1/2} & t>\dfrac{r}{c_i}\end{cases}\quad (i=1,2)\,; \qquad M_i(t)=\begin{cases}0, & 0<t<\dfrac{r}{c_i}\\[2mm] \left(t-\dfrac{r}{c_i}\right)^{1/2}, & t>\dfrac{r}{c_i}\end{cases}\quad (i=1,2,4)
$$

$$N_i(t)=\begin{cases}0, & 0<t<\dfrac{r}{c_i}\\ \left(t-\dfrac{r}{c_i}\right)^{3/2}, & t>\dfrac{r}{c_i}\end{cases}\quad (i=1,2,4);\quad P_i(t)=\begin{cases}0, & 0<t<\dfrac{r}{c_i}\\ \left(t-\dfrac{r}{c_i}\right)^{5/2}, & t>\dfrac{r}{c_i}\end{cases}\quad (i=1,2,4).$$

6. The effect of a suddenly applied body force. Let us consider the case when the body force is suddenly applied at the origin, and acts in the Ox_2 - direction. Such a type of body force has the form (4.1) where $f(t)=H(t)$ ($H(t)$ = Heaviside unit function). The Laplace transform of $f(t)=H(t)$ is $f^{(1)}(p)=1/p$. Substituting $f^{(1)}(p)$ into (4.4) we get

$$(6.1)\quad\begin{cases} u_1^{(2)}(\xi,p)=-\dfrac{F}{2\pi}\dfrac{1}{p}\cdot\dfrac{sq+\gamma^{*2}(\xi^2+\bar r^2+\beta_4^2)}{\Delta}\,\xi_1\xi_2;\\ u_2^{(2)}(\xi,p)=\dfrac{F}{2\pi}\dfrac{1}{p}\cdot\left[\dfrac{1}{\gamma^{*2}\xi^2+\beta_2^2}+\dfrac{\gamma^{*2}(\xi^2+\bar r^2+\beta_4^2)+sq}{\Delta}\cdot\xi_1^2\right];\\ \varphi_3^{(2)}(\xi,p)=-\dfrac{qF}{2\pi}\dfrac{1}{p}\cdot\dfrac{1}{(\xi^2+\beta_2^2)(\xi^2+\bar r^2+\beta_4^2)-sq\xi^2}\cdot i\xi_1.\end{cases}$$

Performing the inverse Fourier transforms with respect to ξ_1, ξ_2 we obtain

$$(6.2)\quad u_1^{(1)}(x_1p)=\frac{F}{(2\pi)^2}\cdot\frac{1}{p}\frac{\partial^2 I_1}{\partial x_1\partial x_2};\qquad u_2^{(1)}(x_1p)=-\frac{F}{(2\pi)^2}\cdot\frac{1}{p}\left[I_2-\frac{\partial^2 I_1}{\partial x_1^2}\right];$$
$$\varphi_3^{(1)}(x_1p)=\frac{qF}{(2\pi)^2}\cdot\frac{1}{p}\frac{\partial I_3}{\partial x_1},$$

where I_1, I_2 and I_3 are given by the relations (5.3).

Using the results obtained in the section 5 and in [4], we find

$$(6.3)\quad\begin{cases} u_1^{(1)}(x_1p)=\dfrac{F}{2\pi}\cdot\dfrac{c_2^2}{c_1^2}\dfrac{x_1x_2}{r^2}\cdot\dfrac{1}{p}\left[\dfrac{sq+\gamma^{*2}(\beta_4^2+\bar r^2-\beta_1^2)}{(\beta_1^2+\lambda_1^2)(\beta_1^2+\lambda_2^2)}\cdot\beta_1^2K_2(\beta_1r)-\right.\\ \qquad-\dfrac{sq+\gamma^{*2}(\beta_4^2+\bar r^2+\lambda_1^2)}{(\beta_1^2+\lambda_1^2)(\lambda_1^2-\lambda_2^2)}\cdot\lambda_1^2;\,K_2(i\lambda_1r)+\\ \qquad\left.+\dfrac{sq+\gamma^{*2}(\beta_4^2+\bar r^2+\lambda_2^2)}{(\beta_1^2+\lambda_1^2)(\lambda_1^2-\lambda_2^2)}\cdot\lambda_2^2K_2(i\lambda_2r)\right];\\ u_2^{(1)}(x,p)=\dfrac{F}{2\pi}\cdot\dfrac{c_2^2}{c_1^2}\dfrac{1}{p}K_0(\beta,r)-\dfrac{x_1}{x_2}u_1^{(1)}(x,p);\\ \varphi_3^{(1)}(x_1p)=-\dfrac{iqF}{2\pi}\cdot\dfrac{1}{\lambda_1^2-\lambda_2^2}\cdot\dfrac{1}{p}[\lambda_1K_1(i\lambda_1r)-\lambda_2K_1(i\lambda_2r)]\end{cases}$$

where K_0, K_1, K_2, λ_1^2, λ_2^2 have the same meaning as in section 5.

We confine to the case of small time approximations; adopting the same method as in section 5; we obtain the following displacement and

rotation distributions:

$$(6.4)\quad\begin{cases} u_1(x_1t)=\dfrac{F}{2\pi}\sqrt{\dfrac{\pi}{2}}\dfrac{c_2^2}{c_1^2}\cdot\dfrac{x_1x_2}{r\sqrt{r}}\left\{-\dfrac{\gamma^{*2}}{a^2-b^2}\dfrac{1}{c_1\sqrt{c_1}}\cdot M_1(t)-\dfrac{15\gamma^{*2}}{8(a^2-b^2)\sqrt{c_1}r}\cdot N_1(t)+\right.\\ \qquad+\left[\dfrac{sq+\gamma^{*2}r^{-2}}{a^2(a^2-b^2)c_1\sqrt{c_1}}-\dfrac{105\gamma^{*}\sqrt{c_1}}{128(a^2-b^2)r^2}\right]\cdot P_1(t)+\dfrac{1}{(a^2-b^2)\sqrt{c_2}}\cdot\\ \qquad\cdot\left[\dfrac{\gamma^{*2}}{c_2}M_2(t)+\dfrac{15\gamma^{*2}}{8r}N_2(t)+\left(\dfrac{105\gamma^{*2}}{128r^2}-\dfrac{sq+\gamma^{*2}\bar r^2}{b^2c_2}\right)\cdot P_2(t)\right]+\\ \qquad\left.+\dfrac{sq+\gamma^{*2}r^{-2}}{a^2b^2}\cdot\dfrac{1}{c_4\sqrt{c_4}}\cdot P_4(t)\right\};\\ u_2(x,t)=\dfrac{F}{2\pi}\sqrt{\dfrac{\pi}{2}}\dfrac{c_2^2}{c_1\sqrt{c_1}r}\left[M_1(r)-\dfrac{8c_1}{r}N_1(t)+\dfrac{9c_1^2}{128r^2}P_1(t)\right]-\dfrac{x_1}{x_2}u_1(x,t);\\ \varphi_3(x_1t)=\dfrac{qF}{2\pi}\sqrt{\dfrac{\pi}{2}}\dfrac{1}{b^2\sqrt{r}}\left\{\dfrac{1}{\sqrt{c_2}}\left[N_2(t)+\dfrac{3c_2}{r}P_2(t)\right]-\dfrac{1}{\sqrt{c_4}}\left[N_4(t)+\dfrac{3c_4}{r}P_4(t)\right]\right\}.\end{cases}$$

where $M_i(t)$ $(i=1,2)$, $N_i(t)$, $P_i(t)$ $(i=1,2,4)$ have the expressions from the section 5.

7. The effect of periodic body force. In this case the body force acts at the origin along the positive Ox_2 - direction and varies harmonically with the time. Such a body force has the form (4.1) where $f(t)=e^{i\omega t}$ (ω = frequency). Since the perturbation force is of harmonic type, the body reply is also harmonically i.e.

$$(7.1)\quad u_i(x,t)=\dot u_i(x)e^{i\omega t}\quad (i=1,2),\qquad \varphi_3(x,t)=\dot\varphi_3(x)e^{i\omega t}.$$

Substituting (7.1) into (3.1) we get

$$(7.2)\quad\begin{cases}(\mu+\alpha)\nabla^2\dot u_1+(\lambda+\mu-\alpha)(\dot u_{1,11}+\dot u_{2,21})+2\alpha\dot\varphi_{3,2}=-\rho\omega^2\dot u_1;\\ (\mu+\alpha)\nabla^2\dot u_2+(\lambda+\mu-\alpha)(\dot u_{1,11}+\dot u_{2,21})-2\alpha\dot\varphi_{3,1}+\dot F_2=-\rho\omega^2\dot u_2;\\ [(\gamma+\varepsilon)\nabla^2-4\alpha]\dot\varphi_3+2\alpha(\dot u_{2,1}-\dot u_{1,2})=-J\omega^2\dot\varphi_3,\end{cases}$$

where $\dot F_2=F\delta(x_1)\,\delta(x_2)$.

We notice that the systems (7.2) and the Laplace transform of (3.1) are similar. For this reason, the solutions for displacement and rotation can be derived from (5.4) directly by replacing $u_i^{(1)}$ by $\dot u_i$, $\varphi_3^{(1)}$ by $\dot\varphi_3$, p by $i\omega$, β_i^2 by $-\beta_i^{*2}=-\omega^2/c_i^2$ $(i=1,2,4)$. Thus

$$(7.3)\quad \dot u_1(x)=\frac{F}{2\pi}\frac{c_2^2}{c_1^2}\frac{x_1x_2}{r^2}\cdot\left[\frac{\gamma^{*2}(\beta_1^{*2}-\beta_4^{*2}+\bar r^2)+sq}{(\beta_1^{*2}-a_1^2)(a_2^2-\beta_1^{*2})}\cdot\beta_1^{*2}K_2(\beta_1^{*}r)-\right.$$
$$-\frac{\gamma^{*2}(a_1^2+\bar r^2-\beta_4^{*2})+sq}{(a_1^2-\beta_1^{*2})(a_1^2-a_2^2)}\cdot a_1^2K_2(ia_1r)+$$
$$\left.+\frac{\gamma^{*2}(a_2^2+\bar r^{-2}-\beta_4^{*2})+sq}{(a_2^2-\beta_2^{*2})(a_1^2-a_2^2)}\cdot a_2^2K_2(ia_2r)\right];$$

$$u_2^*(x) = \frac{F}{2\pi}\frac{c_2^2}{c_1^2} K_0(i\beta_1^* r) - \frac{x_1}{x_2} u_1^*(x) ;$$

(7.3)

$$\varphi_3^*(x) = -\frac{iqF}{2\pi} \cdot \frac{x_1}{a_1^2 - a_2^2} [a_1 K_1(ia_1 r) - a_2 K_1(ia_2 r)]$$

where a_1^2, a_2^2 are the roots of the equation :

$$(\xi^2 - \beta_2^{*2})(\xi^2 + \bar{r}^2 - \beta_4^{*2})) - sq\xi^2 = 0.$$

Substituting (7.3) in (7.1) we obtain the final expressions for the displacements and rotation field.

8. Final conclusions. In the case of impulsive body force we observe from (5.6) that the displacements u_1 an u_2 have discontinuities at the moments $t = r/c_1$, $t = r/c_2$. At the moment $t = r/c_4$, the displacements u_1, u_2 are continuous functions. The rotation component φ_3 has no discontinuities at $t = r/c_2$ and $t = r/c_4$. When the body force is applied suddenly we notice from (6.4) that the displacements and rotation are continuous functions. The same thing is observed in the case of periodic body force.

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POLYNOMIAL SOLUTIONS IN THE AXIAL COORDINATE FOR BEAMS WITH MICROSTRUCTURE HAVING A SYMMETRIC STRESS TENSOR

BY

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Let us consider a beam limited by two planes $x_3 = 0$, $x_3 = h$ $(h > 0)$ and by a cylindrical surface $\mathcal{F}$, the unit vector along the outer normal to $\mathcal{F}$ being $\mathbf{n}(n_1, n_2, 0)$.

We will denote by $\mathcal{V}$ the doumain occupied by the beam ; by S we will denote the domain of a cross-section of the beam and by ∂S its boundary.

We will take the axes of x_1, x_2, x_3 such that

$$\int_S x_\rho d\sigma = 0, \quad \int_S x_1 x_2 d\sigma = 0.$$

The material of the beam has the following constitutive equations

$$(1) \qquad \begin{cases} t_{ij} = \lambda\varepsilon_{mm}\delta_{ij} + 2\mu\varepsilon_{ij} + \omega\varkappa_{mm}\delta_{ij} + \pi(\varkappa_{ij} + \varkappa_{ji}), \\ m_{ij} = \omega\varepsilon_{mm}\delta_{ij} + 2\pi\varepsilon_{ij} + \alpha\varkappa_{mm}\delta_{ij} + \beta\varkappa_{ij} + \gamma\varkappa_{ji}, \\ \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}).,\ \varkappa_{ij} = \varphi_{i,j}, \end{cases}$$

where t_{ij} are the components of the stress tensor, m_{ij} the components of the couple-stress tensor, u_i the displacements, φ_i the rotation vector components, δ_{ij} — Kronecker's delta ; λ, μ, ω, π, α, β γ are constants which characterize the properties of the material.

The constitutive equations (1) are given in [1]; they can be obtained also from [4].

We shall use the summation convention over repeated indices. The index j after a comma indicates partial differentiation with respect to x_j. The Latin indices take the values 1, 2, 3 and the Greek indices take the values 1, 2.

The model defined by the constitutive equations (1) has a symmetric stress tensor $(t_{ij} = t_{ji})$.

In this model a neutral motion (without stresses and couple-stresses) is given by

$$u_1 = \omega_2 x_3 - \omega_3 x_2 + a_1,\ u_2 = \omega_3 x_1 - \omega_1 x_3 + a_2,\ u_3 = \omega_1 x_2 - \omega_2 x_1 + a_3,\ \varphi_i = b_i$$

where ω_i, a_i, b_i are the same at all points of the body.