$$
(7.3)\quad
\begin{aligned}
u_2^{\bullet}(x) &= \frac{F}{2\pi}\,\frac{c_2^2}{c_1^2}\,K_0(i\beta_1^{\bullet}r) - \frac{x_1}{x_2}\,u_1^{\bullet}(x)\ ;\\
\varphi_3^{\bullet}(x) &= -\frac{iqF}{2\pi}\cdot\frac{x_1}{a_1^2-a_2^2}\,[a_1K_1(ia_1 r) - a_2K_1(ia_2 r)]
\end{aligned}
$$

where a_1^2, a_2^2 are the roots of the equation :

$$(\xi^2-\beta_2^{\bullet 2})\,(\xi^2+\bar{r}^2-\beta_4^{\bullet 2}))-sq\xi^2=0.$$

Substituting (7.3) in (7.1) we obtain the final expressions for the displacements and rotation field.

8. Final conclusions. In the case of impulsive body force we observe from (5.6) that the displacements u_1 an u_2 have discontinuities at the moments $t=r/c_1$, $t=r/c_2$. At the moment $t=r/c_4$, the displacements u_1, u_2 are continuous functions. The rotation component φ_3 has no discontinuities at $t=r/c_2$ and $t=r/c_4$. When the body force is applied suddenly we notice from (6.4) that the displacements and rotation are continuous functions. The same thing is observed in the case of periodic body force.

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Department of Mathematics
Polytechnic Institute of Iaşi
6600 — Iaşi, R. S. Romania

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POLYNOMIAL SOLUTIONS IN THE AXIAL COORDINATE FOR BEAMS WITH MICROSTRUCTURE HAVING A SYMMETRIC STRESS TENSOR

BY

C. I. BORŞ

Let us consider a beam limited by two planes $x_3=0$, $x_3=h$ $(h>0)$ and by a cylindrical surface $\mathcal{F}$, the unit vector along the outer normal to $\mathcal{F}$ being $\mathbf{n}(n_1, n_2, 0)$.

We will denote by $\mathcal{V}$ the doumain occupied by the beam ; by S we will denote the domain of a cross-section of the beam and by ∂S its boundary.

We will take the axes of x_1, x_2, x_3 such that

$$\int_S x_\rho d\sigma=0,\quad \int_S x_1x_2 d\sigma=0.$$

The material of the beam has the following constitutive equations

$$
(1)\quad
\begin{cases}
t_{ij}=\lambda\varepsilon_{mm}\delta_{ij}+2\mu\varepsilon_{ij}+\omega\varkappa_{mm}\delta_{ij}+\pi(\varkappa_{ij}+\varkappa_{ji}),\\
m_{ij}=\omega\varepsilon_{mm}\delta_{ij}+2\pi\varepsilon_{ij}+\alpha\varkappa_{mm}\delta_{ij}+\beta\varkappa_{ij}+\gamma\varkappa_{ji},\\
\varepsilon_{ij}=\dfrac{1}{2}(u_{i,j}+u_{j,i}).,\quad \varkappa_{ij}=\varphi_{i,j},
\end{cases}
$$

where t_{ij} are the components of the stress tensor, m_{ij} the components of the couple-stress tensor, u_i the displacements, φ_i the rotation vector components, δ_{ij} — Kronecker's delta ; $\lambda, \mu, \omega, \pi, \alpha, \beta\ \gamma$ are constants which characterize the properties of the material.

The constitutive equations (1) are given in [1]; they can be obtained also from [4].

We shall use the summation convention over repeated indices. The index j after a comma indicates partial differentiation with respect to x_j. The Latin indices take the values 1, 2, 3 and the Greek indices take the values 1, 2.

The model defined by the constitutive equations (1) has a symmetric stress tensor $(t_{ij}=t_{ji})$.

In this model a neutral motion (without stresses and couple-stresses) is given by

$$u_1=\omega_2x_3-\omega_3x_2+a_1,\quad u_2=\omega_3x_1-\omega_1x_3+a_2,\quad u_3=\omega_1x_2-\omega_2x_1+a_3,\quad \varphi_i=b_i$$

where ω_i, a_i, b_i are the same at all points of the body.

1. Formulation of the problem. The lateral surface of the beam is free from tractions and from surface couples. The body forces and the body couples are absent. The beam is loaded only on the ends in such a way that it is in equilibrium, the displacements and the rotations being of the form.

$$u_i=\sum_{k=1}^{n}U_i^{(k)}(x_1,x_2)x_3^k,\quad \varphi_i=\sum_{k=1}^{n}\Phi_i^{(k)}(x_1,x_2)x_3^k, \tag{2}$$

where the functions $U_i^{(k)}(.,.)$, $\Phi_i^{(k)}(.,.)$ and the integer n must be determined.

Then, the equilibrium equations and the lateral boundary condition are

$$t_{ji,j}=0,\quad m_{ji,j}=0\quad \text{in } \mathcal{V}, \tag{3}$$

$$t_{\nu i}n_\nu=0,\quad m_{\nu i}n_\nu=0\quad \text{on } \mathcal{F}. \tag{4}$$

Our purpose, for such a beam, is to look for the most general solution u_i, φ_i of (1), (3), (4) which is polynomial in x_3 and to determine the end tractions and couple-stress compatible with it.

2. General equations. In order to solve our problem, we will use up the same technics of calculus as in [2]. From (1) and (2) we obtain

$$\left\{\begin{aligned}
&t_{\rho\rho}=\Sigma t_{\rho\rho}^{(k)}x_3^k+\Sigma k(\lambda U_3^{(k)}+\omega\Phi_3^{(k)})x_3^{k-1}\ \text{(not summed)},\\
&t_{33}=\Sigma t_{33}^{(k)}x_3^k+\Sigma k[(\lambda+2\mu)U_3^{(k)}+(\omega+2\pi)\Phi_3^{(k)}]x_3^{k-1},\\
&t_{12}=\Sigma t_{12}^{(k)}x_3^k,\\
&t_{\rho 3}=\Sigma(\mu U_{3,\rho}^{(k)}+\pi\Phi_{3,\rho}^{(k)})x_3^k+\Sigma k(\mu U_\rho^{(k)}+\pi\Phi_\rho^{(k)})x_3^{k-1}
\end{aligned}\right. \tag{5}$$

and

$$\left\{\begin{aligned}
&m_{\rho\rho}=\Sigma m_{\rho\rho}^{(k)}x_3^k+\Sigma k(\omega U_3^{(k)}+\alpha\Phi_3^{(k)})x_3^{k-1}\ \text{(not summed)},\\
&m_{33}=\Sigma m_{33}^{(k)}x_3^k+\Sigma k(\omega+2\pi)U_3^{(k)}+(\alpha+\beta+\gamma)\Phi_3^{(k)}]x_3^{k-1},\\
&m_{\rho\nu}=\Sigma m_{\rho\nu}^{(k)}x_3^k\quad (\rho\neq\nu),\\
&m_{\rho 3}=\Sigma(\pi U_{3,\rho}^{(k)}+\gamma\Phi_{3,\rho}^{(k)})x_3^k+\Sigma k(\pi U_\rho^{(k)}+\beta\Phi_\rho^{(k)})x_3^{k-1},\\
&m_{3\rho}=\Sigma(\pi U_{3,\rho}^{(k)}+\beta\Phi_{3,\rho}^{(k)})x_3^k+\Sigma k(\pi U_\rho^{(k)}+\gamma\Phi_\rho^{(k)})x_3^{k-1},
\end{aligned}\right. \tag{6}$$

where $t_{\rho\nu}^{(k)}$, $t_{33}^{(k)}$ and $m_{\rho\nu}^{(k)}$, $m_{33}^{(k)}$ are the components of the stress and the components of the couple-stress of the plane strain problem defined by the displacement $(U_1^{(k)}, U_2^{(k)}, 0)$ and the rotation $(\Phi_1^{(k)}, \Phi_2^{(k)}, 0)$.

The equilibrium equations (3) and the boundary conditions (4) will be satisfied if and only if, for arbitrary k,

$$\left\{\begin{aligned}
&t_{\rho\nu,\nu}^{(k)}+(k+1)[(\lambda+\mu)U_{3,\rho}^{(k+1)}+(\omega+2\pi)\Phi_{3,\rho}^{(k+1)}]+\\
&+(k+1)(k+2)(\mu U_\rho^{(k+2)}+\pi\Phi_\rho^{(k+2)})=0,\\
&m_{\nu\rho,\nu}^{(k)}+(k+1)[(\omega+\pi)U_{3,\rho}^{(k+1)}+(\alpha+\beta)\Phi_{3,\rho}^{(k+1)}]+\\
&+(k+1)(k+2)(\pi U_\rho^{(k+2)}+\gamma\Phi_\rho^{(k+2)})=0\quad \text{in } S;
\end{aligned}\right. \tag{7}$$

$$\left\{\begin{aligned}
&t_{\rho\nu}^{(k)}n_\nu=-(k+1)(\lambda U_3^{(k+1)}+\omega\Phi_3^{(k+1)})n_\rho,\\
&m_{\nu\rho}^{(k)}n_\nu=-(k+1)(\omega U_3^{(k+1)}+\alpha\Phi_3^{(k+1)})n_\rho\quad \text{on } \partial S
\end{aligned}\right. \tag{8}$$

and

$$\begin{aligned}
&(\mu\gamma-\pi^2)\Delta U_3^{(k)}+(k+1)\{(\mu\gamma-\pi^2)U_{\nu,\nu}^{(k+1)}+\pi(\gamma-\beta)\Phi_{\nu,\nu}^{(k-1)}+\gamma t_{33}^{(k+1)}-\pi m_{33}^{(k+1)}\}+\\
&+(k+1)(k+2)\{[(\lambda+2\mu)\gamma-(\omega+2\pi)\pi]U_3^{(k+2)}+[(\omega+2\pi)\gamma-\\
&-(\alpha+\beta+\gamma)\pi]\Phi_3^{(k+2)}\}=0\quad \text{in } S,
\end{aligned} \tag{9}$$

$$(\mu\gamma-\pi^2)DU_3^{(k)}=-(k+1)[(\mu\gamma-\pi^2)U_\nu^{(k+1)}n_\nu+\pi(\gamma-\beta)\Phi_\nu^{(k+1)}n_\nu]\quad \text{on } \partial S, \tag{10}$$

$$\begin{aligned}
&(\mu\gamma-\pi^2)\Delta\Phi_3^{(k+1)}+(k+1)[(\beta\mu-\pi^2)\Phi_{\nu,\nu}^{(k+1)}+\mu m_{33}^{(k+1)}-\pi t_{33}^{(k+1)}]+\\
&+(k+1)(k+2)\{(\mu\omega-\lambda\pi)U_3^{(k+2)}+[(\alpha+\beta+\gamma)\mu-(\omega+2\pi)\pi]\Phi_3^{(k+2)}\}=0\ \text{in } S,
\end{aligned} \tag{11}$$

$$(\mu\gamma-\pi^2)D\Phi_3^{(k)}=-(k+1)(\beta\mu-\pi^2)\Phi_\nu^{(k+1)}n_\nu\quad \text{on } \partial S, \tag{12}$$

where

$$\Delta=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2},\quad D=n_1\frac{\partial}{\partial x_1}+n_2\frac{\partial}{\partial x_2}.$$

The problem defined by (7), (8) is a plane problem with body forces and body couples for the displacements $U_1^{(k)}$, $U_2^{(k)}$ and the rotations $\Phi_1^{(k)}$, $\Phi_2^{(k)}$.

The necessary and sufficient conditions for the existence of the solution for this problem require that

$$\int_S[\mu U_{3,\rho}^{(k+1)}+\pi\Phi_{3,\rho}^{(k+1)}+(k+2)(\mu U_\rho^{(k+2)}+\pi\Phi_\rho^{(k+2)})]d\sigma=0, \tag{13 a}$$

$$\begin{aligned}
&\int_S\{(\mu U_{3,2}^{(k+1)}+\pi\Phi_{3,2}^{(k+1)})x_1-(\mu U_{3,1}^{(k+1)}+\pi\Phi_{3,1}^{(k+1)})x_2+\\
&+(k+2)[(\mu U_2^{(k+2)}+\pi\Phi_2^{(k+2)})x_1-(\mu U_1^{(k+2)}+\pi\Phi_1^{(k+2)})x_2]\}d\sigma=0,
\end{aligned} \tag{13 b}$$

$$\int_S[\pi U_{3,\rho}^{(k+1)}+\beta\Phi_{3,\rho}^{(k+1)}+(k+2)(\pi U_\rho^{(k+2)}+\gamma\Phi_\rho^{(k+2)})]d\sigma=0. \tag{13 c}$$

The problem for $U_3^{(k)}$ and $\Phi_3^{(k)}$ are of the Neumann type and the existence of solutions require that

$$\begin{aligned}
&\int_S\{\gamma t_{33}^{(k+1)}-\pi m_{33}^{(k+1)}+(k+2)[(\lambda+2\mu)\gamma-(\omega+2\pi)\pi]U_3^{(k+2)}+\\
&+(k+2)[(\omega+2\pi)\gamma-(\alpha+\beta+\gamma)\pi]\Phi_3^{(k+2)}\}d\sigma=0,
\end{aligned} \tag{14 a}$$

$$\begin{aligned}
&\int_S\{\mu m_{33}^{(k)}-\pi t_{33}^{(k+1)}+(k+2)(\mu\omega-\lambda\pi)U_3^{(k+2)}+\\
&+(k+2)[(\alpha+\beta+\gamma)\mu-(\omega+2\pi)\pi]\Phi_3^{(k+2)}\}d\sigma=0.
\end{aligned} \tag{14 b}$$

3. The solution of the problem. Now, let us to solve the above problem for $k=n, n-1, n-2, \ldots$

In the following $A_\rho^{(k)}$, $B_\rho^{(k)}$, C_i, τ_k $(k=1, 2, \ldots)$ denote arbitrary constants; $A_\nu u^{(\nu)}=A_1u^{(1)}+A_2u^{(2)}$.

The case $k=n$: The corresponding equations are given by

$$t_{\rho\nu,\nu}^{(n)}=0,\quad m_{\nu\rho,\nu}^{(n)}=0\quad \text{in } S;\quad t_{\rho\nu}^{(n)}n_\nu=0,\quad m_{\nu\rho}^{(n)}n_\nu=0\ \text{on } \partial S,$$

and

$$\Delta U_3^{(n)}=0,\ \Delta\Phi_3^{(n)}=0 \text{ in } S\ ;\ DU_3^{(n)}=0,\ D\Phi_3^{(n)}=0 \quad \text{on } \partial S.$$

It results

(15) $$U_\rho^{(n)}=A_\rho^{(n)}+(-1)^\rho\tau_n x_\rho,\ U_3^{(n)}=C_n,\ \Phi_i^{(n)}=B_i^{(n)} \quad (\delta\neq\rho).$$

The case $k=n-1$: The equations (7)—(12) are

$$t_{\rho\nu,\nu}^{(n-1)}=0,\ m_{\nu\rho,\nu}^{(n-1)}=0 \text{ in } S,$$

$$t_{\rho\nu}^{(n-1)}n_\nu=-n(C_n\lambda+B_3^{(n)}\omega)n_\rho,\ m_{\nu\rho}^{(n-1)}n_\nu=-n(C_n\omega+B_3^{(n)}\alpha)n_\rho \quad \text{on } \partial S\ ;$$

$$\Delta U_3^{(n-1)}=0,\ \Delta\Phi_3^{(n-1)}=0 \quad \text{in } S,$$

$$DU_3^{(n-1)}=-n[(A_1^{(n)}-\tau_n x_2)n_1+(A_2^{(n)}+\tau_n x_1)n_2+\frac{\pi(\gamma-\beta)}{\mu\gamma-\pi^2}B_\nu^{(n)}n_\nu,$$

$$D\Phi_3^{(n-1)}=-n\,\frac{\beta\mu-\pi^2}{\mu\gamma-\pi^2}B_\nu^{(n)}n_\nu.$$

For the corresponding solution we have :

(16) $$\begin{cases} U_\rho^{(n-1)}=n(C_n a+B_3^{(n)}A)x_\rho+A_\rho^{(n-1)}+(-1)^\rho\tau_{n-1}x_\delta, \\ \Phi_\rho^{(n-1)}=n(C_n b+B_3^{(n)}B)x_\rho+B_\rho^{(n-1)}, \\ U_3^{(n-1)}=n[\tau_n\varphi-A_\nu^{(n)}x_\nu-\dfrac{\pi(\gamma-\beta)}{\pi\mu-\pi^2}B_\nu^{(n)}x_\nu]+C_{n-1}, \\ \Phi_3^{(n-1)}=-n\,\dfrac{\beta\mu-\pi^2}{\mu\gamma-\pi^2}B_\nu^{(n)}x_\nu+B_3^{(n-1)} \quad (\delta\neq\rho), \end{cases}$$

where φ is the function of torsion defined by

$$\Delta\varphi=0 \text{ in } S,\ D\varphi=x_2 n_1-x_1 n_2 \text{ on } \partial S$$

and a, A, b, B are constants given by

$$2(\lambda+\mu)\,a+2(\omega+\pi)\,b=-\lambda,\ 2(\omega+\pi)\,a+(2\alpha+\beta+\gamma)\,b=-\omega,$$

$$2(\lambda+\mu)\,A+2(\omega+\pi)\,B=-\omega,\ 2(\omega+\pi)\,A+(2\alpha+\beta+\gamma)\,B=-\alpha.$$

The case $k=n-2$: Because the rigidity of torsion

$$D=\int_S(x_1^2+x_2^2+x_1\,\varphi_{,2}-x_2\,\varphi_{,1})d\sigma>0,$$

the conditions (13), (14) will be fulfilled if we take

(17) $$\tau_n=B_i^{(n)}=C_n=0.$$

Taking into account (17), the equations (7)—(12) become

$$t_{\rho\nu,\nu}^{(n-2)}-n(n-1)A_\rho^{(n)}\lambda=0,\ m_{\nu\rho,\nu}^{(n-2)}-n(n-1)A_\rho^{(n)}\omega=0 \text{ in } S,$$

$$t_{\rho\nu}^{(n-2)}n_\nu=(n-1)[(n\lambda A_\nu^{(n)}x_\nu-C_{n-1}\lambda-B_3^{(n-1)}\omega)n_\rho,$$

$$m_{\nu\rho}^{(n-2)}n_\nu=(n-1)(n\omega A_\nu^{(n)}x_\nu-C_{n-1}\omega-B_3^{(n-1)}\alpha)n_\rho \quad \text{on } \partial S$$

and

$$\Delta U_3^{(n-2)}=0,\ \Delta\Phi_3^{(n-3)}=0 \quad \text{in } S,$$

$$DU_3^{(n-2)}=-(n-1)\left[(A_1^{(n-1)}-\tau_{n-1}x_2)n_1+(A_2^{(n-2)}+\tau_{n-1}x_1)n_2+\frac{\pi(\gamma-\beta)}{\mu\gamma-\pi^2}B_\nu^{(n-1)}n_\nu\right],$$

$$D\Phi_3^{(n-2)}=-(n-1)\,\frac{\beta\mu-\pi^2}{\mu\gamma-\pi^2}B_\nu^{(n-1)}n_\nu \quad \text{on } \partial S.$$

The solutions of these problems are given by

(18) $$\begin{cases} U_\rho^{(n-2)}=(n-1)[nA_\nu^{(n)}u_\rho^{(\nu)}+(C_{n-1}a+B_3^{(n-1)}A)x_\rho]+A_\rho^{(n-2)}+(-1)^\rho\tau_{n-2}x_\delta, \\ \Phi_\rho^{(n-2)}=(n-1)[nA_\nu^{(n)}\varphi_\rho^{(\nu)}+(C_{n-1}b+B_3^{(n-1)}B)x_\rho]+B_\rho^{(n-2)}, \\ U_3^{(n-2)}=(n-1)[\tau_{n-1}\varphi-A_\nu^{(n-1)}x_\nu-\dfrac{\pi(\gamma-\beta)}{\mu\gamma-\pi^2}B_\nu^{(n-1)}x_\nu]+C_{n-2}, \\ \Phi_3^{(n-2)}=-(n-1)\,\dfrac{\beta\mu-\pi^2}{\mu\gamma-\pi^2}B_\nu^{(n-1)}x_\nu+B_3^{(n-2)}, \end{cases}$$

where $u_\rho^{(1)}$, $\varphi_\rho^{(1)}$ and $u_\rho^{(2)}$, $\varphi_\rho^{(2)}$ are the solutions of two auxiliary plane problems defined by

(19) $$\begin{cases} \tau_{\rho\nu,\nu}^{(\delta)}-\lambda\delta_{\rho\delta}=0,\ \mu_{\nu\rho,\nu}^{(\delta)}-\omega\delta_{\rho\delta}=0 \quad \text{in } S; \\ \tau_{\rho\nu}^{(\delta)}n_\nu=\lambda x_\delta n_\rho,\ \mu_{\nu\rho}^{(\delta)}n_\nu=\omega x_\delta n_\rho \quad \text{on } \partial S. \end{cases}$$

We have

(20) $$\int_S\tau_{33}^{(\delta)}d\sigma=0,\ \int_S\mu_{33}^{(\delta)}d\sigma=0,\ \int_S\tau_{33}^{(1)}x_2 d\sigma=\int_S\tau_{33}^{(2)}x_1 d\sigma$$

and

(22) $$\begin{cases} t_{33}^{(n-2)}=n(n-1)A_\nu^{(n)}\tau_{33}^{(\nu)}+2(n-1)[(\lambda a+\omega b)C_{n-1}+(\lambda A+\omega B)B_3^{(n-1)}], \\ m_{33}^{(n-2)}=n(n-1)A_\nu^{(n)}\mu_{33}^{(\nu)}+2(n-1)[(\omega a+\alpha b)C_{n-1}+\omega A+\alpha B)B_3^{(n-1)}]. \end{cases}$$

The case $k=n-3$: From 13 (a)—(14 b) we have

(23) $$\tau_{n-1}=B_i^{(n-1)}=C_{n-1}=0,$$

and the equations (7)—(12) are

$$t_{\rho\nu,\nu}^{(n-3)}-(n-1)(n-2)A_\rho^{(n-1)}\lambda=0,\ m_{\nu\rho,\nu}^{(n-3)}-(n-1)(n-2)A_\rho^{(n-1)}\omega=0 \text{ in } S,$$

$$t_{\rho\nu}^{(n-3)}n_\nu=(n-2)[(n-1)\lambda A_\nu^{(n-1)}x_\nu-C_{n-2}\lambda-B_3^{(n-2)}\omega]n_\rho,$$

$$m_{\nu\rho}^{(n-3)}n_\nu=(n-2)[(n-1)\omega A_\nu^{(n-1)}x_\nu-C_{n-2}\omega-B_3^{(n-2)}\alpha]n_\rho \quad \text{on } \partial S\ ;$$

$$(\mu\gamma-\pi^2)\Delta U_3^{(n-3)}+n(n-1)(n-2)A_\rho^{(n)}\{(\mu\gamma-\pi^2)u_{\nu,\nu}^{(\rho)}+\pi(\gamma-\beta)\varphi_{\nu,\nu}^{(\rho)}+$$

$$+\gamma\tau_{33}^{(\rho)}-\pi\mu_{33}^{(\rho)}-[(\lambda+2\pi)\gamma-(\omega+2\pi)\pi]x_\rho\}=0 \quad \text{in } S,$$

$$DU_3^{(n-3)}=-n(n-1)(n-2)A_\rho^{(n)}[u_\nu^{(\rho)}+\pi(\beta-\gamma)\varphi_\nu^{(\rho)}]n_\nu-(n-2)[(A_1^{(n-2)}-$$

$$-\tau_{n-2}x_2)n_1+(A_2^{(n-2)}+\tau_{n-2}x_1)n_2]+\frac{\pi(\gamma-\beta)}{\mu\gamma-\pi^2}B_\nu^{(n-1)}n_\nu \quad \text{on } \partial S;$$

$$(\mu\gamma-\pi^2)\Delta\Phi_3^{(n-3)}+n(n-1)(n-2)A_\rho^{(n)}[(\mu\beta-\pi^2)\varphi^{(\rho)}_{\nu,\nu}+\mu\mu^{(\rho)}_{33}-\pi\tau^{(\rho)}_{33}-$$
$$-(\mu\omega-\lambda\pi)x_\rho]=0 \quad \text{in } S,$$

$$D\Phi_3^{(n-3)}=-\frac{\beta\mu-\pi^2}{\mu\gamma-\pi^2}(n-2)[n(n-1)A_\rho^{(n)}\varphi_\nu^{(\rho)}+B_\nu^{(n-2)}]n_\nu \quad \text{on } \partial S.$$

The above problems have the solutions

$$(24)\quad \begin{cases} U_\rho^{(n-3)}=(n-2)[(n-1)A_\nu^{(n-1)}u_\rho^{(\nu)}+(C_{n-2}a+B_3^{(n-2)}A)x_\rho]+\\ +A_\rho^{(n-3)}+(-1)^\rho\tau_{n-3}x_\rho \quad (\delta\neq\rho),\\ \Phi_\rho^{(n-3)}=(n-2)[(n-1)A_\nu^{(n-1)}\varphi_\rho^{(\nu)}+(C_{n-2}b+B_3^{(n-2)}B)x_\rho]+B_\rho^{(n-3)},\\ U_3^{(n-3)}=n(n-1)(n-2)A_\nu^{(n)}\chi_\nu+(n-2)\Big[\tau_{n-2}\varphi-A_\nu^{(n-2)}x_\nu-\\ -\dfrac{\pi(\gamma-\beta)}{\mu\gamma-\pi^2}B_\nu^{(n-2)}x_\nu\Big]+C_{n-3},\\ \Phi_3^{(n-3)}=n(n-1)(n-2)A_\nu^{(n)}\Psi_\nu-(n-2)\dfrac{\beta\mu-\pi^2}{\mu\gamma-\pi^2}B_\nu^{(n-2)}x_\nu+B_3^{(n-3)}. \end{cases}$$

The functions χ_ρ and Ψ_ρ are defined by the following value problems

$$(25)\quad \begin{cases} (\mu\gamma-\pi^2)\Delta\chi_\rho+(\mu\gamma-\pi^2)u^{(\rho)}_{\nu,\nu}+\pi(\gamma-\beta)\varphi^{(\rho)}_{\nu,\nu}+\\ +\gamma\tau^{(\rho)}_{33}-\pi\mu^{(\rho)}_{33}-[(\lambda+2\mu)\gamma-(\omega+2\pi)\pi]x_\rho=0 \quad \text{in } S,\\ D\chi_\rho=-\Big[u_\nu^{(\rho)}+\dfrac{\pi(\gamma-\beta)}{\mu\gamma-\pi^2}\varphi_\nu^{(\rho)}\Big]n_\nu \quad \text{on } \partial S; \end{cases}$$

$$(26)\quad \begin{cases} (\mu\gamma-\pi^2)\Delta\Psi_\rho+(\mu\beta-\pi^2)\varphi^{(\rho)}_{\nu,\nu}+\mu\mu^{(\rho)}_{33}-\pi\tau^{(\rho)}_{33}-(\mu\omega-\lambda\pi)x_\rho=0 \quad \text{in } S,\\ D\Psi_\rho=\dfrac{\mu\beta-\pi^2}{\mu\gamma-\pi^2}\varphi_\nu^{(\rho)}n_\nu \quad \text{on } \partial S. \end{cases}$$

The functions χ_ρ and Ψ_ρ exist. We call them the bending functions.

The case $k=n-4$. From the conditions (13 a) we get

$$(27)\qquad A_\nu^{(n)}L_{\nu\rho}=0,$$

where

$$(28)\qquad L_{\nu\rho}=\int_S[\mu(\chi_{\nu,\rho}+u_\rho^{(\nu)})+\pi(\Psi_{\nu,\nu}+\varphi_\rho^{(\nu)})]d\sigma.$$

Because

$$(29)\qquad L_{11}L_{22}-L_{12}^2=L_{11}L_{22}-L_{21}^2\neq 0$$

and the rigidity of torsion $D>0$, from (27) and (13 b), we obtain

$$(30)\qquad A_\nu^{(n)}=0,\quad \tau_{n-2}=0 \quad (n\geq 4).$$

Taking into account all the above results we find that the displacements and the rotations, up to a rigid motion, are given by

$$(30\text{ a})\quad \begin{cases} u_\rho=(-1)^\rho x_\delta x_3+a_\nu u_\rho^{(\nu)}+(a_3a+b_3A)x_\rho+A_\nu u_\rho^{(\nu)}x_3+\dfrac{1}{2}a_\rho x_3^2+\dfrac{1}{6}A_\rho x_3^3,\\ u_3=\tau\varphi+A_\nu\chi_\nu-\dfrac{\pi(\gamma-\beta)}{\mu\gamma-\pi^2}b_\nu x_\nu-(a_\nu x_\nu+a_3)x_3-\dfrac{1}{2}A_\nu x_\nu x_3^2; \end{cases}$$

$$(30\text{ b})\quad \begin{cases} \varphi_\rho=a_\nu\varphi_\rho^{(\nu)}+(a_3b+b_3B)x_\rho+A_\nu\varphi_\rho^{(\nu)}x_3+b_\rho x_3,\\ \varphi_3=A_\nu\Psi_\nu-\dfrac{\beta\mu-\pi^2}{\mu\gamma-\pi^2}b_\nu x_\nu+b_3x_3, \end{cases}$$

where

$$a_\rho=2A_\rho^{(2)},\quad A_\rho=6A_\rho^{(3)},\quad b_i=B_i^{(1)},\quad a_3=C_1,\quad \tau=\tau_1$$

are arbitrary constants.

The stresses and the couple stresses, corresponding to (30 a) and (30 b), are given by

$$(31\text{ a})\quad \begin{cases} t_{\rho\rho}=(a_\nu+A_\nu x_3)(\tau^{(\nu)}_{\rho\rho}-\lambda x_\nu) \quad (\text{not summed over } \rho),\\ t_{33}=(a_\nu+A_\nu x_3)[\tau^{(\nu)}_{33}-(\lambda+2\mu)x_\nu]+(2\lambda a+2\omega b+\lambda+2\mu)a_3+\\ +(2\lambda A+2\omega B+\omega+2\pi)b_3,\\ t_{12}=(a_\nu+A_\nu x_3)\tau^{(\nu)}_{12},\\ t_{\rho3}=\tau\mu[\varphi_{,\rho}+(-1)^\rho x_\delta]+A_\nu[\mu(\chi_{\nu,\rho}+u_\rho^{(\nu)})+\pi(\Psi_{\nu,\rho}+\varphi_\rho^{(\nu)})]; \end{cases}$$

$$(31\text{ b})\quad \begin{cases} m_{\rho\rho}=(a_\nu+A_\nu x_3)(\mu^{(\nu)}_{\rho\rho}-\omega x_\nu) \quad (\text{not summed over } \rho),\\ m_{33}=(a_\nu+A_\nu x_3)[\mu^{(\nu)}_{33}-(\omega+2\pi)x_\nu]+(2\omega a+2\alpha b+\omega+2\pi)a_3+(2\omega A+\\ +2\alpha B+\alpha+\beta+\gamma)b_3,\\ m_{\rho\delta}=(a_\nu+A_\nu x_3)\mu^{(\nu)}_{\rho\delta} \quad (\rho\neq\delta),\\ m_{\rho3}=\tau\pi[\varphi_{,\rho}+(-1)^\rho x_\delta]+A_\nu[\pi(\chi_{\nu,\rho}+u_\rho^{(\nu)})+\gamma\Psi_{\nu,\rho}+\beta\varphi_\rho^{(\nu)}],\\ m_{3\rho}=\tau\pi[\varphi_{,\rho}+(-1)^\rho x_\delta]+A_\nu[\pi(\chi_{\nu,\rho}+u_\rho^{(\nu)})+\beta\Psi_{\nu,\rho}+\gamma\varphi_\rho^{(\nu)}]+\\ +\dfrac{(\gamma-\beta)[\mu(\beta+\gamma)-2\pi^2]}{\mu\gamma-\pi^2}b_\rho. \end{cases}$$

In order to prove (29) let us consider in (30 a), (30 b) and (31 a) (31 b), $a_\rho\neq 0$ and $A_\rho=\tau=b_i=a_3=0$. In this case the deformation energy of the beam is given by

$$W(a_1,a_2)=-h(I_{11}a_1^2+2I_{12}a_1a_2+I_{22}a_2^2),$$

where

$$I_{\rho\nu}=\int_S[\tau^{(\rho)}_{33}-(\lambda+2\mu)x_\rho]x_\nu d\sigma,\quad I_{\rho\nu}=I_{\nu\rho}.$$

It is a positive definite quadratic form in a_1 and a_2. Consequently, we have

$$I_{11}I_{22}-I_{12}^2\neq 0.$$

On the other hand, from the equilibrium equations, we obtain

$$\int_S t_{\rho 3}d\sigma=\int_S[(t_{13}x_\rho)_{,1}+(t_{23}x_\rho)_{,2}+(t_{33}x_\rho)_{,3}]d\sigma=\int_S t_{33,3}x_\rho d\sigma=A_\nu\int_S[\tau_{33}^{(\nu)}-(\lambda+2\mu)x_\nu]x_\rho d\sigma,$$

and at last, using (31), we get

$$A_\nu L^{**}_{\nu\rho}=A_\nu I_{\nu\rho},\qquad L_{\nu\rho}=I_{\nu\rho}.$$

In this way (29) is proved.

The arbitrary constants a_ρ, A_ρ, b_i, a_3, τ can be uniquely determined by the end conditions

$$R_i=\int_S t_{3i}d\sigma,\quad M_\rho=(-1)^\rho\int_S t_{33}x_\delta\, d\sigma\quad(\delta\neq\rho),$$

$$M_3=\int_S(t_{13}x_2-t_{23}x_1)\,d\sigma,\quad N_i=\int_S m_{3i}\,d\sigma\quad\text{on } x_3=0,$$

R_i, M_i, N_i being given.

Then, the relaxed Saint-Venant problem has a unique solution. It coincides with the solution given in [3].

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Faculty of Mathematics
University of Iaşi
6600 — Iaşi, R. S. România

Analele ştiinţifice ale Universităţii „Al. I. Cuza" din Iaşi
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RECENZII

A. F. MONNA : *Methods, Concepts and Ideas in Mathematics: Aspect of an Evolution*, CWI Tract, nr. 23, Mathematical Centrum, Amsterdam, 1986 : IV+170 p., Dfl 26,40.

Cartea prezintă unele aspecte ale evoluţiei matematicii considerînd tendinţa de algebrizare ca fundamentală. De aceea, prima parte a cărţii este consacrată studiului acestei tendinţe, pentru ca abia în celelalte două părţi să se studieze istoria unor probleme de existenţă şi să se caracterizeze perioada modernă de dezvoltare a matematicii. Comentariile sînt interesante, argumentaţia este bine condusă de-a lungul celor 170 de pagini.

Mirela Ştefănescu

JEAN-PERRE SERRE : *Oeuvres-Collected Papers*, Springer-Verlag, Berlin, Heidelberg, New York, Tokyo, 1986, vol. I, XVIII+596 p., vol. II, IV+740 p., vol. III, IV+728 p., DM 398.

Apariţia acestor trei volume conţinînd opera matematică a cunoscutului matematician francez J. P. Serre, membru al Academiei Franceze din 1977, se încadrează în linia promovată în ultimii ani de cunoscuta editură vestgermană Springer de a colecta în volume articolele şi memoriile publicate în diverse reviste de specialitate de către matematicieni cu contribuţii notabile la dezvoltarea matematicii contemporane. Cele trei volume de faţă, realizate în condiţii grafice excelente, conţin aproape în intregime notele, articolele, memoriile şi unele seminarii ale autorului. Din motive evidente nu sînt incluse în aceste volume cărţile scrise de către autor. Primul volum conţine rezultatele autorului publicate între 1949—1959. Aici trebuie să menţionăm teza de doctorat intitulată „Omologia singulară a spaţiilor fibrate. Aplicaţii", prezentată şi publicată în 1951. În continuare preocupările autorului se axează tot pe probleme de topologie algebrică, teoria categoriilor, geometrie algebrică. O menţiune specială valabilă pentru toate cele trei volume constă în prezentarea rezumatelor cursurilor ţinute de autor la College de France de-a lungul anilor. În volumul al doilea, conţinînd lucrările publicate între 1960—1971 sînt continuate preocupările anterioare dar observăm şi o deplasare a interesului autorului către funcţiile de mai multe variabile complexe şi geometria algebrică p-adică. În sfîrşit, în al treilea volum sînt adunate articolele publicate între 1972—1984. Menţionăm că între materialele neincluse în aceste volume sînt cele 11 cărţi publicate sau apărute sub formă multiplicată, foarte puţine note şi articole (unele au fost deja publicate în volume asemănătoare ale lui A. Borel şi S. S. Chern), textele conferinţelor în seminariile Bourbaki, H. Cartan, C. Chevalley, Delange-Pisot-Poitou, Grothendieck, S. Lie, inmulţirea complexă.

V. Oproiu

S. SHELAH : *Around Classification Theory of Models*, Lecture Notes in Mathematics, vol. 1182 ; Springer-Verlag, Berlin, Heidelberg, New York, Tokyo, 1986 ; VII+279 p. ; DM 42,50

Cartea extinde teoria clasificării în diferite direcţii, de exemplu pentru cuantificatorii generalizaţi, pentru predicate. Se studiază apoi diverse alte probleme legate de clasificare, cum ar fi existenţa algebrelor Boole endorigide, mulţimilor de