We also have with a similar proof the following

Theorem 4.2. *If the mean curvature for totally real non-degenerate q-planes of nonnull index is independent of the choice of such q-planes at each point for* $2 \leq q \leq m$, *then M is an indefinite quaternion-space-form.*

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Received 25.04.1985

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ANALELE ŞTIINŢIFICE ALE UNIVERSITĂŢII „AL. I. CUZA" — IAŞI
Tomul XXXII, s. I a, Matematică, 1986 fasc. 2

GEOMETRY OF FINSLER SUBSPACES (I)

BY

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Introduction. As it is well-known, contrary to the Riemannian geometry the theory of Finsler subspaces was not embraced by many people. This happened because of the tremendous computation which was involved with this theory. Since 1934, when M. H a i m o v i c i wrote the first paper on hypersurfaces in Finsler spaces we have few papers published on this matter. However, important results were obtained by G. M. B r o w n [4], [5], I. C o m i c [6], [7], E. T. D a v i e s [8], M. H a i m o v i c i [9], [10], H. H o m b u [12], S. K i k u c h i [13], M. M a t s u m o t o [14], R. M i r o n [18], R. M i r o n-A. B e j a n c u [20], A. R a p c s á k [22], H. R u n d [23], [24], O. V a r g a [25], etc. But we do not have a complete theory of geometry of Finsler subspaces in a Finsler space.

It is the purpose of these series to give a new method for studying geometry of Finsler subspaces of arbitrary codimension in a Finsler space and to obtain a complete theory for this topic. Our study is mainly based on an arbitrary Finsler connection in the ambient Finsler space and on the induced Finsler connections on various vector bundles.

In the present paper we introduce in §1 the concept of vectorial Finsler connection on a vector bundle over the tangent bundle to a manifold. We show that in fact, each classical Finsler connection on a Finsler space is a vectorial Finsler connection on a certain vector bundle. Then in §2 we define the Finsler normal bundle to a Finsler subspace and obtain a local field of Finsler frames on the ambient space along the Finsler subspace. The induced Finsler connection on a Finsler subspace and the induced vectorial Finsler connection on a certain vector bundle are constructed in §3. It is important to note that a non-linear connection on the ambient space induces a natural non-linear connection on the subspace (see Theorem 3.1). Also, we obtain here Gauss' formulas for a Finsler subspace. The §4 is dedicated to the Finsler normal connection on the Finsler normal bundle. We get Weingarten's formulas for a Finsler subspace and finally, in §5 we obtain all the structure equations of Gauss, Codazzi and Ricci for a Finsler subspace.

The author would like to express his hearty thanks to Professor Makoto Matsumoto for his patience to read the first version of the paper and for his valuable suggestions on the matter.

§1. Vectorial Finsler connections. Let M be a differentiable manifold of real dimension n. Denote by TM the tangent bundle to M and by π

the canonical projection of TM to M. Also, denote by $d\pi : TTM \to TM$ the differential of π and define the vertical subbundle VTM of TTM as the kernel of $d\pi$. A complementary distribution N to VTM in TTM is called a *non-linear connection* on TM (see M a t s u m o t o [15]). The fibres of both vector bundles VTM and N are of the same dimension n. In this section we have the following range of indices: $\alpha, \beta, \gamma, \ldots = 1, \ldots, n$ and $a, b, c, \ldots = 1, \ldots, p$.

Now, we consider a canonical chart (U, φ, R^{2n}) on TM, where U is an open set of TM and $\varphi : U \to R^{2n}$ is a diffeomorphism of U on $\varphi(U)$. For each $z \in U$ denote by $(u^1, \ldots, u^n ; v^1, \ldots, v^n)$ its coordinates in this chart. Then the non-linear connection N is determined by n^2 differentiable functions N^β_α on each U satisfying

$$N^{\beta'}_{\alpha'} = \frac{\partial u^{\beta'}}{\partial u^\beta}\left(\frac{\partial u^\alpha}{\partial u^{\alpha'}} N^\beta_\alpha + \frac{\partial^2 u^\beta}{\partial u^{\alpha'} \partial u^{\gamma'}} v^{\gamma'}\right), \tag{1.1}$$

where $N^{\beta'}_{\alpha'}$ are the corresponding functions on the domain U' of another local chart (U', φ', R^{2n}) with $U \cap U' \neq \varnothing$. Moreover, if $\{\partial/\partial u^\alpha, \partial/\partial v^\alpha\}$ is the natural local field of frames on TM then $\{\delta/\delta u^\alpha\}$ given by

$$\frac{\delta}{\delta u^\alpha} = \frac{\partial}{\partial u^\alpha} - N^\beta_\alpha \frac{\partial}{\partial v^\beta}, \quad \alpha = 1, \ldots, n, \tag{1.2}$$

is a local field of frames on the distribution N.

Next, let $\pi_E : E \to TM$ be a vector bundle over TM with standard fibre R^p. Denote by $\Gamma(E)$ and $\Gamma(TTM)$ the module of differentiable sections of E and respectively TTM. Suppose $\nabla : \Gamma(TTM) \times \Gamma(E) \to \Gamma(E)$ is a linear connection on E, that is, we have

$$\begin{cases} \nabla_X(fV + W) = (Xf)\, V + f\nabla_X V + \nabla_X W, \\ \nabla_{fX+Y}(V) = f\nabla_X V + \nabla_Y V, \end{cases} \tag{1.3}$$

for any differentiable function f on TM, $X, Y \in \Gamma(TTM)$ and $V, W \in \Gamma(E)$.

The pair $VFC = (N, \nabla)$ of a non-linear connection on TM and a linear connection on a vector bundle E over TM is called a *vectorial Finsler connection* on E.

Taking into account that for many cases in Finslerian geometry the computations are locally made we need to express locally a vectorial Finsler connection. In order to do this we consider a vectorial chart (U, Φ, R^p) on the vector bundle E, where U is an open set of TM and $\Phi : \pi_E^{-1}(U) \to U \times R^p$ is a diffeomorphism. Let (U', Φ', R^p) be another vectorial chart such that $U \cap U' \neq \varnothing$. Denote by $\{S_a\}$ (resp. $\{S_{a'}\}$) the local basis of differentiable sections of E on U (resp. U'). Then there exist p^2 differentiable functions $G^a_{a'}$ defined on $U \cap U'$ and satisfying

$$S_{a'} = G^a_{a'}\, S_a. \tag{1.4}$$

Now, suppose (N, ∇) be a vectorial Finsler connection on the vector bundle E. Then since N is a non-linear connection on TM there exist n^2 differentiable functions N^β_α on U satisfying (1.1). On the other hand, taking into account that $\{\delta/\delta u^\alpha, \partial/\partial v^\alpha\}$ is a local field of frames on TM, we define on U the differentiable functions $F_a{}^b{}_\alpha$ and $C_a{}^b{}_\alpha$ by

$$\nabla_{\frac{\delta}{\delta u^\alpha}} S_a = F_a{}^b{}_\alpha S_b; \qquad \nabla_{\frac{\partial}{\partial v^\alpha}} S_a = C_a{}^b{}_\alpha S_b. \tag{1.5}$$

Thus (N, ∇) induces a triplet of differentiable functions $(N^\beta_\alpha, F_a{}^b{}_\alpha, C_a{}^b{}_\alpha)$ on each domain of a canonical chart of TM. By a similar computation with that one for linear connections we obtain that the existence of a vectorial Finsler connection (N, ∇) on E is equivalent with the existence of a triplet of differentiable functions $(N^\beta_\alpha, F_a{}^b{}_\alpha, C_a{}^b{}_\alpha)$ on each domain U of TM satisfying (1.1) and

$$\begin{cases} F_{a'}{}^{b'}{}_{\alpha'} = \dfrac{\partial u^\alpha}{\partial u^{\alpha'}} \bar{G}^{b'}_b \left(G^a_{a'} F_a{}^b{}_\alpha + \dfrac{\delta G^b_{a'}}{\delta u^\alpha}\right), \\[2ex] C_{a'}{}^{b'}{}_{\alpha'} = \dfrac{\partial u^\alpha}{\partial u^{\alpha'}} \bar{G}^{b'}_b \left(G^a_{a'} C_a{}^b{}_\alpha + \dfrac{\partial G^b_{a'}}{\partial v^\alpha}\right), \end{cases} \tag{1.6}$$

for any change of both, the canonical chart and the vectorial chart on TM and respectively E, where $[\bar{G}^{b'}_b]$ is the inverse matrix of $[G^b_{b'}]$.

Now, let $VFC = (N, \nabla)$ be a vectorial Finsler connection on E and R^* be the curvature tensor field of ∇ given by

$$R^*(X, Y)\, V = \nabla_X \nabla_Y V - \nabla_Y \nabla_X V - \nabla_{[X,Y]} V, \tag{1.7}$$

for any $X, Y \in \Gamma(TTM)$ and $V \in \Gamma(E)$. Then the local components $R_a{}^b{}_{\alpha\beta}$, $P_a{}^b{}_{\alpha\beta}$ and $S_a{}^b{}_{\alpha\beta}$ given by

$$\begin{aligned} &R^*\left(\frac{\delta}{\delta u^\beta}, \frac{\delta}{\delta u^\alpha}\right) S_a = R_a{}^b{}_{\alpha\beta} S_b; \qquad R^*\left(\frac{\partial}{\partial v^\beta}, \frac{\delta}{\delta u^\alpha}\right) S_a = P_a{}^b{}_{\alpha\beta} S_b, \\ &R^*\left(\frac{\partial}{\partial v^\beta}, \frac{\partial}{\partial v^\alpha}\right) S_a = S_a{}^b{}_{\alpha\beta} S_b \end{aligned} \tag{1.8}$$

are called the *Finsler curvature tensors* of the vectorial Finsler connection VFC. By a direct computation using (1.5), (1.7) and (1.8) we obtain

$$\begin{aligned} R_a{}^b{}_{\alpha\beta} &= \frac{\delta F_a{}^b{}_\alpha}{\delta u^\beta} - \frac{\delta F_a{}^b{}_\beta}{\delta u^\alpha} + F_a{}^c{}_\alpha F_c{}^b{}_\beta - F_a{}^c{}_\beta F_c{}^b{}_\alpha + C_a{}^b{}_\gamma R^\gamma{}_{\alpha\beta}, \\ P_a{}^b{}_{\alpha\beta} &= \frac{\partial F_a{}^b{}_\alpha}{\partial v^\beta} - \frac{\delta C_a{}^b{}_\beta}{\delta u^\alpha} + F_a{}^c{}_\alpha C_c{}^b{}_\beta - C_a{}^c{}_\beta F_c{}^b{}_\alpha + \frac{\partial N^\gamma_\alpha}{\partial v^\beta} C_a{}^b{}_\gamma, \\ S_a{}^b{}_{\alpha\beta} &= \frac{\partial C_a{}^b{}_\alpha}{\partial v^\beta} - \frac{\partial C_a{}^b{}_\beta}{\partial v^\alpha} + C_a{}^c{}_\alpha C_c{}^b{}_\beta - C_a{}^c{}_\beta C_c{}^b{}_\alpha, \end{aligned} \tag{1.9}$$

where $R^\gamma{}_{\alpha\beta}$ is given by

$$R^\gamma{}_{\alpha\beta} = \frac{\delta N^\gamma_\alpha}{\delta u^\beta} - \frac{\delta N^\gamma_\beta}{\delta u^\alpha}. \tag{1.10}$$

Next, we show that each Finsler connection on M is a vectorial Finsler connection on VTM. In fact, a Finsler connection on M is given by triplets $(N^\beta_\alpha, F_\alpha{}^\gamma{}_\beta, C_\alpha{}^\gamma{}_\beta)$ defined on each $U \subset TM$ and satisfying (1.1) and

$$
(1.11)\qquad \begin{cases} F_{\alpha'}{}^{\gamma'}{}_{\beta'} = \dfrac{\partial u^{\gamma'}}{\partial u^{\gamma}}\left(\dfrac{\partial u^{\alpha}}{\partial u^{\alpha'}} F_{\alpha}{}^{\gamma}{}_{\beta} \dfrac{\partial u^{\beta}}{\partial u^{\beta'}} + \dfrac{\partial^2 u^{\gamma}}{\partial u^{\alpha'}\partial u^{\beta'}}\right). \\ C_{\alpha'}{}^{\gamma'}{}_{\beta'} = \dfrac{\partial u^{\gamma'}}{\partial u^{\gamma}} \dfrac{\partial u^{\alpha}}{\partial u^{\alpha'}} \dfrac{\partial u^{\beta}}{\partial u^{\beta'}} C_{\alpha}{}^{\gamma}{}_{\beta}. \end{cases}
$$

On the other hand, in this case a local basis in VTM is just $\{\partial/\partial v^1, \ldots, \partial/\partial v^n\}$ Hence we have

$$
(1.12)\qquad G^{a}_{a'} = G^{\alpha}_{\alpha'} = \frac{\partial u^{\alpha}}{\partial u^{\alpha'}}.
$$

Thus taking account of (1.11) and (1.12) we obtain (1.6). Also, each vectorial Finsler connection on VTM is a Finsler connection on M. In fact, in this case, from (1.6), by using (1.12) we get (1.11). We have to remark that such a passing from Finsler connections on M to linear connections on VTM was also studied by Oproiu [21].

We finish this section with some remarks on covariant derivatives induced by vectorial Finsler connections. Suppose $VFC=(N^{\beta}{}_{\alpha}, F_a{}^b{}_{\alpha}, C_a{}^b{}_{\alpha})$ is a vectorial Finsler connection on E and $W=W^a S_a$ is a differentiable section on E. Then the *(h)-covariant derivative* and the *(v)-covariant derivative* of W are given by

$$
(1.13)\qquad W^a{}_{|\alpha} = \frac{\delta W^a}{\delta u^{\alpha}} + F_b{}^a{}_{\alpha} W^b \quad \text{and respectively}
$$

$$
(1.14)\qquad W^a|_{\alpha} = \frac{\partial W^a}{\partial v^{\alpha}} + C_b{}^a{}_{\alpha} W^b.
$$

For a differentiable functions f on TM we define

$$
(1.15)\qquad f_{|\alpha} = \frac{\delta f}{\delta u^{\alpha}}; \quad f|_{\alpha} = \frac{\partial f}{\partial v^{\alpha}}.
$$

Now suppose there are defined $(np)^2$ differentiable functions $T^{\alpha a}_{\beta b}$ on the domain U of each canonical chart (U, φ, R^{2n}) of TM. We say that $T^{\alpha a}_{\beta b}$ define a *mixed Finsler tensor field* on M with respect to the vector bundle E if for a change of both the canonical chart on TM and the vectorial chart on E we have

$$
(1.16)\qquad T^{\alpha' a'}_{\beta' b'} = \frac{\partial u^{\alpha'}}{\partial u^{\alpha}} \frac{\partial u^{\beta}}{\partial u^{\beta'}} G^{a'}_{a} G^{b}_{b'} T^{\alpha a}_{\beta b}.
$$

Now, we consider a Finsler connection $FC=(N^{\beta}_{\alpha}, F_{\alpha}{}^{\beta}{}_{\gamma}, C_{\alpha}{}^{\beta}{}_{\gamma})$ on M and a vectorial Finsler connection $VFC=(N^{\beta}{}_{\alpha}, F_a{}^b{}_{\alpha}, C_a{}^b{}_{\alpha})$ on the vector bundle E. We have to remark that these Finsler connections have the same non linear connection $N=(N^{\beta}_{\alpha})$. Then we define the *(h)-covariant derivative* of $T^{\alpha a}_{\beta b}$ by

$$
(1.17)\qquad T^{\alpha a}_{\beta b|\gamma} = \frac{\delta T^{\alpha a}_{\beta b}}{\delta u^{\gamma}} + T^{\varphi a}_{\beta b} F_{\varphi}{}^{\alpha}{}_{\gamma} - T^{\alpha a}_{\varphi b} F_{\beta}{}^{\varphi}{}_{\gamma} + T^{\alpha c}_{\beta b} F_c{}^a{}_{\gamma} - T^{\alpha a}_{\beta c} F_b{}^c{}_{\gamma}
$$

and the *(v)-covariant derivative* of $T^{\alpha a}_{\beta b}$ by

$$
(1.18)\qquad T^{\alpha a}_{\beta b}|_{\gamma} = \frac{\partial T^{\alpha a}_{\beta b}}{\partial v^{\gamma}} + T^{\varphi a}_{\beta b} C_{\varphi}{}^{\alpha}{}_{\gamma} - T^{\alpha a}_{\varphi b} C_{\beta}{}^{\varphi}{}_{\gamma} + T^{\alpha c}_{\beta b} C_c{}^a{}_{\gamma} - T^{\alpha a}_{\beta c} C_b{}^c{}_{\gamma}.
$$

Thus each pair (VFC, FC) of a vectorial Finsler connection on E and a Finsler connection on M defines a *relative differentiation* given by (1.17) and (1.18). Moreover, by a direct computation, using (1.13), (1.14), (1.17) and (1.18) we obtain the following commutation formulas

$$
(1.19)\qquad \begin{cases} W^a{}_{|\alpha|\beta} - W^a{}_{|\beta|\alpha} = R_b{}^a{}_{\alpha\beta} W^b - R^{\varphi}{}_{\alpha\beta} W^a|_{\varphi} - T_{\alpha}{}^{\varphi}{}_{\beta} W^a{}_{|\varphi}, \\ W^a{}_{|\alpha}|_{\beta} - W^a|_{\beta|\alpha} = P_b{}^a{}_{\alpha\beta} W^b - P^{\varphi}{}_{\alpha\beta} W^a|_{\varphi} - C_{\alpha}{}^{\varphi}{}_{\beta} W^a{}_{|\varphi}, \\ W^a|_{\alpha}|_{\beta} - W^a|_{\beta}|_{\alpha} = S_b{}^a{}_{\alpha\beta} W^b - S^{\varphi}{}_{\alpha\beta} W^a|_{\varphi}, \end{cases}
$$

where $T_{\alpha}{}^{\varphi}{}_{\beta}$, $P^{\varphi}{}_{\alpha\beta}$ and $S^{\varphi}_{\alpha\beta}$ are three from the torsion tensors of FC and they are given by

$$
(1.20)\qquad \begin{cases} T_{\alpha}{}^{\varphi}{}_{\beta} = F_{\alpha}{}^{\varphi}{}_{\beta} - F_{\beta}{}^{\varphi}{}_{\alpha}, \quad P^{\varphi}{}_{\alpha\beta} = \dfrac{\partial N^{\varphi}{}_{\alpha}}{\partial v^{\beta}} - F_{\beta}{}^{\varphi}{}_{\alpha}, \\ S^{\varphi}_{\alpha\beta} = C_{\alpha}{}^{\varphi}{}_{\beta} - C_{\beta}{}^{\varphi}{}_{\alpha}. \end{cases}
$$

§2. The Finsler normal bundle to a Finsler subspace. Let $F^m = (\tilde{M}, \tilde{L}(x, y))$ be a Finsler space, where $\tilde{M}$ is a real m-dimensional differentiable manifold and $\tilde{L}(x, y)$ is the fundamental function of F^m (see Matsumoto [15]). Denote by $g_{ij} = (1/2)(\partial^2 L^2/\partial y^i \partial y^j)$ the fundamental tensor field of $\tilde{M}$. In the present paper we only suppose $g_{ij}(x, y)$ be positive definite. Thus our theory holds good also for subspaces of generalized Finsler spaces (see Miron [19], Hashiguchi [11], Watanabe-Ikeda [26]).

Next, let M be an n-dimensional submanifold of $\tilde{M}$ locally given by equations

$$
(2.1)\qquad x^i = x^i(u^1, \ldots, u^n); \quad \text{rank}\left[\frac{\partial x^i}{\partial u^{\alpha}}\right] = n.
$$

In this section and in the sequel we use the following range of indices: $i, j, k, \ldots = 1, \ldots, m$; $\alpha, \beta, \gamma, \ldots = 1, \ldots, n$; $a, b, c, \ldots = n+1, \ldots, m$.

The immersion i of M in $\tilde{M}$ induces an immersion di of TM in $T\tilde{M}$. Thus locally, if (u, v) of TM is carried by di in $(x^i(u), y^i(u, v))$ of $T\tilde{M}$ we have

$$
(2.2)\qquad y^i = B^i_{\alpha} v^{\alpha} \quad \text{where} \quad B^i_{\alpha} = \frac{\partial x^i}{\partial u^{\alpha}}.
$$

Since (y^i) is the supporting element of $\tilde{M}$ we call (v^{α}) the supporting element of M. We denote

$$
(2.3)\qquad B^i_{\alpha\beta} = \frac{\partial^2 x^i}{\partial u^{\alpha}\partial u^{\beta}}; \quad B^i_{0\beta} = B^i_{\alpha\beta} v^{\alpha}.
$$

The natural local field of frames $\{\partial/\partial u^{\alpha}, \partial/\partial v^{\alpha}\}$ and $\{\partial/\partial x^i, \partial/\partial y^i\}$ on TM and respectively $T\tilde{M}$ are related by

$$
(2.4)\qquad \begin{cases} \dfrac{\partial}{\partial u^\alpha} = B^i_\alpha \dfrac{\partial}{\partial x^i} + B^i_{0\alpha}\dfrac{\partial}{\partial y^i}\,. \\[2mm] \dfrac{\partial}{\partial v^\alpha} = B^i_\alpha \dfrac{\partial}{\partial y^i}\,. \end{cases}
$$

Next, the fundamental function $\tilde{L}$ on F^m induces a fundamental function L on M by

$$
(2.5)\qquad L(u, v) = \tilde{L}(x(u),\ y(u, v)),
$$

and further we have a fundamental tensor field $g_{\alpha\beta}(u, v)$ on M given by

$$
(2.6)\qquad g_{\alpha\beta}(u, v) = g_{ij}(x(u),\ y(u, v))\, B^i_\alpha B^j_\beta = \frac{1}{2}\,\frac{\partial^2 L^2}{\partial v^\alpha \partial v^\beta}\,.
$$

In this way M inherites a Finsler structure and from now on we shall investigate the geometry of the Finsler space $F^n = (M,\ L(u, v))$ immersed in F^m.

First, it is easy to see that in fact the fundamental tensor $g_{ij}(x, y)$ defines a Riemannian metric g on the vertical subbundle $VT\tilde{M}$ of $TT\tilde{M}$ by

$$
(2.7)\qquad g(X, Y) = g_{ij} X^i Y^j,
$$

for any $X = X^i \partial/\partial y^i$ and $Y = Y^i \partial/\partial y^i$. We denote by $VT\tilde{M}^*$ the restriction of the vertical subbundle $VT\tilde{M}$ to the points of TM. Then by the second line of equations in (2.4) we see that VTM is a vector subbundle of $VT\tilde{M}^*$. The complementary orthogonal subbundle $VTM^\perp$ to VTM in $VT\tilde{M}^*$ with respect to the Riemannian metric g is called the *Finsler normal bundle* to F^n.

Let $\{B_a = B^i_a\, \partial/\partial y^i\}$ be a local field of orthonormal frames in the Finsler normal bundle $VTM^\perp$. Thus we have

$$
(2.8)\qquad g_{ij}(x(u),\ y(u, v))\, B^i_a(u, v) B^j_b(u, v) = \delta_{ab}.
$$

On the other hand, since $VTM^\perp$ is orthogonal to VTM, by the second line of equations (2.4) we get

$$
(2.9)\qquad g_{ij}(x(u),\ y(u, v))\, B^i_\alpha(u)\, B^j_a(u, v) = 0.
$$

The local basis $\{\partial/\partial v^\alpha = B^i_\alpha\, \partial/\partial y^i,\ B_a = B^i_a\, \partial/\partial y^i\}$ in $VT\tilde{M}^*$ is called *the Finsler field of frames on F^m along to F^n*. We denote by $[\tilde{B}^\alpha_i\ \tilde{B}^a_j]$ the inverse matrix of $[B^i_\alpha\ B^j_a]$. Thus we have

$$
(2.10)\qquad \begin{cases} \tilde{B}^\alpha_i B^i_\beta = \delta^\alpha_\beta\,; \quad \tilde{B}^\alpha_i B^i_a = 0\,; \quad \tilde{B}^a_i B^i_\alpha = 0\,; \quad \tilde{B}^a_i \tilde{B}^i_b = \delta^a_b\,; \\ B^i_\alpha \tilde{B}^\alpha_j + B^i_a \tilde{B}^a_j = \delta^i_j\,. \end{cases}
$$

Then by (2.6), (2.9) and (2.10) we get

$$
(2.11)\qquad g_{\alpha\beta}\, \tilde{B}^\alpha_i = g_{ij} B^j_\beta\,.
$$

§3. The induced Finsler connections by a Finsler connection on the ambient space. Let $F^n = (M, g_{\alpha\beta})$ be a Finsler subspace of $F^m = (\tilde{M}, g_{ij})$. Suppose $FC = (\tilde{N}, \tilde{\nabla}) = (N^i_j,\ F^i_{jk},\ C^i_{jk})$ is a Finsler connection on F^m, where $\tilde{N}$ is a non-linear connection on $T\tilde{M}$ given locally by N^i_j and $\tilde{\nabla}$ is a linear connection on $VT\tilde{M}$ given locally by F^i_{jk} and C^i_{jk}.

First we prove

Theorem 3.1. *Let F^n be a Finsler subspace of a Finsler space F^m endowed with a Finsler connection $FC = (\tilde{N}, \tilde{\nabla})$. Then there exists a unique non-linear conection N on TM satisfying*

$$
(3.1)\qquad N_z \subset VTM^\perp_z \oplus \tilde{N}^*_z \quad (\text{direct sum}), \quad \forall z \in TM,
$$

where $\tilde{N}^*$ is the restriction of $\tilde{N}$ to TM.

Proof. We define locally the differentiable functions

$$
(3.2)\qquad N^\beta_\alpha = \tilde{B}^\beta_i (B^i_{0\alpha} + N^i_j B^j_\alpha),
$$

and it is easy to check (1.1). Thus we have a non-linear connection N on TM defined locally by N^β_α. Next, from the second line of equation in (2.4) we get

$$
(3.3)\qquad \frac{\partial}{\partial y^i} = \tilde{B}^\alpha_i \frac{\partial}{\partial v^\alpha} + \tilde{B}^a_i B_a.
$$

The distribution $\tilde{N}$ on $T\tilde{M}$ is spanned locally by $\dfrac{\delta}{\delta x^i}$ given by

$$
(3.4)\qquad \frac{\delta}{\delta x^i} = \frac{\partial}{\partial x^i} - N^j_i \frac{\partial}{\partial y^j}\,.
$$

Then by using (1.2), (1.4), (3.2) — (3.4) we obtain

$$
(3.5)\qquad \frac{\delta}{\delta u^\alpha} = B^i_\alpha \frac{\delta}{\delta x^i} + H^a_\alpha B_a\,,
$$

where H^a_α are given by

$$
(3.6)\qquad H^a_\alpha = \tilde{B}^a_j (B^i_\alpha N^j_i + B^j_{0\alpha}).
$$

Thus by (3.5) we have the existence of N satisfying (3.1).

Now suppose $\bar{N} = (\bar{N}^\beta_\alpha)$ is another non-linear connection on TM satisfying (3.1), that is, (3.5) with H^a_α as arbitrary functions. Then by using (1.2) for $\bar{N}^\beta_\alpha$, (2.4) and (3.4) we have

$$
(3.7)\qquad \frac{\delta}{\delta u^\alpha} = B^i_\alpha \frac{\delta}{\delta x^i} + (N^j_i B^i_\alpha + B^j_{0\alpha} - \bar{N}^\beta_\alpha B^j_\beta)\frac{\partial}{\partial y^j}\,.
$$

Thus from (3.5) and (3.7) we obtain

$$
(3.8)\qquad H^a_\alpha B^j_a = N^j_i B^i_\alpha + B^j_{0\alpha} - \bar{N}^\beta_\alpha B^j_\beta.
$$

Finally, by contracting (3.8) with $\tilde{B}^{\gamma}_{j}$ we obtain $\tilde{N}^{\beta}_{\alpha}=N^{\beta}_{\alpha}$ given by (3.2) and by contracting (3.8) with $\tilde{B}^{b}_{j}$ we get (3.6). Therefore, we have the uniqueness of the non-linear connection given by (3.2).

The non-linear connection N on TM given by (3.2) is called *the induced non-linear connection* by $\tilde{N}$ on $T\tilde{M}$.

Further, the linear connection $\tilde{\nabla}=(F_{j}{}^{i}{}_{k},\ C_{j}{}^{i}{}_{k})$ on $VT\tilde{M}$ induces a linear connection $\overset{*}{\nabla}$ on $VT\tilde{M}^{*}$ given by

$$\overset{*}{\nabla}_{\frac{\delta}{\delta u^{\alpha}}}\frac{\partial}{\partial y^{i}}=\tilde{\nabla}_{\frac{\delta}{\delta u^{\alpha}}}\frac{\partial}{\partial y^{i}}=F_{i}{}^{j}{}_{\alpha}\frac{\partial}{\partial y^{j}} \quad \text{and} \tag{3.9}$$

$$\overset{*}{\nabla}_{\frac{\partial}{\partial v^{\alpha}}}\frac{\partial}{\partial y^{i}}=\tilde{\nabla}_{\frac{\partial}{\partial v^{\alpha}}}\frac{\partial}{\partial y^{i}}=C_{i}{}^{j}{}_{\alpha}\frac{\partial}{\partial y^{j}}, \tag{3.10}$$

where $\dfrac{\delta}{\delta u^{\alpha}}$ are given by (1.2) with N^{β}_{α} from (3.2). Therefore we have a vectorial Finsler connection $IVFC=(N^{\beta}_{\alpha},\ F_{i}{}^{j}{}_{\alpha},\ C_{i}{}^{j}{}_{\alpha})$ on $VT\tilde{M}^{*}$ which we call *the induced vectorial Finsler connection*. Of course, it is important for us to find the coefficients $F_{i}{}^{j}{}_{\alpha}$ and $C_{i}{}^{j}{}_{\alpha}$ by means of the elements of the Finsler connection on F^{m}. Thus we prove

Theorem 3.2. *Let F^{n} be a Finsler subspace of a Finsler space F^{m} endowed with a Finsler connection $FC=(\tilde{N},\tilde{\nabla})$. Then there exists a vectorial Finsler connection $IVFC=(N^{\beta}_{\alpha},\ F_{i}{}^{j}{}_{\alpha},\ C_{i}{}^{j}{}_{\alpha})$ on the vector bundle $VT\tilde{M}^{*}$, where N^{β}_{α} are given by (3.2) and $F_{i}{}^{j}{}_{\alpha}$, $C_{i}{}^{j}{}_{\alpha}$ are given by*

$$F_{i}{}^{j}{}_{\alpha}=F_{i}{}^{j}{}_{k}B^{k}_{\alpha}+C_{i}{}^{j}{}_{k}B^{k}_{a}H^{a}_{\alpha}, \tag{3.11}$$

$$C_{i}{}^{j}{}_{\alpha}=C_{i}{}^{j}{}_{k}B^{k}_{\alpha}. \tag{3.12}$$

Proof. First, we see that the linear connection $\tilde{\nabla}$ on $VT\tilde{M}$ is given by

$$\tilde{\nabla}_{\frac{\delta}{\delta x^{i}}}\frac{\partial}{\partial y^{j}}=F_{j}{}^{k}{}_{i}\frac{\partial}{\partial y^{k}} \quad \text{and} \tag{3.13}$$

$$\tilde{\nabla}_{\frac{\partial}{\partial y^{i}}}\frac{\partial}{\partial y^{j}}=C_{j}{}^{k}{}_{i}\frac{\partial}{\partial y^{k}}. \tag{3.14}$$

Then by using (3.5), (3.13) and (3.14) we get

$$\tilde{\nabla}_{\frac{\delta}{\delta u^{\alpha}}}\frac{\partial}{\partial y^{i}}=(B^{k}_{\alpha}F_{i}{}^{j}{}_{k}+H^{a}_{\alpha}B^{k}_{a}C_{i}{}^{j}{}_{k})\frac{\partial}{\partial y^{j}}. \tag{3.15}$$

Hence (3.11) follows from (3.9) and (3.15). In a similar way, by using the second line of equations (2.4) and (3.14) we infer

$$\tilde{\nabla}_{\frac{\partial}{\partial v^{\alpha}}}\frac{\partial}{\partial y^{i}}=B^{k}_{\alpha}C_{i}{}^{j}{}_{k}\frac{\partial}{\partial y^{j}}. \tag{3.16}$$

Thus, (3.12) follows from (3.16) and (3.10).

By the definition of the Finsler normal bundle we have

$$VT\tilde{M}^{*}=VTM\oplus VTM^{\perp}. \tag{3.17}$$

Then we put

$$\overset{*}{\nabla}_{\frac{\delta}{\delta u^{\alpha}}}\frac{\partial}{\partial v^{\beta}}=F_{\beta}{}^{\gamma}{}_{\alpha}\frac{\partial}{\partial v^{\gamma}}+H^{a}{}_{\beta\alpha}B_{a} \quad \text{and} \tag{3.18}$$

$$\overset{*}{\nabla}_{\frac{\partial}{\partial v^{\alpha}}}\frac{\partial}{\partial v^{\beta}}=C_{\beta}{}^{\gamma}{}_{\alpha}\frac{\partial}{\partial v^{\gamma}}+K^{a}{}_{\beta\alpha}B_{a}. \tag{3.19}$$

Thus we have a linear connection ∇ on VTM defined by

$$\nabla_{\frac{\delta}{\delta u^{\alpha}}}\frac{\partial}{\partial v^{\beta}}=F_{\beta}{}^{\gamma}{}_{\alpha}\frac{\partial}{\partial v^{\gamma}} \quad \text{and} \tag{3.20}$$

$$\nabla_{\frac{\partial}{\partial v^{\alpha}}}\frac{\partial}{\partial v^{\beta}}=C_{\beta}{}^{\gamma}{}_{\alpha}\frac{\partial}{\partial v^{\gamma}}. \tag{3.21}$$

Next we state

Theorem 3.3. *Let F^{n} be a Finsler subspace of a Finsler space F^{m} endowed with a Finsler connection $FC=(\tilde{N},\tilde{\nabla})$. Then there exists a Finsler connection $IFC=(N,\nabla)=(N^{\beta}_{\alpha},\ F_{\alpha}{}^{\beta}{}_{\gamma},\ C_{\alpha}{}^{\beta}{}_{\gamma})$ on M where N^{β}_{α} are given by (3.2) and $F_{\alpha}{}^{\beta}{}_{\gamma}$, $C_{\alpha}{}^{\beta}{}_{\gamma}$ are given by*

$$F_{\alpha}{}^{\beta}{}_{\gamma}=\tilde{B}^{\beta}_{j}(B^{j}_{\alpha\gamma}+F_{i}{}^{j}{}_{\gamma}B^{i}_{\alpha}), \text{ respectively} \tag{3.22}$$

$$C_{\alpha}{}^{\beta}{}_{\gamma}=\tilde{B}^{\beta}_{j}C_{i}{}^{j}{}_{\gamma}B^{i}_{\alpha}. \tag{3.23}$$

Proof. Certainly, $IFC=(N,\nabla)$ is a Finsler connection on M since it is a vectorial Finsler connection on VTM. Moreover, by using (3.9) and (3.3) we get

$$\begin{aligned}\overset{*}{\nabla}_{\frac{\delta}{\delta u^{\alpha}}}\frac{\partial}{\partial v^{\beta}}&=\overset{*}{\nabla}_{\frac{\delta}{\delta u^{\alpha}}}\left(B^{i}_{\beta}\frac{\partial}{\partial y^{i}}\right)=(B^{j}_{\alpha\beta}+F_{i}{}^{j}{}_{\alpha}B^{i}_{\beta})\frac{\partial}{\partial y^{j}}=\\&=\tilde{B}^{\gamma}_{j}(B^{j}_{\alpha\beta}+F_{i}{}^{j}{}_{\alpha}B^{i}_{\beta})\frac{\partial}{\partial v^{\gamma}}+\tilde{B}^{a}_{j}(B^{j}_{\alpha\beta}+F_{i}{}^{j}{}_{\alpha}B^{i}_{\beta})B_{a}.\end{aligned} \tag{3.24}$$

By comparing (3.24) with (3.18) we get (3.22) and

$$H^{a}{}_{\beta\alpha}=\tilde{B}^{a}_{j}(B^{j}_{\alpha\beta}+F_{i}{}^{j}{}_{\alpha}B^{i}_{\beta}). \tag{3.25}$$

In a similar way, by using (3.10) and (3.3) we infer

$$\overset{*}{\nabla}_{\frac{\partial}{\partial v^{\alpha}}}\frac{\partial}{\partial v^{\beta}}=\overset{*}{\nabla}_{\frac{\partial}{\partial v^{\alpha}}}\left(B^{j}_{\beta}\frac{\partial}{\partial y^{j}}\right)=B^{j}_{\beta}C_{j}{}^{k}{}_{\alpha}\frac{\partial}{\partial y^{k}}=B^{j}_{\beta}C_{j}{}^{k}{}_{\alpha}\tilde{B}^{\gamma}_{k}\frac{\partial}{\partial v^{\gamma}}+B^{j}_{\beta}C_{j}{}^{k}{}_{\alpha}\tilde{B}^{a}_{k}B_{a}. \tag{3.26}$$

Hence, by comparing (3.26) with (3.19) we get (3.23) and

$$K^{a}{}_{\beta\alpha}=B^{j}_{\beta}C_{j}{}^{k}{}_{\alpha}\tilde{B}^{a}_{k}. \tag{3.27}$$

The Finsler connection $IFC=(N^{\beta}_{\alpha},\ F_{\alpha}{}^{\beta}{}_{\gamma},\ C_{\alpha}{}^{\beta}{}_{\gamma})$ on M is called *the induced Finsler connection* on F^{n}. The tensor fields $H^{a}{}_{\alpha\beta}$ and $K^{a}{}_{\alpha\beta}$ $(a=n+1,\ldots,m)$, given by (3.25) and (3.27) are called the *(h)-second fundamental forms* and respectively the *(v)-second fundamental forms* of the Finsler subspace F^{n}. Also, according to the theory of Riemannian subspaces we call (3.18) and (3.19) *Gauss' formulas* for the immersion of F^{n} in F^{m}.

Remark 3.1. If in particular $n=m-1$, then our induced Finsler connection is just that one obtained by Matsumoto in [14] for hypersurfaces in Finsler spaces.

Thus summing up our results, the Finsler connection $FC=(\tilde{N}, \tilde{\nabla})=(N^i_j, F_j{}^i{}_k, C_j{}^i{}_k)$ on the ambient Finsler space F^m induces in a natural way a vectorial Finsler connection $IVFC=(N, \overset{*}{\nabla})=(N^\beta_\alpha, F_i{}^j{}_\alpha, C_i{}^j{}_\alpha)$ on $VT\tilde{M}^*$ and a Finsler connection $IFC=(N, \nabla)=(N^\beta_\alpha, F_\alpha{}^\beta{}_\gamma, C_\alpha{}^\beta{}_\gamma)$ on F^n. Thus by the theory from §1 we have a relative differentiation of tensor fields $T^{\alpha i}_{\beta j}$ given by

$$(3.28)\qquad T^{\alpha i}_{\beta j|\gamma}=\frac{\delta T^{\alpha i}_{\beta j}}{\delta u^\gamma}+T^{\varphi i}_{\beta j}F_\varphi{}^\alpha{}_\gamma-T^{\alpha i}_{\varphi j}F_\beta{}^\varphi{}_\gamma+T^{\alpha k}_{\beta j}F_k{}^i{}_\gamma-T^{\alpha i}_{\beta k}F_j{}^k{}_\gamma$$

$$(3.29)\qquad T^{\alpha i}_{\beta j}|_\gamma=\frac{\partial T^{\alpha i}_{\beta j}}{\partial v^\gamma}+T^{\varphi i}_{\beta j}C_\varphi{}^\alpha{}_\gamma-T^{\alpha i}_{\varphi j}C_\beta{}^\varphi{}_\gamma+T^{\alpha k}_{\beta j}C_k{}^i{}_\gamma-T^{\alpha i}_{\beta k}C_j{}^k{}_\gamma .$$

As an application of (3.28)and (3.29), taking account of (3.25), (3.27) and (3.6) we obtain

$$(3.30)\qquad B^i_{\alpha|\beta}=H^a{}_{\alpha\beta}B^i_a\ ;\quad B^i_\alpha|_\beta=K^a{}_{\alpha\beta}B^i_a \quad\text{and}$$

$$(3.31)\qquad \begin{aligned} g_{ij|\alpha}&=g_{ij|k}B^k_\alpha+g_{ij}|_k B^k_a H^a_\alpha ,\\ g_{ij}|_\alpha&=g_{ij}|_k B^k_\alpha .\end{aligned}$$

§4. The Finsler normal connection. Let F^n be a Finsler subspace of a Finsler space F^m endowed with a Finsler connection $FC=(\tilde{N}, \tilde{\nabla})=(N^i_j, C_j{}^i{}_k, C_j{}^i{}_k)$. Taking account that $\overset{*}{\nabla}$ is a linear connection on $VT\tilde{M}^*$ by (3.17) we have

$$(4.1)\qquad \overset{*}{\nabla}_{\frac{\delta}{\delta u^\alpha}} B_a=-\tilde{H}_a{}^\varphi{}_\alpha\frac{\partial}{\partial v^\varphi}+F_a{}^b{}_\alpha B_b \quad\text{and}$$

$$(4.2)\qquad \overset{*}{\nabla}_{\frac{\partial}{\partial v^\alpha}} B_a=-\tilde{K}_a{}^\varphi{}_\alpha\frac{\partial}{\partial v^\varphi}+C_a{}^b{}_\alpha B_b .$$

Thus we have a linear connection $\nabla^\perp$ on $VTM^\perp$ defined by

$$(4.3)\qquad \nabla^\perp_{\frac{\delta}{\delta u^\alpha}} B_a=F_a{}^b{}_\alpha B_b\ ;\qquad \nabla^\perp_{\frac{\partial}{\partial v^\alpha}} B_a=C_a{}^b{}_\alpha B_b .$$

Theorem 4.1. *Let F^n be a Finsler subspace of a Finsler space F^m endowed with a Finsler connection $FC=(\tilde{N}, \tilde{\nabla})$. Then there exists a vectorial Finsler connection $FNC=(N, \nabla^\perp)=(N^\beta_\alpha, F_a{}^b{}_\alpha, C_a{}^b{}_\alpha)$ on the Finsler normal bundle $VTM^\perp$, where N^β_α are given by (3.2) and $F_a{}^b{}_\alpha$, $C_a{}^b{}_\alpha$ are given by*

$$(4.4)\qquad F_a{}^b{}_\alpha=\tilde{B}^b_i\left(\frac{\delta B^i_a}{\delta u^\alpha}+F_j{}^i{}_\alpha B^j_a\right)=\tilde{B}^b_i B^i_{a|\alpha} .$$

$$(4.5)\qquad C_a{}^b{}_\alpha=\tilde{B}^b_i\left(\frac{\partial B^i_a}{\partial v^\alpha}+C_j{}^i{}_\alpha B^j_a\right)=\tilde{B}^b_i B^i_a|_\alpha .$$

Proof. By using (3.3) and (3.9) we get

$$(4.6)\qquad \overset{*}{\nabla}_{\frac{\delta}{\delta u^\alpha}} B_a=\left(\frac{\delta B^i_a}{\delta u^\alpha}+B^j_aF_j{}^i{}_\alpha\right)\tilde{B}^\beta_i\frac{\partial}{\partial v^\beta}+\left(\frac{\delta B^i_a}{\delta u^\alpha}+B^j_aF_j{}^i{}_\alpha\right)\tilde{B}^b_iB_b .$$

Hence by comparing (4.6) and (4.1) we obtain (4.4) and

$$(4.7)\qquad \tilde{H}_a{}^\beta{}_\alpha=-\tilde{B}^\beta_i\left(\frac{\delta B^i_a}{\delta u^\alpha}+B^j_aF_j{}^i{}_\alpha\right)=-\tilde{B}^\beta_iB^i_{a|\alpha} .$$

In a similar way we infer (4.5) and

$$(4.8)\qquad \tilde{K}_a{}^\beta{}_\alpha=-\tilde{B}^\beta_i\left(\frac{\partial B^i_a}{\partial v^\alpha}+B^j_aC_j{}^i{}_\alpha\right)=-\tilde{B}^\beta_iB^i_a|_\alpha .$$

The vectorial Finsler connection $FNC=(N, \nabla^\perp)$ whose coefficients are given by (3.2), (4.4) and (4.5) is called the *Finsler normal connection* on $TM^\perp$. Also, according to the Riemannian case, we call (4.1) and (4.2) the *Weingarten's formulas.*

Next, by differentiating (h)-covariantly (2.9) and taking account of (3.30) we obtain

$$(4.9)\qquad g_{ij|\beta}B^i_aB^j_\alpha+g_{ij}H^b{}_{\alpha\beta}B^i_bB^j_a+g_{ij}B^i_\alpha B^j_{a|\beta}=0.$$

Now we put

$$(4.10)\qquad \begin{cases} H_{a\alpha\beta}=\delta_{ab}H^b{}_{\alpha\beta},\\ K_{a\alpha\beta}=\delta_{ab}K^b{}_{\alpha\beta},\end{cases}\qquad \begin{cases} H_a{}^\beta{}_\alpha=g^{\beta\varphi}H_{a\varphi\alpha},\\ K_a{}^\beta{}_\alpha=g^{\beta\varphi}K_{a\varphi\alpha}.\end{cases}$$

Then by using (2.11) and (4.10) in (4.9) we get

$$(4.11)\qquad g_{ij|\beta}B^i_\alpha B^j_a+H_{a\alpha\beta}+g_{\varphi\alpha}\tilde{B}^\varphi_jB^j_{a|\beta}=0.$$

Hence by (4.7), (4.10) and (4.11) we obtain

$$(4.12)\qquad \tilde{H}_a{}^\varphi{}_\beta=H_a{}^\varphi{}_\beta+g^{\alpha\varphi}g_{ij|\beta}B^i_\alpha B^j_a.$$

In a similar way we obtain

$$(4.13)\qquad \tilde{K}_a{}^\varphi{}_\beta=K_a{}^\varphi{}_\beta+g^{\alpha\varphi}g_{ij}|_\beta B^i_\alpha B^j_a.$$

Finally, taking account of (4.1), (4.3), (4.4), (4.5), (4.7) and (4.8) we obtain

$$(4.14)\qquad B^i_{a|\alpha}=-\tilde{H}_a{}^\varphi{}_\alpha B^i_\varphi+F_a{}^b{}_\alpha B^i_b \quad\text{and}$$

$$(4.15)\qquad B^i_a|_\alpha=-\tilde{K}_a{}^\varphi{}_\alpha B^i_\varphi+C_a{}^b{}_\alpha B^i_b .$$

§5. The structure equations of Gauss, Codazzi and Ricci for Finsler subspaces. Let F^n be a Finsler subspace of a Finsler space F^m endowed with a Finsler connection $FC=(\tilde{N}, \tilde{\nabla})=(N^i_j, F_j{}^i{}_k, C_j{}^i{}_k)$. In the previous sections we obtain three vectorial Finsler connection $IVFC=(N, \overset{*}{\nabla})=$

$=(N^{\beta}_{\alpha},\ F_{j}{}^{i}{}_{\alpha},\ C_{j}{}^{i}{}_{\alpha})$, $IFC=(N,\nabla)=(N^{\beta}_{\alpha},\ F_{\alpha}{}^{\beta}{}_{\gamma},\ C_{\alpha}{}^{\beta}{}_{\gamma})$ and $FNC=(N,\nabla^{\perp})=$ $=(N^{\beta}_{\alpha},\ F_{a}{}^{b}{}_{\alpha},\ C_{a}{}^{b}{}_{\alpha})$ on the vector bundles $VT\tilde{M}^{*}$, VTM and respectively $VTM^{\perp}$. Hence we have two types of relative differentiation given by the pairs $(IFC,\ IVFC)$ and $(IFC,\ FNC)$. In this section we need both these relative covariant derivatives. Thus, for a tensor field $T^{\beta j}_{\alpha i}$ we denote by $T^{\beta j}_{\alpha i|\gamma}$ and $T^{\beta j}_{\alpha i}|_{\gamma}$ its (h)-and respectively (v)-covariant derivatives with respect to the pair $(IFC,\ IVFC)$. Also, for a tensor field $T^{\beta b}_{\alpha a}$ we denote by $T^{\beta b}_{\alpha a\perp\gamma}$ and $T^{\beta b}_{\alpha a}\perp_{\gamma}$ its (h)- and respectively (v)-covariant derivatives with respect to the pair $(IFC,\ FNC)$.

By a direct computation, using (1.9), (1.10), (1.17), (1.18) (1.20) and (1.21) we obtain the following commutation formulas :

$$(5.1)\qquad \begin{cases} B^{i}_{\alpha|\beta|\gamma}-B^{i}_{\alpha|\gamma|\beta}=B^{j}_{\alpha}R_{j}{}^{i}{}_{\beta\gamma}-B^{i}_{\varphi}R_{\alpha}{}^{\varphi}{}_{\beta\gamma}-(H^{a}{}_{\alpha\varphi}T_{\beta}{}^{\varphi}{}_{\gamma}+K^{a}{}_{\alpha\varphi}R^{\varphi}{}_{\beta\gamma})B^{i}_{a}, \\ B^{i}_{\alpha|\beta}|_{\gamma}-B^{i}_{\alpha}|_{\gamma|\beta}=B^{j}_{\alpha}P_{j}{}^{i}{}_{\beta\gamma}-B^{i}_{\varphi}P_{\alpha}{}^{\varphi}{}_{\beta\gamma}-(H^{a}{}_{\alpha\varphi}C_{\beta}{}^{\varphi}{}_{\gamma}+K^{a}{}_{\alpha\varphi}P^{\varphi}{}_{\beta\gamma})B^{i}_{a}, \\ B^{i}_{\alpha}|_{\beta}|_{\gamma}-B^{i}_{\alpha}|_{\gamma}|_{\beta}=B^{j}_{\alpha}S_{j}{}^{i}{}_{\beta\gamma}-B^{i}_{\varphi}S_{\alpha}{}^{\varphi}{}_{\beta\gamma}-K^{a}{}_{\alpha\varphi}S^{\varphi}{}_{\beta\gamma}B^{i}_{a}, \end{cases}$$

$$(5.2)\qquad \begin{cases} B^{i}_{a|\alpha|\beta}-B^{i}_{a|\beta|\alpha}=B^{j}_{a}R_{j}{}^{i}{}_{\alpha\beta}+(R^{\varphi}{}_{\alpha\beta}\tilde{K}_{a}{}^{\lambda}{}_{\varphi}+T_{\alpha}{}^{\varphi}{}_{\beta}\tilde{H}_{a}{}^{\lambda}{}_{\varphi})B^{i}_{\lambda}- \\ -(R^{\varphi}{}_{\alpha\beta}C_{a}{}^{b}{}_{\varphi}+T_{\alpha}{}^{\varphi}{}_{\beta}F_{a}{}^{b}{}_{\varphi})B^{i}_{b}, \\ B_{a|\alpha}|_{\beta}-B^{i}_{a}|_{\beta|\alpha}=B^{j}_{a}P_{j}{}^{i}{}_{\alpha\beta}+(C_{\alpha}{}^{\varphi}{}_{\beta}\tilde{H}_{a}{}^{\lambda}{}_{\varphi}+P^{\varphi}{}_{\alpha\beta}\tilde{K}_{a}{}^{\lambda}{}_{\varphi})B^{i}_{\lambda}- \\ -(C_{\alpha}{}^{\varphi}{}_{\beta}F_{a}{}^{b}{}_{\varphi}+P^{\varphi}{}_{\alpha\beta}C_{a}{}^{b}{}_{\varphi})B^{i}_{b}, \\ B^{i}_{a}|_{\alpha}|_{\beta}-B^{i}_{a}|_{\beta}|_{\alpha}=B^{j}_{\alpha}S_{j}{}^{i}{}_{\alpha\beta}+S^{\varphi}{}_{\alpha\beta}\tilde{K}_{a}{}^{\lambda}{}_{\varphi}B^{i}_{\lambda}-S^{\varphi}{}_{\alpha\beta}C_{a}{}^{b}{}_{\varphi}B^{i}_{b}. \end{cases}$$

where $R_{j}{}^{i}{}_{\beta\gamma}$, $P_{j}{}^{i}{}_{\beta\gamma}$, $S_{j}{}^{i}{}_{\beta\gamma}$ and $R_{\alpha}{}^{\varphi}{}_{\beta\gamma}$, $P_{\alpha}{}^{\varphi}{}_{\beta\gamma}$, $S_{\alpha}{}^{\varphi}{}_{\beta\gamma}$ are the Finsler curvature tensors of $IVFC$ and respectively IFC.

Proposition 5.1. *The curvature tensors of IVFC are given by*

$$(5.3)\qquad \begin{cases} R_{j}{}^{i}{}_{\beta\gamma}=R_{j}{}^{i}{}_{kh}B^{k}_{\beta}B^{h}_{\gamma}+P_{j}{}^{i}{}_{kh}(H^{a}_{\gamma}B^{k}_{\beta}-H^{a}_{\beta}B^{k}_{\gamma})B^{h}_{a}+S_{j}{}^{i}{}_{kh}B^{k}_{a}B^{h}_{b}H^{a}_{\beta}H^{b}_{\gamma}, \\ P_{j}{}^{i}{}_{\beta\gamma}=P_{j}{}^{i}{}_{kh}B^{k}_{\beta}B^{h}_{\gamma}+S_{j}{}^{i}{}_{kh}B^{k}_{a}H^{a}_{\beta}B^{h}_{\gamma}, \\ S_{j}{}^{i}{}_{\beta\gamma}=S_{j}{}^{i}{}_{kh}B^{k}_{\beta}B^{h}_{\gamma}. \end{cases}$$

Proof. By using (3.5), (3.8), (3.11) and (3.12) we obtain

$$(5.4)\qquad \begin{aligned} &\frac{\delta}{\delta u^{\gamma}}(F_{j}{}^{i}{}_{\beta})-\frac{\delta}{\delta u^{\beta}}(F_{j}{}^{i}{}_{\gamma})+C_{j}{}^{i}{}_{\varphi}R^{\varphi}{}_{\beta\gamma}=\Bigg(\frac{\delta F_{j}{}^{i}{}_{k}}{\delta x^{h}}-\frac{\delta F_{j}{}^{i}{}_{h}}{\delta x^{k}}+ \\ &+C_{j}{}^{i}{}_{s}R^{s}{}_{kh}\Bigg)B^{k}_{\beta}B^{h}_{\gamma}+\left(\frac{\partial C_{j}{}^{i}{}_{k}}{\partial y^{h}}-\frac{\partial C_{j}{}^{i}{}_{h}}{\partial y^{k}}\right)B^{h}_{b}B^{k}_{a}H^{b}_{\gamma}H^{a}_{\beta}+ \\ &+\left(\frac{\partial F_{j}{}^{i}{}_{k}}{\partial y^{h}}-\frac{\delta F_{j}{}^{i}{}_{h}}{\delta x^{k}}+C_{j}{}^{i}{}_{s}\frac{\partial N^{s}{}_{k}}{\partial y^{h}}\right)(H^{a}_{\gamma}B^{k}_{\beta}-H^{a}_{\beta}B^{k}_{\gamma})B^{h}_{a}, \quad \text{and} \end{aligned}$$

$$(5.5)\qquad \begin{aligned} &F_{j}{}^{k}{}_{\beta}F_{k}{}^{i}{}_{\gamma}-F_{j}{}^{k}{}_{\gamma}F_{k}{}^{i}{}_{\beta}=(F_{j}{}^{s}{}_{k}F_{s}{}^{i}{}_{h}-F_{j}{}^{s}{}_{h}F_{s}{}^{i}{}_{k})B^{k}_{\beta}B^{h}_{\gamma}+ \\ &+(F_{j}{}^{s}{}_{k}C_{s}{}^{i}{}_{h}-C_{j}{}^{s}{}_{h}F_{s}{}^{i}{}_{k})(B^{k}_{\beta}H^{a}_{\gamma}-B^{k}_{\gamma}H^{a}_{\beta})B^{h}_{a}+ \\ &+(C_{j}{}^{s}{}_{k}C_{s}{}^{i}{}_{h}-C_{j}{}^{s}{}_{h}C_{s}{}^{i}{}_{k})B^{k}_{a}B^{h}_{b}H^{a}_{\beta}H^{b}_{\gamma}. \end{aligned}$$

Thus by adding (5.4) and (5.5) and taking account of (1.9) for vectorial Finsler connections $IVFC$ and FC we obtain the first line of equations in (5.3). In a similar way we get the remaining equations in (5.3).

Now we can state the main result of this section as follows

Theorem 5.1. *Let F^{n} be a Finsler subspace of the Finsler space F^{m} endowed with the Finsler connection $FC=(\tilde{N},\tilde{\nabla})$. Then the curvature tensors of vectorial Finsler connection IVFC, IFC and FNC are related by the following structure equations*

$$(5.6)\qquad \begin{cases} R_{j}{}^{i}{}_{\beta\gamma}B^{j}_{\alpha}\tilde{B}^{\varphi}_{i}=R_{\alpha}{}^{\varphi}{}_{\beta\gamma}+H^{a}{}_{\alpha\gamma}\tilde{H}_{a}{}^{\varphi}{}_{\beta}-H^{a}{}_{\alpha\beta}\tilde{H}_{a}{}^{\varphi}{}_{\gamma}, \\ P_{j}{}^{i}{}_{\beta\gamma}B^{j}_{\alpha}\tilde{B}^{\varphi}_{i}=P_{\alpha}{}^{\varphi}{}_{\beta\gamma}+K^{a}{}_{\alpha\gamma}\tilde{H}_{a}{}^{\varphi}{}_{\beta}-H^{a}{}_{\alpha\beta}\tilde{K}_{a}{}^{\varphi}{}_{\gamma}, \\ S_{j}{}^{i}{}_{\beta\gamma}B^{j}_{\alpha}\tilde{B}^{\varphi}_{i}=S_{\alpha}{}^{\varphi}{}_{\beta\gamma}+K^{a}{}_{\alpha\gamma}\tilde{K}_{a}{}^{\varphi}{}_{\beta}-K^{a}{}_{\alpha\beta}\tilde{K}_{a}{}^{\varphi}{}_{\gamma}, \end{cases}$$

$$(5.7)\qquad \begin{cases} R_{j}{}^{i}{}_{\beta\gamma}B^{j}_{\alpha}\tilde{B}^{a}_{i}=H^{a}{}_{\alpha\beta\perp\gamma}-H^{a}{}_{\alpha\gamma\perp\beta}+H^{a}{}_{\alpha\varphi}T_{\beta}{}^{\varphi}{}_{\gamma}+K^{a}{}_{\alpha\varphi}R^{\varphi}{}_{\beta\gamma}, \\ P_{j}{}^{i}{}_{\beta\gamma}B^{j}_{\alpha}\tilde{B}^{a}_{i}=H^{a}{}_{\alpha\beta}\perp_{\gamma}-K^{a}{}_{\alpha\gamma\perp\beta}+C_{\beta}{}^{\varphi}{}_{\gamma}H^{a}{}_{\alpha\varphi}+P^{\varphi}{}_{\beta\gamma}K^{a}{}_{\alpha\varphi}, \\ S_{j}{}^{i}{}_{\beta\gamma}B^{j}_{\alpha}\tilde{B}^{a}_{i}=K^{a}{}_{\alpha\beta}\perp_{\gamma}-K^{a}{}_{\alpha\gamma}\perp_{\beta}+K^{a}{}_{\alpha\varphi}S^{\varphi}{}_{\beta\gamma}, \end{cases}$$

$$(5.8)\qquad \begin{cases} R_{j}{}^{i}{}_{\alpha\beta}B^{j}_{a}\tilde{B}^{b}_{i}=R_{a}{}^{b}{}_{\alpha\beta}+\tilde{H}_{a}{}^{\varphi}{}_{\beta}H^{b}{}_{\varphi\alpha}-\tilde{H}_{a}{}^{\varphi}{}_{\alpha}H^{b}{}_{\varphi\beta}, \\ P_{j}{}^{i}{}_{\alpha\beta}B^{j}_{a}\tilde{B}^{b}_{i}=P_{a}{}^{b}{}_{\alpha\beta}+\tilde{K}_{a}{}^{\varphi}{}_{\beta}H^{b}{}_{\varphi\alpha}-\tilde{H}_{a}{}^{\varphi}{}_{\alpha}K^{b}{}_{\varphi\beta}, \\ S_{j}{}^{i}{}_{\alpha\beta}B^{j}_{a}\tilde{B}^{b}_{i}=S_{a}{}^{b}{}_{\alpha\beta}+\tilde{K}_{a}{}^{\varphi}{}_{\beta}K^{b}{}_{\varphi\alpha}-\tilde{K}_{a}{}^{\varphi}{}_{\alpha}K^{b}{}_{\varphi\beta}, \end{cases}$$

$$(5.9)\qquad \begin{cases} R_{j}{}^{i}{}_{\alpha\beta}B^{j}_{a}\tilde{B}^{\varphi}_{i}=\tilde{H}_{a}{}^{\varphi}{}_{\gamma\perp\alpha}-\tilde{H}_{a}{}^{\varphi}{}_{\alpha\perp\beta}-T_{\alpha}{}^{\lambda}{}_{\beta}\tilde{H}_{a}{}^{\varphi}{}_{\lambda}-R^{\lambda}{}_{\alpha\beta}\tilde{K}_{a}{}^{\varphi}{}_{\lambda}, \\ P_{j}{}^{i}{}_{\alpha\beta}B^{j}_{a}\tilde{B}^{\varphi}_{i}=\tilde{K}_{a}{}^{\varphi}{}_{\beta\perp\alpha}-\tilde{H}_{a}{}^{\varphi}{}_{\alpha}\perp_{\beta}-C_{\alpha}{}^{\lambda}{}_{\beta}\tilde{H}_{a}{}^{\varphi}{}_{\lambda}-P^{\lambda}{}_{\alpha\beta}\tilde{K}_{a}{}^{\varphi}{}_{\lambda}, \\ S_{j}{}^{i}{}_{\alpha\beta}B^{j}_{a}\tilde{B}^{\varphi}_{i}=\tilde{K}_{a}{}^{\varphi}{}_{\beta}\perp_{\alpha}-\tilde{K}_{a}{}^{\varphi}{}_{\alpha}\perp_{\beta}-S^{\lambda}{}_{\alpha\beta}\tilde{K}_{a}{}^{\varphi}{}_{\lambda}, \end{cases}$$

where $R_{a}{}^{b}{}_{\alpha\beta}$, $P_{a}{}^{b}{}_{\alpha\beta}$, $S_{a}{}^{b}{}_{\alpha\beta}$ are the Finsler curvature tensors of the Finsler normal connection.

Proof. By using (3.30), (4.14) and (4.15) in the first line of commutation formulas (5.1) we get

$$(5.10)\qquad \begin{aligned} &B^{j}_{\alpha}R_{j}{}^{i}{}_{\beta\gamma}-B^{i}_{\varphi}R_{\alpha}{}^{\varphi}{}_{\beta\gamma}=(H^{b}{}_{\alpha\beta|\gamma}-H^{b}{}_{\alpha\gamma|\beta}+H^{a}{}_{\alpha\beta}F_{a}{}^{b}{}_{\gamma}- \\ &-H^{a}{}_{\alpha\gamma}F_{a}{}^{b}{}_{\beta}+H^{b}{}_{\alpha\varphi}T_{\beta}{}^{\varphi}{}_{\gamma}+K^{b}{}_{\alpha\varphi}R^{\varphi}{}_{\beta\gamma})B^{i}_{b}-(H^{a}{}_{\alpha\beta}\tilde{H}_{a}{}^{\varphi}{}_{\gamma}-H^{a}_{\alpha\gamma}\tilde{H}_{a}{}^{\varphi}{}_{\beta})B^{i}_{\varphi}. \end{aligned}$$

By contracting (5.10) with $\tilde{B}^{\lambda}_{i}$ we obtain the first line of equations in (5.6) Also, by contracting (5.10) with $\tilde{B}^{a}_{i}$ we get

$$(5.11)\qquad \begin{aligned} B^{j}_{\alpha}\tilde{B}^{a}_{i}R_{j}{}^{i}{}_{\beta\gamma}=&(H^{a}{}_{\alpha\beta|\gamma}+H^{c}{}_{\alpha\beta}F_{c}{}^{a}{}_{\gamma})-(H^{a}{}_{\alpha\gamma|\beta}+H^{c}{}_{\alpha\gamma}F_{c}{}^{a}{}_{\beta})+ \\ &+T_{\beta}{}^{\varphi}{}_{\gamma}H^{a}{}_{\alpha\varphi}+R^{\varphi}{}_{\beta\gamma}K^{a}{}_{\alpha\varphi}. \end{aligned}$$

Next, by using both relative differentiations discussed in this section we obtain

$$(5.12)\qquad H^{a}{}_{\alpha\beta\perp\gamma}=H^{a}{}_{\alpha\beta|\gamma}+H^{b}{}_{\alpha\beta}F_{b}{}^{a}{}_{\gamma}.$$

Thus by (5.12) we see that (5.11) becomes just the first line of equations in (5.7). In a similar way we get the second and the third line of equations in (5.6) and (5.7) from the second and respectively the third line of equations in (5.1). Finally, by using again (3.30), (4.14) and (4.15) in equations (5.2) we infer

$$(5.13)\quad \begin{cases} R_j{}^i{}_{\alpha\beta}B^j_a = (R_a{}^b{}_{\alpha\beta}+\tilde{H}_a{}^\varphi{}_\beta H^b{}_{\varphi\alpha}-\tilde{H}_a{}^\varphi{}_\alpha H^b{}_{\varphi\beta})B^i_b+(\tilde{H}_a{}^\varphi{}_{\beta\perp\alpha}-\tilde{H}_a{}^\varphi{}_{\alpha\perp\beta}- \\ \qquad -R^\lambda{}_{\alpha\beta}\tilde{K}_a{}^\varphi{}_\lambda - T_\alpha{}^\lambda{}_\beta\tilde{H}_a{}^\varphi{}_\lambda)B^i_\varphi, \\ P_j{}^i{}_{\alpha\beta}B^j_a = (P_a{}^b{}_{\alpha\beta}+\tilde{K}_a{}^\varphi{}_\beta H^b{}_{\varphi\alpha}-\tilde{H}_a{}^\varphi{}_\alpha K^b{}_{\varphi\beta})B^i_b+(\tilde{K}_a{}^\varphi{}_{\beta\perp\alpha}-\tilde{H}_a{}^\varphi{}_\alpha\perp_\beta- \\ \qquad -C_\alpha{}^\lambda{}_\beta\tilde{H}_a{}^\varphi{}_\lambda - P^\lambda{}_{\alpha\beta}\tilde{K}_a{}^\varphi{}_\lambda)B^i_\varphi, \\ S_j{}^i{}_{\alpha\beta}B^j_a = (S_a{}^b{}_{\alpha\beta}+\tilde{K}_a{}^\varphi{}_\beta K^b{}_{\varphi\alpha}-\tilde{K}_a{}^\varphi{}_\alpha K^b{}_{\varphi\beta})B^i_b+(\tilde{K}_a{}^\varphi{}_\beta\perp_\alpha-\tilde{K}_a{}^\varphi{}_\alpha\perp_\beta- \\ \qquad -S^\lambda{}_{\alpha\beta}\tilde{K}_a{}^\varphi{}_\lambda)B^i_\varphi. \end{cases}$$

Then the equations (5.8) and (5.9) are obtained by contracting (5.13) with $\tilde{B}^c_i$ and respectively $\tilde{B}^\varepsilon_i$.

The equations (5.6), (5.7) and (5.8) are the equations of Gauss, Codazzi and Ricci for the immersion of a Finsler space F^n in F^m. For a metric Finsler connection FC on F^m we have $g_{ij|k}=g_{ij}|_k=0$. Hence by using (3.31), (4.12) and (4.13) we obtain the equivalence of (5.7) and (5.9). However, for other Finsler connections of F^m the equations (5.9) represent something which shows how far from the geometry of Riemannian subspaces is the geometry of Finsler subspaces.

Now, let f be a Finsler scalar field on F^m, that is, f is a differentiable function on $T\tilde{M}$. Then by using (3.28) and (3.29) we get the following commutation formulas for IFC

$$(5.14)\quad \begin{cases} f_{|\beta|\gamma}-f_{|\gamma|\beta} = -f_{|\alpha}T_\beta{}^\alpha{}_\gamma - f|_\alpha R^\alpha{}_{\beta\gamma}, \\ f_{|\beta}|_\gamma - f|_{\gamma|\beta} = -f_{|\alpha}C_\beta{}^\alpha{}_\gamma - f|_\alpha P^\alpha{}_{\beta\gamma}, \\ f|_\beta|_\gamma - f|_\gamma|_\beta = -f|_\alpha S^\alpha{}_{\beta\gamma}. \end{cases}$$

By using (5.14) and similar formulas for FC on F^m we obtain

Theorem 5.2. *Let F^n be a Finsler subspace of the Finsler space F^m endowed with the Finsler connection $FC=(\tilde{N}, \tilde{\nabla})$. Then the torsion tensors of IFC and FC are related by*

$$(5.15)\quad \begin{cases} R^\alpha{}_{\beta\gamma} = R^i{}_{jk}B^j_\beta B^k_\gamma\tilde{B}^\alpha_i + P^i{}_{jk}B^k_a\tilde{B}^\alpha_i(B^j_\beta H^a_\gamma - B^j_\gamma H^a_\beta) + H^a_\beta\tilde{H}_a{}^\alpha{}_\gamma - H^a_\gamma\tilde{H}_a{}^\alpha{}_\beta\,+ \\ \qquad + S^i_{jk}H^a_\beta H^b_\gamma B^j_a B^k_b\tilde{B}^\alpha_i \\ P^\alpha{}_{\beta\gamma} = H^a_\beta\tilde{K}_a{}^\alpha{}_\gamma + \tilde{B}^\alpha_i(S^i{}_{jk}H^a_\beta B^j_a B^k_\gamma + P^i{}_{jk}B^j_\beta B^k_\gamma), \\ T_\beta{}^\alpha{}_\gamma = \tilde{B}^\alpha_i\{T_j{}^i{}_k B^j_\beta B^k_\gamma + C_j{}^i{}_k(B^j_\beta H^a_\gamma - B^j_\gamma H^a_\beta)B^k_a\}, \\ S^\alpha{}_{\beta\gamma} = S^i{}_{jk}\tilde{B}^\alpha_i B^j_\beta B^k_\gamma;\quad C_\beta{}^\alpha{}_\gamma = C_j{}^i{}_k\tilde{B}^\alpha_i B^j_\beta B^k_\gamma, \\ D^\alpha{}_\beta = C_j{}^i{}_k\tilde{B}^\alpha_i B^k_a H^a_\beta y^j + \tilde{B}^\alpha_i B^j_\beta D^i{}_j, \end{cases}$$

where $D^\alpha{}_\beta$ and $D^i{}_j$ are the deflection tensor fields of IFC and respectively FC.

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Received 13.V.1985

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