Démonstration. L'assertion (a) est contenue dans la proposition 2.3, par (2.5) et (2.7). Pour démontrer l'assertion (b) on tient compte que (1.5) avec $\overset{c}{\nabla}=\nabla$ a la propriètè (2.5). Vu que $\tilde{D}_X g=0$, un calcul direct montre que (2.8) a aussi la propriété (2.6).

On appellera $\overset{*}{\nabla}$ donné par (2.8) c.m.p.t. *canonique.*

Si on cherche toutes les c.m.p.t. sous la forme $\nabla_X=\overset{*}{\nabla}_X+A_X$, on obtient facilement le systéme d'équations $\underset{1}{Q}\circ A=0$, $\underset{2}{\wedge}\circ A=0$, qui est évidemment compatible et donc on peut formuler

Théorème 2.2. *L'ensemble des c.m.p.t. est donné par*

$$\nabla_X=\overset{*}{\nabla}_X+(\underset{1}{Q}\circ\underset{1}{\wedge})(W_X), \quad W\in\mathcal{T}^1_2(M), \tag{2.9}$$

où $\overset{*}{\nabla}$ est la c.m.p.t. canonique.

3. Soit $R\in\mathcal{T}^1_3(M)$ le champ de tenseurs de courbure de la connexion linéaire ∇. Alors pour tout $a\in\mathcal{T}^0_2(M)$, $u\in\mathcal{T}^1_1(M)$, $X, Y, Z, U, V\in\mathcal{X}(M)$ on a

$$([D_X, D_Y]u)(Z)=u(R(X, Y)Z)-R(X, Y)(uZ)-(D_{[X, Y]}u)(Z), \tag{3.1}$$

$$([\tilde{D}_X, \tilde{D}_Y]a)(U, V)=-a(R(X, Y)U, V)-a(U, R(X, Y)V)+ (D_{[X, Y]}a)(U, V). \tag{3.2}$$

Les relations (3.1) et (3.2) nous permettent de formuler

Théorème 3.1. *Pour toute c.m.p.t. on a*

$$\underset{2}{Q}D_{X_p}\dots D_{X_1}R(X, Y)=0, \qquad \underset{2}{\wedge}\tilde{D}_{X_p}\dots\tilde{D}_{X_1}R(X, Y)=0$$
$$p=0, 2, \dots, X, Y, X_p, \dots, X_1\in\mathcal{X}(M). \tag{3.3}$$

Remarques 1°. Pour l'intégrabilité de la s.m.p.t. on peut consulter [5], [7], [8]. 2°. Si on renonce à ce que $g\in\mathcal{T}^0_2(M)$ soit positivement défini, alors on prendre en consideration la s.m.p.t. pour laquelle (2.1) est remplace par: (2.1) $'f\circ g\circ f=-g\circ h$. Leur étude se fait de la même maniére.

RÉFÉRENCES

1. Clark R. S. et Bruckheimer M. R. — C. R. Acad. Sci. Paris, *251* (1960), 627—629.
2. Eliopoulos H. A., C.R. Acad. Sci. Paris, *255* (1962), 1 563—1 565.
3. Grifone J., Ann. Inst. Fourier, Grenoble, I, *22 (1)* (1972), 287—334; II, *22 (3)* (1972), 291—338.
4. Gadea P. M., Tensor, N.S., *31* (1980), 30—34.
5. Lehmann J.-Lejeune J., C. R. Acad. Sci. Paris, *258* (1964), 5 326—5 329.
6. Libermann P., C. R. Acad. Sci. Paris, *231* (1952), 1 030—1 032.
7. Munteanu Gh., Anal. St. Univ. „Al. I. Cuza" Iaşi, **XXXIII**, 2 (1987), 151—158.
8. Yano K. and Ako M., Kodai Sem. Rep. *25* (1973), 63—94.

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ON PSEUDO SYMMETRIC AND PSEUDO RICCI SYMMETRIC P-SASAKIAN MANIFOLD

BY

M. TARAFDAR

Introduction. In a previous paper [1], M. C. Chaki introduced a type of non-flat Riemannian manifold (M^n, g) $(n\geqslant 2)$ whose curvature tensor R satisfies the condition

$$(\nabla_X R)(Y, Z)W=2A(X)R(Y, Z)W+A(Y)R(X, Z)W+A(Z)R(Y, X)W+ +A(W)R(Y, Z)X+g(R(Y, Z)W, X)P \tag{1}$$

where A is a non-zero 1-form

$$g(X, P)=A(X) \tag{2}$$

for every vector field x and ∇ denotes the operator of covariant differentiation with respect to the metric g. Such a manifold was called a pseudo symmetric manifold, A was called its associated 1-form and an n-dimensional manifold of this kind was denoted by $(PS)_n$. In a subsequent paper [2] Chaki introduced another type of non-flat Riemannian manifold (M^n, g) $(n\geqslant 3)$ whose Ricci tensor S of type (0, 2) satisfies the condition:

$$(\nabla_X S)(Y, Z)=2A(X)S(Y, Z)+A(Y)S(X, Z)+A(Z)S(Y, X) \tag{3}$$

where A and ∇ have meaning already stated. Such a manifold was called a pseudo-Ricci symmetric manifold and an n-dimensional manifold of this kind was denoted by $(PRS)_n$. On the other hand, *I* Sato [3] and Matsumoto [4] introduced the notion of a P — Sasakian manifold as follows:

Let (M, g) be an n-dimensional Riemannian manifold admitting a 1—form η, a vector field ξ and an (1—1) tensor field Φ which satisfy the following conditions

$$(\nabla_X\eta)(Y)-(\nabla_Y\eta)(X)=0, \tag{4}$$

$$(\nabla_X\nabla_Y\eta)(Z)=-g(X, Z)\eta(Y)-g(X, Y)\eta(Z)+2\eta(X)(Y)\eta(Z), \tag{5}$$

$$g(X, \xi)=\eta(X), \text{ for all vector fields } X, \tag{6}$$

$$\eta(\xi)=1, \tag{7}$$

$$\nabla_X\xi=\Phi X. \tag{8}$$

Such a manifold was called a Para-Sasakian manifold or briefly a P-Sasakian manifold. In the present paper the question whether a Pseudo symmetric or a Pseudo Ricci symmetric manifold may be a P-Sasakian manifold has been answered in the negative.

1. Preliminaries. In this section we first consider some formulas which hold in a $(PS)_n$. Let ν denote the scalar curvature and L denote the symmetric endomorphism of the tangent space at each point of a Riemannian manifold (M^n, g) corresponding to the Ricci tensor S i.e.

$$gS(LX, Y)=S(X, Y) \tag{1.1}$$

for any vector fields X, Y. From 1) we get

$$(\nabla_X S)(Y, Z)=2A(X)S(Y, Z)+A(Y)S(X, Z)+A(Z)S(X, Y)+ \\ +A(R(X, Y)Z)+A(R(X, Z)Y) \tag{1.2}$$

Contracting (1.2) we get

$$d\nu(X)=2A(X)\nu+4A(LX) \tag{1.3}$$

Next we consider the case of a $(PSR)_n$ [2]. In this case, let B be the 1-form defined by

$$B(X)=A(LX),$$

where L has the meaning already given. Contracting 3) we get

$$d\nu(X)=2A(X)\nu+2B(X). \tag{1.4}$$

Again from 3) we have

$$(\nabla_X S)(Y, Z)-(\nabla_Z S)(Y, X)=A(X)S(Y, Z)-A(Z)S(Y, X). \tag{1.5}$$

Contracting (1.5) we get

$$d\nu(X)=2A(X)\nu-2B(X). \tag{1.6}$$

From (1.4) and (1.6) it follows that

$$B(X)=0. \tag{1.7}$$

In virtue of (1.7), the equation (1.6) takes the form

$$d\nu(X)=2A(X)\nu. \tag{1.8}$$

Further we get

$$\nu dA(X, Y)=0. \tag{1.9}$$

We shall use these formulas later.

2. Pseudo symmetric P-Sasakian manifold $(PS)_n$ $(n\geq 3)$. In this section we suppose that an n-dimensional $(PS)_n$ $(n\geq 3)$ is a P-Sasakian manifold. It is known [3, 4] that in a P-Sasakian manifold, besides 4) — 8) the following relations hold.

(2.1) $\Phi\xi=0$, (2.2) $R(\xi, X)Y=-g(X, Y)\xi+\eta(Y)X$,

(2.3) $S(X, \xi)=-(n-1)\eta(X)$, (2.4) $g(\Phi X, Y)=g(X, \Phi Y)$,

(2.5) $S(\Phi X, Y)=S(X, \Phi Y)$, (2.6) $(\nabla_X\eta)Y=g(\Phi X, Y)$.

Now

$$(\nabla_X S)(Y, \xi)=\nabla_X S(Y, \xi)-S(\nabla_X Y, \xi)-S(Y, \nabla_X\xi). \tag{2.7}$$

Using 8), (2.3), (2.6), the equations (2.7) can be written as

$$(\nabla_X S)(Y, \xi)=-(n-1)g(\nabla_X\xi, Y)-S(Y, \Phi X). \tag{2.8}$$

Again from (1.2) and (2.3), we get

$$(\nabla_X S)(Y, \xi)=-2(n-1)A(X)\eta(Y)-(n-1)A(Y)\eta(X)+ \\ +A(\xi)S(Y, X)+A(R(X, Y)\xi)+A(R(X, \xi)Y). \tag{2.9}$$

Now $A(R(X, Y)\xi)=g(R(X, Y)\xi, P)=g(R(\xi, Y)X-R(\xi, X)Y, P)$.

Using (2.2) we find

$$A(R(X, Y)\xi=\eta(X)A(Y)-\eta(Y)A(X). \tag{2.10}$$

Also

$$A(R(X, \xi)Y)=-A(X)\eta(Y)+g(X, Y)A(\xi). \tag{2.11}$$

Thus (2.9) reduces to

$$(\nabla_X S)(Y, \xi)=-2nA(X)\eta(Y)-(n-2)A(Y)\eta(X)+\eta(P)\{S(X, Y)+ \\ +g(X, Y)\}. \tag{2.12}$$

From (2.8) and (2.12) we get

$$-2nA(X)\eta(Y)-(n-2)A(Y)\eta(X)+\eta(P)\{S(X, Y)+g(X, Y)\}= \\ =-S(\Phi X, Y)-(n-1)g(\Phi X, Y). \tag{2.13}$$

Putting $X=\xi$ in the above equation, we get

$$-(3n-2)\eta(P)\eta(Y)-(n-2)A(Y)=0. \tag{2.14}$$

Next putting $Y=\xi$ in (2.14) we get

$$-4(n-1)\eta(P)=0. \tag{2.15}$$

Since $n\geq 3$, we get

$$\eta(P)=0. \tag{2.16}$$

Hence from (2,14) we get

$$(n-2)A(Y)=0. \tag{2.17}$$

Thus we get $A(Y)=0$, which is inadmissible by the definition of a $(PS)_n$. This leads to the following theorem.

Theorem 1. *A* $(PS)_n$ $(n \geq 3)$ *cannot be a P-Sasakian manifold.*

3. Pseudo Ricci Symmetric P-Sasakian manifold $(PRS)_n$ $(n \geq 3)$. In this section we suppose that a $(PRS)_n$ $(n \geq 3)$ is P-Sasakian. From 3) we get

$$(\nabla_X S)(Y, \xi) = 2A(X)S(Y, \xi) + A(Y)S(X, \xi) + A(\xi)S(Y, X).$$

In virtue of (2.3) the above equation can be written as

$$(\nabla_X S)(Y, \xi) = -2(n-1)A(X)\eta(Y) - (n-1)A(Y)\eta(X) + A(\xi)S(Y, X). \tag{3.1}$$

From (2.8) and (3.1) we get

$$-2(n-1)A(X)\eta(Y) - (n-1)A(Y)\eta(X) + A(\xi)S(Y, X) = - \tag{3.2}$$
$$-(n-1)g(\Phi X, Y) - S(Y, \Phi X).$$

Putting $X = \xi$ in (3.2) and taking account of (2.1) and (2.3) we obtain

$$3A(\xi)\eta(Y) + A(Y) = 0, \quad \text{because} \quad n \geq 3. \tag{3.3}$$

Now putting $Y = \xi$ in above we get

$$A(\xi) = 0. \tag{3.4}$$

Hence from (3.3) we obtain

$$A(Y) = 0, \tag{3.5}$$

which is inadmissible by the definition of $(PRS)_n$. Thus we can state

Theorem 2. *A* $(PRS)_n$ $(n \geq 3)$ *cannot be a P-Sasakian manifold.*

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REFERENCES

1. Chaki, M. C. — *On pseudo symmetric manifolds*, An. Şt. Univ. „AL. I. CUZA". Iaşi, XXXIII (1987), 54—58.
2. Chaki, M. C. — *On pseudo Ricci symmetric manifolds*, Bulg. J. Phys, 15 (1988), 526—530.
3. Sato, I. — *On a structure similar to the almost contact structure*, Tensor N. S., Vol. 30 (1976), 219—224.
4. Matsumoto, K. — *On P-Sasakian manifolds satisfying certain conditions*, Tensor N.S. Vol. 33 (1979), 173—178.

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COSYMPLECTIC QUASI-SASAKIAN MANIFOLDS WITH $a\Phi$—STRUCTURE VECTOR FIELD ξ

BY

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1. Introduction and summary. *A quassi-Sasakian manifold* $\tilde{M}(\Phi, \tilde{\Omega}, \tilde{\eta}, \xi, \tilde{g})$ is a normal almost contact metric manifold with closed structure 2-form $\tilde{\Omega}$ [11] (see also section 2 below). If in addition the structure 1-form $\tilde{\eta}$ is closed, then $\tilde{M}$ is called *cosymplectic quasi-Sasakian manifold*.

General cosymplectic manifolds and their submanifolds have been studied by many authors (see, e.g. Blair [2], Eum [6], Goldberg [7], Ludden [10]). However very little is known about submanifolds of a cosymplectic manifold endowed with a Sasakian structure, although certain aspects of this problem have been discussed by Kim [8].

We denote by a the *anticanonical operator* [18] of square $+1$ that anticommutes with Φ and by $d\tilde{p}$ the line element of $\tilde{M}$. In this paper, we consider cosymplectic quasi Sasakian manifolds $\tilde{M}$, such that the covariant derivative of the structure vector field ξ satisfies $\tilde{\nabla}\xi = c(a \circ \Phi)d\tilde{p}$, where c is a nonvanishing constant. ξ will then be called a $_a\Phi$—*structure vector field*.

In section 3 it will be proved that $\tilde{M}$ is foliated by symplectic minimal hypersurfaces M_ξ orthogonal to ξ, which have constant total curvature $(-1)^m c^{2m}$. Further for any tangent vector field X of M_ξ, $aX \pm \Phi X$ are principal vector fields on M_ξ.

In section 4, contact CR submanifolds M of $\tilde{M}$ [20] will be considered. The vertical distribution $D^\perp$ on M is always involutive. Hence M enjoys one of the basic properties of Kähler manifolds [1], Sasakian [20] and pseudo-Sasakian manifolds [14]. In addition, ξ is an umbilical section for the leaves of $D^\perp$.

In the last section, we study a manifold $\tilde{M}$ with the additional property that the structure vector field ξ is exterior concurrent [15], [16]. In this case the second covariant differential of ξ satisfies $\tilde{\nabla}^2\xi = c^2\tilde{\eta} \wedge d\tilde{p}$. If $\tilde{\mathcal{D}}$ denotes the tangential antiinvariant distribution of dimension $m+1$ on $\tilde{M}$ [20], then the necessary and sufficient condition that the above equation be satisfied, is that $\tilde{\mathcal{D}}$ and $\Phi\tilde{\mathcal{D}}$ are both involutive. In this case $\tilde{M}$ is foliate and $\tilde{M} = M_A \times M_A^\perp$, where M_A is a tangential antiinvariant submanifold tangent to $\tilde{\mathcal{D}}$ and $M_A^\perp$ is a normal antiinvariant submanifold tangent to $\Phi\tilde{\mathcal{D}}$. Furthermore M_A is