REFERENCES

1. J. N. Mather — *Stability of C^∞-mappings*, Ann. Math., vol. 89 (1969).
2. P. W. Michor — *Manifolds of Differentiable Mappings*, Shiva mathematics series, no. 3, pag. 18.
3. L. Răileanu — *Varietăţi topologice*, Centrul de multipliere al Univ. „Al. I. Cuza", Iaşi, 1989
4. L. Răileanu, R. Miron — *Geometrie diferenţială*, Centrul de multiplicare al Universităţii „Al. I. Cuza" Iaşi, 1989

Received 24.V.1990

Faculty of Mathematics
University of Iaşi
6600 — Iaşi, România

ANALELE ŞTIINŢIFICE ALE UNIVERSITĂŢII „AL. I. CUZA" DIN IAŞI
Tomul XXXVII, fasc. 2, 1991, secţ. I — Matematică

ASYMPTOTIC STABILITY IN THE NONSIMPLE VISCOELASTICITY

BY

RAMÓN QUINTANILLA

1. **Introduction.** The basic theory of nonsimple materials has been studied by several authors [1, 9—11]. In [11] was established an existence and uniqueness theorem for the elastostatic case. Recently, Iesan and the author [1] have proved some theorems for the elastodynamic case. In this paper we establish some results for the linear nonsimple viscoelastic materials. First, we obtain an existence, uniqueness and continuous dependence result. Then we prove the asymptotic stability of the solutions in the dissipative case.

The method used rests on the semigroup theory of linear operators and some results of topological dynamics for semigroups of contractions and is inspired by the work of Dafermos [5] in the classical linear viscoelasticity. Similar results can be obtained by the methods used in [12]. We also remark that some existence results could be obtained following the methods of Duvaut-Lions [13, pp. 184]. In fact, they use weaker hypotheses to obtain well-posed results.

In the next section, we present the basic equations of the linear nonsimple viscoelasticity. The third section is devoted to the theorem about well-posedness and in the fourth section we prove the asymptotic stability of solutions.

2. **Basic equations.** We consider a body that at time $t=0$ occupies the region B of Euclidean three-dimensional space and is bounded by a smooth surface ∂B. The motion of the body is referred to a fixed system of rectangular Cartesian axes $Ox_i (i=1, 2, 3)$. We designate by **n** the outward unit normal of B. Letters in boldface stand for tensors of an order $p \geq 1$ and if **v** has order p, we write $v_{ij\ldots k}$ (p subscripts) for the components of **v** in the Cartesian coordinate frame. We shall employ the usual summation convention and differentation conventions: Latin subscripts are understood to range over the integer (1, 2, 3), summation over repeated subscripts is implied and subscripts preceded by a comma denote partial differentiation with respect to the corresponding Cartesian coordinate. In all that follows, we use a superposed dot to denote partial differentiation with respect to the time.

By $\ll C \gg$ we denote the closed vectorial space generates by a set C.

We consider the linear theory of second order viscoelastic bodies. In the mechanical theory the equation of motion is given by ([9], [10])

$$\rho\ddot{\mathbf{u}} = Div[\tau - Div\mu] + \rho\mathbf{F}, \ (\mathbf{x}, t) \in B \times \mathbb{R}^+ \tag{2.1}$$

where $\mathbf{u}$ is the displacement vector, ρ is the density in the reference configuration, $\rho\mathbf{F}$ is the body force per unit volume in the reference configuration, μ is the hyperstress tensor and τ is defined by

$$\tau = t + Div\mu, \tag{2.2}$$

t being the stress tensor. We know the symmetry

$$\tau = \tau^t. \tag{2.3}$$

We restrict our attention to the homogeneous conditions

$$\mathbf{u}=0, \text{ and } D\mathbf{u}=0 \text{ on } \partial B\times\mathbb{R} \tag{2.4}$$

where $D\mathbf{u}=\nabla\mathbf{u}\cdot\mathbf{n}$.

In the linear second order theory of viscoelasticity the constitutive equations may be obtained in a similar way to [7, 8]. Thus,

$$\tau(t)=\mathbf{G}(0)\mathbf{e}[(t)]+\mathbf{H}(0)[\nabla^2\mathbf{u}(t)]+\int_0^\infty[\mathbf{G}'(s)[\mathbf{e}(t-s)]+\mathbf{H}'(s)[\nabla^2\mathbf{u}(t-s)]]ds$$

$$\mu(t)=\mathbf{H}(0)\mathbf{e}[(t)]+\mathbf{K}(0)[\nabla^2\mathbf{u}(t)]+\int_0^\infty[\mathbf{H}'(s)[\mathbf{e}(t-s)]+\mathbf{K}'(s)[\nabla^2\mathbf{u}(t-s)]]ds, \tag{2.5}$$

where $\mathbf{e}=(\nabla\mathbf{u}+\nabla\mathbf{u}^t)/2$ and $\nabla^2\mathbf{u}=\{u_{r,ij}\}$.

The tensor $\mathbf{K}$, $\mathbf{H}$ and $\mathbf{G}$ satisfy the symmetries

$$G_{ijrs}=G_{rsij}=G_{jirs},\ H_{ijpqr}=H_{jipqr}=H_{ijqpr},\ K_{ijkpqr}=K_{pqrijk}=K_{jikpqr}. \tag{2.6}$$

If we introduce the equations (2.5) in the evolution equation (2.1), we obtain the equations of motion for the linear second order viscoelasticity

$$\ddot{\mathbf{u}}(t)=\mathbf{F}(t)+\rho^{-1}Div(\mathbf{G}(0)[\mathbf{e}(t)]+\mathbf{H}(0)[\nabla^2\mathbf{u}(t)]-Div(\mathbf{H}(0)[\mathbf{e}(t)]+\mathbf{K}(0)[\nabla^2\mathbf{u}(t)]))+$$

$$+\rho^{-1}\int_0^\infty Div[\mathbf{G}'(s)[\mathbf{e}(t-s)]+\mathbf{H}'(s)[\nabla^2\mathbf{u}(t-s)]$$

$$-Div(\mathbf{H}'(s)[\mathbf{e}(t-s)]+\mathbf{K}'(s)[\nabla^2\mathbf{u}(t-s)])]ds,\ (\mathbf{x},t)\in B\times\mathbb{R}^+. \tag{2.7}$$

3. An existence theorem. In this section, we use the semigroups theory of linear operators to obtain an existence theorem. We assume that the following assumptions are satisfied:

(i) The density $\rho(\mathbf{x})$ is strictly positive;

(ii) We suppose the existence of

$$\mathbf{G}(\mathbf{x},\infty)=\lim_{s\to\infty}\mathbf{G}(\mathbf{x},s),\ \mathbf{H}(\mathbf{x},\infty)=\lim_{s\to\infty}\mathbf{H}(\mathbf{x},s),\ \mathbf{K}(\mathbf{x},\infty)=\lim_{s\to\infty}\mathbf{K}(\mathbf{x},s); \tag{3.1}$$

(iii) $\mathbf{G}(\mathbf{x},s)$, $\mathbf{H}(\mathbf{x},s)$ and $\mathbf{K}(\mathbf{x},s)$ are continuous on B, are functions of class C^2 on $[0,\infty)$ and they satisfy the symmetry conditions (2.6);

(iv) There exists a positive constant c_0 such that:

$$\int_B([\nabla^2\mathbf{u}]\cdot\mathbf{K}(\infty)[\nabla^2\mathbf{u}]+2[\nabla\mathbf{u}]\cdot\mathbf{H}(\infty)[\nabla^2\mathbf{u}]+[\nabla\mathbf{u}]\cdot\mathbf{G}(\infty)[\nabla\mathbf{u}])dV\geq \tag{3.2}$$

$$\geq c_0\int_B(\nabla^2\mathbf{u}\cdot\nabla^2\mathbf{u}+\nabla\mathbf{u}\cdot\nabla\mathbf{u})dV,$$

for all $\mathbf{u}\in[C_0^\infty(B)]^3$.

(v) There exists a positive function $g(s)$ satisfying the monotonicity condition:

$$-\int_B([\nabla^2\mathbf{u}]\cdot\mathbf{K}'(s)[\nabla^2\mathbf{u}]+2[\nabla\mathbf{u}]\cdot\mathbf{H}'(s)[\nabla^2\mathbf{u}]+[\nabla\mathbf{u}]\cdot\mathbf{G}'(s)[\nabla\mathbf{u}])dV\geq \tag{3.3}$$

$$\geq g(s)\int_B(\nabla^2\mathbf{u}.\nabla^2\mathbf{u}+\nabla\mathbf{u}\cdot\nabla\mathbf{u})dV,$$

for all $\mathbf{u}\in[C_0^\infty(B)]^3$ and for all $s\in[0,\infty)$.

(vi) There exists a positive constant α such that

$$\int_B([\nabla^2\mathbf{u}]\cdot(\mathbf{K}''(s)-\alpha\mathbf{K}'(s))[\nabla^2\mathbf{u}]+2[\nabla\mathbf{u}]\cdot(\mathbf{H}''(s)-\alpha\mathbf{H}'(s))[\nabla^2\mathbf{u}]+ \tag{3.4}$$

$$+[\nabla\mathbf{u}]\cdot(\mathbf{G}''(s)-\alpha\mathbf{G}'(s))[\nabla\mathbf{u}])dV\geq 0,$$

for all $\mathbf{u}\in[C_0^\infty(B)]^3$ and for all $s\in[0,\infty)$.

Before establishing the abstract evolution equation and the Hilbert space we are to work with, we make a remark.

Remark. Let $L^2_{\mathcal{M}}([0,\infty),\ \mathbf{W}_0^{2,2}(B))$ be the space of the functions $\mathbf{z}:[0,\infty)\to\mathbf{W}_0^{2,2}(B)$ such that

$$\left|\int_0^\infty\int_B([\nabla^2\mathbf{z}]\cdot\mathbf{K}'(s)[\nabla^2\mathbf{z}]+2[\nabla\mathbf{z}]\cdot\mathbf{H}'(s)[\nabla^2\mathbf{z}]+[\nabla\mathbf{z}]\cdot\mathbf{G}'(s)[\nabla\mathbf{z}])dVds\right|<\infty, \tag{3.5}$$

where $W_0^{2,2}(B)$ is the Sobolev space ([2]). We may define a structure of Hilbert space in $L^2_{\mathcal{M}}([0,\infty),\ \mathbf{W}_0^{2,2}(B))$ with the inner product

$$J(\mathbf{z},\mathbf{z}^*)=-\int_0^\infty\int_B([\nabla^2\mathbf{z}]\cdot\mathbf{K}'(s)[\nabla^2\mathbf{z}^*]+[\nabla\mathbf{z}]\cdot\mathbf{H}'(s)[\nabla^2\mathbf{z}^*]+ \tag{3.6}$$

$$+[\nabla\mathbf{z}^*]\cdot\mathbf{H}'(s)[\nabla^2\mathbf{z}]+[\nabla\mathbf{z}]\cdot\mathbf{G}'(s)[\nabla\mathbf{z}^*])dVds.$$

Now, we transform our problem in an abstract evolution problem on the Hilbert space $\mathcal{H}=\mathbf{W}_0^{2,2}(B)\times\mathbf{W}_0^{2,2}(B)\times L^2_{\mathcal{M}}([0,\infty),\ \mathbf{W}_0^{2,2}(B))$ with the inner product defined by

$$<(\mathbf{u},\ \mathbf{v},\ \mathbf{z}),(\mathbf{u}^*,\mathbf{v}^*,\mathbf{z}^*)>_*=\int_B\rho\mathbf{v}\mathbf{v}^*dV+\int_B([\nabla^2\mathbf{u}]\cdot\mathbf{K}(\infty)[\nabla^2\mathbf{u}^*]+$$

$$+[\nabla\mathbf{u}]\cdot\mathbf{H}(\infty)[\nabla^2\mathbf{u}^*]+[\nabla\mathbf{u}^*]\cdot\mathbf{H}(\infty)[\nabla^2\mathbf{u}]+[\nabla\mathbf{u}^*]\cdot\mathbf{G}(\infty)[\nabla\mathbf{u}])dV- \tag{3.7}$$

$$-J(\mathbf{u}-\mathbf{z},\mathbf{u}^*-\mathbf{z}^*).$$

Let $\mathcal{C}$, $\mathcal{M}$ and $\mathcal{F}$ be the linear operators:

$$\mathcal{C}\mathbf{u}=\rho^{-1}Div[\mathbf{G}(0)[\nabla\mathbf{u}]+\mathbf{H}(0)[\nabla^2\mathbf{u}]-Div[\mathbf{H}(0)[\nabla\mathbf{u}]+\mathbf{K}(0)[\nabla^2\mathbf{u}]],$$

$$\mathcal{M}\mathbf{z}=\rho^{-1}Div[\int_0^\infty\mathbf{G}'(s)[\nabla\mathbf{z}(s)]+\mathbf{H}'(s)[\nabla^2\mathbf{z}(s)]-Div[\mathbf{H}'(s)[\nabla\mathbf{z}(s)]+$$

$$+\mathbf{K}'(s)[\nabla^2\mathbf{z}(s)]]ds],$$

$$\mathcal{F}z = -\partial z/\partial s, \tag{3.8}$$

and let $\mathcal{A}$ be the matrix operator on $\mathcal{H}$ defined by:

$$\mathcal{A} = \begin{pmatrix} 0 & Id & 0 \\ \mathcal{C} & 0 & \mathcal{M} \\ 0 & 0 & \mathcal{F} \end{pmatrix}, \tag{3.9}$$

where Id is the identity operator. The domain of $\mathcal{A}$ is

$$D(\mathcal{A}) = \{(\mathbf{u}, \mathbf{v}, \mathbf{z}) \in \mathcal{H} / \mathbf{z}(0) = \mathbf{u} \text{ and } \mathcal{A}\begin{pmatrix} \mathbf{u} \\ \mathbf{v} \\ \mathbf{z} \end{pmatrix} \in \mathcal{X}\},$$

wich is dense in $\mathcal{X}$.

Thus our boundary value problem (2.7), (2.4) may be transformed in the following abstract evolution equation in the Hilbert space $\mathcal{X}$

$$\frac{d}{dt}\begin{pmatrix} \mathbf{u} \\ \mathbf{v} \\ \mathbf{z} \end{pmatrix} = \mathcal{A}\begin{pmatrix} \mathbf{u} \\ \mathbf{v} \\ \mathbf{z} \end{pmatrix} + \begin{pmatrix} 0 \\ \mathbf{F} \\ 0 \end{pmatrix}, \quad t \in \mathbb{R}^+. \tag{3.10}$$

Theorem 3. 1. *The operator $\mathcal{A}$ generates a quasi-contractive semigroup on $\mathcal{X}$.*

Proof. We first calculate the product:

$$<\mathcal{A}\begin{pmatrix} \mathbf{u} \\ \mathbf{v} \\ \mathbf{z} \end{pmatrix}, (\mathbf{u}, \mathbf{v}, \mathbf{z})>_* = <(\mathbf{v}, \mathcal{C}\mathbf{u} + \mathcal{M}\mathbf{z}, \mathcal{F}\mathbf{z}), (\mathbf{u}, \mathbf{v}, \mathbf{z})>_*.$$

If we apply the Gauss-Green theorem, the boundary conditions and the symmetries (2.6), we obtain

$$-1/2 \int_B \int_0^\infty ([\nabla^2\mathbf{u} - \nabla^2\mathbf{z}] \cdot \mathbf{K}''(s)[\nabla^2\mathbf{u} - \nabla^2\mathbf{z}] + 2[\nabla\mathbf{u} - \nabla\mathbf{z}] \cdot \mathbf{H}''(s)[\nabla^2\mathbf{u} - \nabla^2\mathbf{z}] +$$
$$+ [\nabla\mathbf{u} - \nabla\mathbf{z}] \cdot \mathbf{G}''(s)[\nabla\mathbf{u} - \nabla\mathbf{z}])\,ds\,dV.$$

By means of the inequality (3.4) we see

$$<\mathcal{A}\begin{pmatrix} \mathbf{u} \\ \mathbf{v} \\ \mathbf{z} \end{pmatrix}, (\mathbf{u}, \mathbf{v}, \mathbf{z})>_* \leq (1/2)\alpha\|(\mathbf{u}, \mathbf{v}, \mathbf{z})\|_*^2.$$

Now, we want to determine the range of $Id - \mathcal{A}$. Then we must consider the system

$$\begin{aligned} \mathbf{u} - \mathbf{v} &= \mathbf{u}_* \\ \mathbf{v} - \mathcal{C}\mathbf{u} - \mathcal{M}\mathbf{z} &= \mathbf{v}_* \\ \mathbf{z} + \partial\mathbf{z}/\partial s &= \mathbf{z}_* \\ \mathbf{z}(0) &= \mathbf{u}, \end{aligned} \tag{3.11}$$

where $(\mathbf{u}_*, \mathbf{v}_*, \mathbf{z}_*) \in \mathcal{X}$. Integrating the third equation of (3.11) we obtain

$$\mathbf{z}(s) = \mathbf{u}e^{-s} + \int_0^s e^{\eta - s}\mathbf{z}_*(\eta)\,d\eta. \tag{3.12}$$

Introducing the first equation in the second one, we get

$$\mathbf{u} - \mathbf{u}_* - \mathcal{C}\mathbf{u} - \mathcal{M}(e^{-s}\mathbf{u}) - \mathcal{M}(\int_0^s e^{\eta - s}\mathbf{z}_*(\eta)\,d\eta) = \mathbf{v}_*.$$

Thus, we obtain

$$\mathbf{u} - \mathcal{C}\mathbf{u} - \mathcal{M}(e^{-s}\mathbf{u}) = \mathbf{u}_* + (\int_0^s e^{\eta - s}\mathbf{z}_*(\eta)\,d\eta) + \mathbf{v}_*. \tag{3.13}$$

An easy calculation shows that the right-hand side of this equality belongs to $\mathbf{W}_0^{-2,2}(B)$. Now, we consider the bilinear form

$$B(\mathbf{u}, \bar{\mathbf{u}}) = \int_B \rho\mathbf{u}\bar{\mathbf{u}}\,dV + \int_B ([\nabla^2\mathbf{u}] \cdot (\mathbf{K}(0) + \int_0^\infty e^{-s}\mathbf{K}'(s)ds)[\nabla^2\bar{\mathbf{u}}] +$$
$$+ [\nabla\mathbf{u}] \cdot (\mathbf{H}(0) + \int_0^\infty e^{-s}\mathbf{H}'(s)ds)[\nabla^2\mathbf{u}] + [\nabla\bar{\mathbf{u}}] \cdot (\mathbf{H}(0) + \int_0^\infty e^{-s}\mathbf{H}'(s)ds)[\nabla^2\mathbf{u}] +$$
$$+ [\nabla\mathbf{u}] \cdot (\mathbf{G}(0) + \int_0^\infty e^{-s}\mathbf{G}'(s)ds)[\nabla\bar{\mathbf{u}}])\,dV,$$

which is a bounded and coercitive symmetric form on $\mathbf{W}_0^{2,2}(B)$. Using the Lax-Milgram theorem [3], we conclude the existence of $\mathbf{u} \in \mathbf{W}_0^{2,2}(B)$ solving (3.13). Then, there exists $\mathbf{v} \in \mathbf{W}_0^{2,2}(B)$ solving the first equation of the system (3.11) Using (3.9) we obtain $\mathbf{z} \in L^2_{\mathcal{M}}([0, \infty), \mathbf{W}_0^{2,2}(B))$. Using the corollary of Lumer-Phillips to the Hille-Yosida theorem [4], our theorem is proved.

This theorem allows us to establish

Theorem 3.2. *Assume that* $\mathbf{F} \in C^1(\mathbb{R}^+, L^2(B)) \cap C^0(+, \mathbf{W}_0^{2,2}(B))$ *and* $(\mathbf{u}_0, \mathbf{v}_0, \mathbf{z}_0) \in D(\mathcal{A})$. *Then, there exists a unique solution* $(\mathbf{u}(t), \mathbf{v}(t), \mathbf{z}(t)) \in C^1(\mathbb{R}^+, \mathcal{X})$ *with values in* $D(\mathcal{A})$ *to the boundary value problem* (2.7), (2.4).

Remark. Since the semigroup defined by the operator $\mathcal{A}$ is quasi-contractive, it follows the estimate for the solutions

$$\|(\mathbf{u}(t), \mathbf{v}(t), \mathbf{z}(t))\|_* \leq e^{\alpha t/2}(\|(\mathbf{u}(0), \mathbf{v}(0), \mathbf{z}(0))\|_* + \int_0^t \|\mathbf{F}(s)\|_{L^2(B)}ds).$$

4. **Asymptotic stability of solutions.** In this section we study the asymptotic behavior of the solutions whose existence have been investigated previously, in the homogeneous case ($\mathbf{F} = 0$). In the former section, we have obtained an existence theorem in a large class for the mechanical coefficients $\mathbf{K}(s)$, $\mathbf{H}(s)$, $\mathbf{G}(s)$. Now, we are going to consider the case in which the semigroup is contractive. We suppose the following hypothesis (related with the usual hypotheses of convexity)

$$\int_B ([\nabla^2\mathbf{u}] \cdot \mathbf{K}''(s)[\nabla^2\mathbf{u}] + 2[\nabla\mathbf{u}] \cdot \mathbf{H}''(s)[\nabla^2\mathbf{u}] + [\nabla\mathbf{u}] \cdot \mathbf{G}''(s)[\nabla u])\,dV \geq 0 \tag{4.1}$$

for all $\mathbf{u} \in [C_0^\infty(B)]^3$ and for all $s \in \mathbb{R}^+$. In order to perform the viscosity effects in the material coefficients, we also suppose that the tensors $\mathbf{K}''$, $\mathbf{H}''$ and $\mathbf{G}''$ are not identically null.

Our proof is inspired by the work of Dafermos [5]. We recall that a sufficient condition to prove asymptotic stability for semigroups of contractions is the precompactness of the orbits jointly with the fact that there are not any eigenvalue $i\lambda(\lambda \in \mathbb{R})$ in the closed subspace $\mathcal{L} = \ll \{(\mathbf{u}, \mathbf{v}, \mathbf{z}) / < \mathcal{A}(\mathbf{u}, \mathbf{v}, \mathbf{z}), (\mathbf{u}, \mathbf{v}, \mathbf{z}) >_* = 0\} \gg$.

Lemma 4.1. *The operator $\mathcal{A}$ has not any eigenvalue $i\lambda(\lambda \in \mathbb{R})$ on $\mathcal{L}$.*

Proof. Let $(\mathbf{u}, \mathbf{v}, \mathbf{z}) \in \mathcal{L}$. By inequality (4.1) we obtain

$$Div[\mathbf{G}''(s)[\nabla\mathbf{u} - \nabla\mathbf{z}] + \mathbf{H}''(s)[\nabla^2\mathbf{u} - \nabla^2\mathbf{z}] - Div([\mathbf{H}''(s)[\nabla\mathbf{u} - \nabla\mathbf{z}] + \tag{4.2}$$
$$+ \mathbf{K}''(s)[\nabla^2\mathbf{u} - \nabla^2\mathbf{z}])] = 0,$$

for all $s \in \mathbb{R}^+$. Let us suppose

$$\mathbf{v} = i\lambda\mathbf{u}, \quad \mathcal{C}\mathbf{u} - \mathcal{M}\mathbf{z} = i\lambda\mathbf{v}, \quad -\partial\mathbf{z}/\partial s = i\lambda\mathbf{z}. \tag{4.3}$$

Integrating by parts (4.2) in $\mathbb{R}^+$ and using the third equality in (4.3) we find

$$\mathcal{M}\mathbf{z} = 0. \tag{4.4}$$

Thus $\mathbf{u}$ is a solution of the equation

$$\mathcal{C}\mathbf{u} + \lambda^2\mathbf{u} = 0, \tag{4.5}$$

and $\mathbf{z}(s) = e^{-i\lambda s}\mathbf{u}$. Introducing these values of $\mathbf{u}$ and $\mathbf{z}$ in the equality (4.2) and using that $\mathbf{K}''(s)$, $\mathbf{H}''(s)$ and $\mathbf{G}''(s)$ are not identically zero we obtain $\mathbf{u} = 0$. Then we find that $\mathbf{v} = 0$ and $\mathbf{z} = 0$. Thus we conclude the lemma.

Lemma 4.2. *The orbits are precompact.*

Proof. Let us denote $\mathcal{H} = \{(\mathbf{u}_*, \mathbf{v}_*, \mathbf{z}_*) \in D(\mathcal{A}) / \mathbf{z}_* \in L^\infty(\mathbb{R}^+, \mathbf{W}^{2,4}(B))\}$ and let $(\mathbf{u}(t), \mathbf{v}(t), \mathbf{z}(t))$ be a solution starting in $(\mathbf{u}_*, \mathbf{v}_*, \mathbf{z}_*) \in \mathcal{H}$. As the semigroup is contractive we know (see [3]) that $(\mathbf{u}(t), \mathbf{v}(t), \mathbf{z}(t)) \in D(\mathcal{A})$ and

$$\left\| \mathcal{A} \begin{pmatrix} \mathbf{u}(t) \\ \mathbf{v}(t) \\ \mathbf{z}(t) \end{pmatrix} \right\|_* \leq \left\| \mathcal{A} \begin{pmatrix} \mathbf{u}_* \\ \mathbf{v}_* \\ \mathbf{z}_* \end{pmatrix} \right\|_*.$$

Thus, we know that $\mathbf{v}(t), \in C^0(\mathbb{R}^+, \mathbf{W}_0^{2,2}(B))$, $\mathbf{z}(t) \in C^0(\mathbb{R}^+, \mathbf{W}^{2,1}_{\mathcal{M}}(\mathbb{R}^+, \mathbf{W}_0^{2,2}(B))$ and $C\mathbf{u}(t) - \mathcal{M}\mathbf{z}(t) \in C^0(\mathbb{R}, \mathbf{L}^2(B))$. Using the standard Volterra iteration scheme (see [12]), we deduce the bound

$$\sup_{\mathbf{R}^+} \|\mathbf{u}\|_{\mathbf{W}_0^{2,4}} \leq r \sup_{\mathbf{R}^+} \|C\mathbf{u} - \mathcal{M}\mathbf{z}\|_{\mathbf{L}^2},$$

where r is a positive constant. Using the compactness of the imbbeding of Sobolev we conclude that $\mathbf{u} \in C^0(\mathbb{R}^+, \mathbf{W}_0^{2,4}(B))$. Then we may conclude the precompactness of the orbits starting in $\mathcal{H}$. To conclude the lemma we may observe that $\mathcal{H}$ is dense in $\mathcal{K}$ jointly with the fact (see [6]) that the set of points wich generate precompact orbits in a semigroup of contractions is closed.

The previous lemmas jointly with the known topological dynamics methods (see [5]) allows to conclude the next theorem.

Theorem 4.1. *Let $(\mathbf{u}(t), \mathbf{v}(t), \mathbf{z}(t))$ be a solution to the boundary value problem (2.7), (2.4). Then*

$$\lim_{t \to \infty} (\mathbf{u}(t), \mathbf{v}(t), \mathbf{z}(t)) = 0,$$

in the norm of the energy given in (3.7).

REFERENCES

1. Iesan D. & Quintanilla R. — *Some Theorems in the Theory of Nonsimple Elastic Bodies*. Preprint. Centre de Recerca Matemàtica. Bellaterra. U.A.B. (1989).
2. Adams R. A., — *Sobolev Spaces*. Academic Press, New York. (1975).
3. H. Brezis H. — *Analyse Fonctionelle*. Masson, Paris. (1983).
4. Marsden J. E. & Hugues T. J. R. — *Mathematical Foundations of Elasticity*. Prentice-Hall Inc. Englewood Cliffs, New Jersey. (1983).
5. Dafermos C. M. — *Contraction semigroups and trend to equilibrium in Continuum Mechanics*. Applications of Methods of Functional Analysis to Problems in Mechanics. Lectures Notes in Mathematics., Springer Verlag, Berlin. (1975).
6. Dafermos C. M. — *Semiflows associated with compact and uniform processes*. Math. Sys. Theory 8 (1974), 142—149.
7. Coleman B. D. & Noll W. — *Foundations of Linear Viscoelasticity*. Rev. Modern Phys. 33 (1961), 239—249.
8. Gurtin M. E. — *Time-Reversal and Symmetry in the Thermodynamics of Materials with Memory*. Arch. Rat. Mech. Anal. 44 (1974), 387—399.
9. Toupin R. A. — *Theories of elasticity with couple-stress*. Arch. Rat. Mech. Anal. 17 (1964), 85—112.
10. Mindlin R. D. — *Microstructure in linear elasticity*. Arch. Rat. Mech. Anal. 16 (1964), 51—78.
11. Hlavacek I. & Hlavacek M. — *On the existence and uniqueness of solutions and some variational principles in linear theories of elasticity with couple-stress*. Aplikace Matematiky 14 (1969), 411—426.
12. Dafermos C. M. — *Asymptotic Stability in Viscoelasticity*. Arch. Rat. Mech. Anal. 37 (1970), 297—308.
13. Duvaut G. & Lions J. L. — *Les Inéquations en Mécanique et en Physique*. Dunod, Paris. (1972).

Received 25.IV.1990

Departament de Matematica Aplicada II
E.T.S. Arquitectura del Vallès U.P.C.,
Spain