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ASYMPTOTIC PROPERTIES OF THE SOLUTIONS OF DIFFERENCE SYSTEMS VIA l^p-DICHOTOMY

BY

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1. **Introduction.** The asymptotic behaviour of the bounded solutions of

$$x(n+1)=A(n)x(n)+f(n, x), \tag{1.1}$$

under the assumption that the solutions of the associated homogeneous (unperturbed) system

$$y(n+1)=A(n)y(n), \tag{1.2}$$

are not all bounded, has been already investigated by many authors. Investigations of the system (1.2) with ordinary or exponential dichotomy have been made by J. S c h i n a s and G. P a p a s c h i n o p o u l o s [4]—[6], [8] and P. P r e d a and M. M e g a n [7]. We are concerned with the extension of such concepts to the l^p-space (l^p-dichotomy), and we shall deal with the existence and asymptotic behaviour of the bounded solutions of (1.1). The results obtained extend some of the results from [1] and [3], and improve some of those from [4]—[8]. In what follows we shall deal, firstly, with the properties of the solutions of (1.2) with l^1-dichotomy. Then, the asymptotic behaviour of the solutions of (1.1) will be investigated, by use of the properties of a suitable operator. In particular, we shall prove that such operator is compact in the space of discrete bounded functions with zero limit as $n\to\infty$. This is true if (1.2) has an l^p-dichotomy with $p>1$, while the preceding statement does not hold if (1.1) has an l^1-dichotomy or an exponential dichotomy.

2. **l^p-dichotomy for linear difference systems.** In the following we shall consider several function spaces or real functions defined on $N=\{0, 1, 2, ...\}$, namely, $B=\{x\,;\ x: N\to R^m,\ x \text{ is bounded}\}$ and $B_0=\{x\,;\ x\in B,\ \lim_{n\to\infty} x(n)=0\}$. We shall denote by $|\cdot|$ the norm in R^m, by $\|\cdot\|$ the supremum norm in B or B_0 and with $\|\cdot\|_p$ the usual l^p-norm, $1\leq p\leq\infty$ ($\|\cdot\|_\infty=\|\cdot\|$).

Let now consider the linear difference system

$$y(n+1)=A(n)y(n),\quad n\in N, \tag{2.1}$$

and let us denote by $Y(\cdot)$ a fundamental matrix of (2.1).

Definition 2.1. *The system (2.1) has an l^p-dichotomy if there exist a positive constant K and an $m\times m$ matrix P_0, with $P_0^2=P_0$, such that*

$$\left[\sum_{s=0}^{n}|Y(n)P_0Y^{-1}(s+1)|^p\right]^{1/p}<K,\quad 0\leq s\leq n, \tag{2.2}$$

$$(2.3)\qquad \left[\sum_{s=n}^{\infty} |Y(n)(I-P_0)Y^{-1}(s+1)|^p\right]^{1/p} < K, \quad 0 \leqslant n \leqslant s,$$

when $1 \leqslant p < \infty$, or

$$(2.2)'\qquad |Y(n)P_0Y^{-1}(s)| < K, \ 0 \leqslant s \leqslant n,$$

$$(2.3)'\qquad |Y(n)(I-P_0)Y^{-1}(s)| < K, \ 0 \leqslant n \leqslant s,$$

when $p = \infty$.

The defined l^∞-dichotomy is the ordinary dichotomy for difference equations [4]–[8]; l^p-dichotomy is a more general case than the:

Definition 2.2. ([4]–[8]). *The system (2.1) has an exponential dichotomy on the set N if there exist a projection P_0, that is a matrix such that $P_0^2 = P_0$, and constants $K > 0$, $0 < r < 1$, such that*

$$(2.2)\qquad |Y(n)P_0Y^{-1}(s)| < K\, r^{n-s}, \quad 0 \leqslant s \leqslant n,$$

$$(2.3)\qquad |Y(n)(I-P_0)Y^{-1}(s)| < K\, r^{s-n}, \quad 0 \leqslant n \leqslant s.$$

If $r = 1$, then (2.1) has an l^∞-dichotomy.

Definition 2.3. ([11], [12]). *The equilibrium $y = 0$ of the equation (2.1) is said to be l^p-stable if it is stable and if $\sum\limits_{s=n_0}^{\infty} |y(s; n_0, y_0)|^p < \infty$ for some $p > 0$.*

It is easy to show that if $y = 0$ is l^p-stable, then any solution of (2.1) is l^p-stable.

Using simple arguments the following proposition can be proved.

Proposition 2.1. *The system (2.1) has:*

— *an l^p-dichotomy, $1 \leqslant p < \infty$, with $P_0 = I$, if and only if it is l^p-stable;*

— *an exponential dichotomy when $P_0 = I$, if and only if it is asymptotically uniformly stable;*

— *an l^∞-dichotomy when $P_0 = I$, if and only if it is uniformly stable.*

To determine the asymptotic behaviour of the solutions of (2.1), we shall use the following two lemmas:

Lemma 2.1. ([10]). *Let $Y(n)$ be a nonsingular matrix defined for $n \in N(n_0) = \{n_0, n_0+1, \ldots\}$. Assume there exist a projection P_0 and a constant $K > 1$ such that for $n \in N(n_0)$, the following inequality*

$$\sum_{s=n_0}^{n} |Y(n)P_0Y^{-1}(s)|^p < K^p, \ p > 1,$$

holds. Then there exists a constant $L > 0$ such that the following inequalities are valid for $n \in N(n_0)$:

$$(2.4)\qquad \begin{aligned} &|Y(n)P_0| \leqslant L(n+1-n_0)^{1-1/p}(1-K^{-1})(1-K^{-1}2^{1/p-1})\ldots \\ &(1-K^{-1}(n+1-n_0)^{1/p-1}) \leqslant L(n+1-n_0)^{1-1/p}\exp[-(n+1-n_0)^{1/p}K^{-1}]. \end{aligned}$$

Remarks. 2.1) From (2.4) we have, obviously, $\lim\limits_{n\to\infty} |Y(n)P_0| = 0$.

2.2) An analogous result may be proved in the case $p = 1$ when the inequality (2.4) becomes

$$|Y(n)P_0| \leqslant \overline{L}\, K^{-1}(K-1)^{n-n_0} \leqslant \overline{L}\exp[-(n-n_0)K^{-1}],$$

where $\overline{L} \geqslant K\,|Y(n_0)P_0|$ and again $\lim\limits_{n\to\infty} |Y(n)P_0| = 0$.

2.3) Under the same hypotheses as in Lemma 2.1 it follows also (see [9]) that $|Y(n)P_0| \in l^p$.

Lemma 2.2. *Let $Y(n)$ be a nonsingular matrix defined for $n \in N(n_0)$. Suppose there exist a projection P_0 and a constant $K > 1$ such that*

$$\sum_{s=n}^{\infty} |Y(n)(I-P_0)Y^{-1}(s)|^p \leqslant K^p, \ \text{for } n \in N(n_0), \ 1 \leqslant p < \infty.$$

Then, for any $\xi \in R^m$ such that $(I-P_0)\xi \neq 0$, *we have*

$$\lim_{n\to\infty} |Y(n)(I-P_0)\xi| = \infty.$$

Proof. If $p = 1$, putting $\varphi(n) = |Y(n)(I-P_0)\xi|^{-1}$, we have $\left[\sum\limits_{s=n}^{\nu}\varphi(s)\right]|Y(n)(I-P_0)\xi| = \sum\limits_{s=n}^{\nu}|Y(n)(I-P_0)Y^{-1}(s)Y(s)(I-P_0)|\varphi(s)$ and hence $[\varphi(n)]^{-1}\sum\limits_{s=n}^{\nu}\varphi(s) \leqslant K$. Thus $\sum\limits_{s=n}^{\infty}\varphi(s)$ exists, which implies $\lim\limits_{n\to\infty}\varphi(n) = 0$.

If $1 < p < \infty$, let us denote by $\varphi(n) = |Y(n)(I-P_0)\xi|^{-p}$. Then we have

$$\begin{aligned} \varphi^{-1/p}(n)\sum_{s=n}^{\nu}\varphi(s) &= \sum_{s=n}^{\nu}|Y(n)(I-P_0)Y^{-1}Y(s)(I-P_0)\xi|\varphi(s) \leqslant \\ &\leqslant \left(\sum_{s=n}^{\nu}|Y(n)(I-P_0)Y^{-1}(s)|^p\right)^{1/p}\left(\sum_{s=n}^{\nu}|Y(s)(I-P_0)\xi\varphi(s)|^q\right)^{1/q} \leqslant \\ &\leqslant K\left(\sum_{s=n}^{\nu}|Y(s)(I-P_0)\xi|^q\varphi(s)^q\right)^{1/q}, \quad p^{-1}+q^{-1} = 1. \end{aligned}$$

Taking into account that

$$\begin{aligned} |Y(s)(I-P_0)\xi|^q\varphi(s)^q &= |Y(s)(I-P_0)\xi|^q|Y(s)(I-P_0)\xi|^{-pq} = \\ &= |Y(s)(I-P_0)\xi|^{q(1-p)} = |Y(s)(I-P_0)\xi|^{-p} = \varphi(s), \end{aligned}$$

we get $[\varphi(n)]^{-1/p}\sum\limits_{s=n}^{\nu}\varphi(s) \leqslant K\left(\sum\limits_{s=n}^{\nu}\varphi(s)\right)^{1/q}$, i.e.,

$$(\varphi(n))^{-1/p}\left(\sum_{s=n}^{\nu}\varphi(s)\right)^{1-1/q} = \varphi(n))^{-1/p}\left(\sum_{s=n}^{\nu}\varphi(s)\right)^{1/p} \leqslant K,$$

or, $(\varphi(n))^{-1}\sum\limits_{s=n}^{\nu}\varphi(s) \leqslant K^p$, for any $\nu \geqslant n$, $\nu \in N$.

From here it follows that $\sum\limits_{s=n}^{\infty}\varphi(s) < \infty$ and this implies that $\lim\limits_{n=\infty}\varphi(n) = 0$.

As a consequence of the preceding Lemmas we have the following

Theorem 1.1. *If the system (2.1) has an l^p-dichotomy, $1 \leqslant p < \infty$, with projection P_0, then there exists a subspace B_k of bounded solutions which tend* *to zero of (2.1) and a subspace U_{m-k} of unbounded solutions of (2.1) such that $S=B_k\oplus U_{m-k}$, where S is the m-dimensional space of all solutions of (2.1). Moreover, we have $\xi\in \mathrm{Im}(P_0)$ if and only if $n\to Y(n)\xi\in B_k$, $n\in N$, $k=\dim \mathrm{Im}(P_0)$.*

Proof. Let $y(n)$ be a solution of (2.1). Then we have

$$y(n)=Y(n)Y^{-1}(0)\eta=Y(n)P_0Y^{-1}(0)\eta+Y(n)(I-P_0)Y^{-1}(0)\eta$$

and from Lemmas 2.1 and 2.2, for $\xi=Y^{-1}(0)\eta$, and $y(n)$ is a bounded solution if and only if $(I-P_0)Y^{-1}(0)\eta=0$, i.e., if and only if $Y^{-1}(0)\eta\in \mathrm{Im}(P_0)$. Moreover, from Lemma 2.1 we get $\lim\limits_{n\to\infty} x(n)=0$, $\forall x\in B_k$.

3. l^p-dichotomy for perturbed difference systems. Let now consider the homogeneous system

$$y(n+1)=A(n)y(n), \tag{3.1}$$

and its corresponding perturbed system

$$x(n+1)=A(n)x(n)+f(n,x), \tag{3.2}$$

where $f:N\times R^m\to R^m$ is a continuous function. The following result holds:

Proposition 3.1. *Assume that the following hypotheses hold;*

i) The linear system (3.1) has an l^p-dichotomy, $1\leqslant p\leqslant\infty$;

ii) There exists a function $\omega(n,v): N\times R_+\to R_+$ continuous and non-decreasing in v for each $n\in N$ such that

$$\omega(n,c)\in l^q, \text{ for every constant } c,\ 0<c<\infty,\ p^{-1}+q^{-1}=1, \tag{3.3}$$
$$(p=1,\ q=\infty;\ p=\infty,\ q=1);$$

$$|f(n,u)|\leqslant\omega(n,|u|),\ n\in N,\ u\in R^m. \tag{3.4}$$

Then, a function $u\in B$ is a solution of (3.2) if and only if it is a B-solution of

$$z(n)=Y(n)P_0Y^{-1}(0)z(0)+\sum_{s=0}^{n-1}Y(n)P_0Y^{-1}(s+1)f(s,z(s))- \tag{3.5}$$
$$-\sum_{s=n}^{\infty}Y(n)(I-P_0)Y^{-1}(s+1)f(s,z(s)),\ n\geqslant 1.$$

Proof. Let $u(n)$ be a solution of (3.5). It is easily to verify that $u(n)$ is a bounded solution (B-solution) of (3.2).

Let u be a B-solution ($|u|\leqslant c$) of (3.2); from the variation of constant formula we have

$$u(n)=Y(n)Y^{-1}(0)u(0)+\sum_{s=0}^{n-1}Y(n)Y^{-1}(s+1)f(s,u(s))=$$
$$=Y(n)P_0Y^{-1}(0)u(0)+Y(n)(I-P_0)Y^{-1}(0)u(0)+\sum_{s=0}^{n-1}Y(n)P_0Y^{-1}(s+1) \tag{3.6}$$
$$f(s,u(s))+\sum_{s=0}^{\infty}Y(n)(I-P_0)Y^{-1}(s+1)f(s,u(s))-\sum_{s=n}^{\infty}Y(n)(I-$$
$$-P_0)Y^{-1}(s+1)f(s,u(s)),\ n\geqslant 1.$$

The sums in the above formula exist because of from i) and ii) we have:

$$p=1:\ \left|\sum_{s=n}^{\infty}Y(n)(I-P_0)Y^{-1}(s+1)f(s,u(s))\right|\leqslant\sum_{s=n}^{\infty}|Y(n)(I-P_0)Y^{-1}(s+1)|\sup_{n\in N}\omega(n,c);$$

$$1<p<\infty:\ \left|\sum_{s=n}^{\infty}Y(n)(I-P_0)Y^{-1}(s+1)f(s,u(s))\right|\leqslant\left[\sum_{s=n}^{\infty}|Y(n)(I-P_0)Y^{-1}(s)|^p\right]^{1/p}\cdot$$
$$\left[\sum_{s=n}^{\infty}\omega^q(s,c)\right]^{1/q}\leqslant K\,\|\omega(n,c)\|_q,\quad n\in N;$$

$$p=\infty:\ \left|\sum_{s=n}^{\infty}Y(n)(I-P_0)Y^{-1}(s+1)f(s,u(s))\right|\leqslant K\sum_{s=n}^{\infty}|f(s,u(s))|\leqslant K\sum_{s=n}^{\infty}\omega(s,c)\leqslant$$
$$\leqslant K\|\omega(n,c)\|_1,\ n\in N.$$

Reasoning as above it follows that $\sum\limits_{s=0}^{n-1}Y(n)P_0Y^{-1}(s+1)f(s,u(s))$, $n\in N$, is bounded. Moreover, because of i) we have $Y(n)P_0Y^{-1}(0)u(0)\in B_k\subset B$, $n\in N$. Therefore, from (3.6) it follows that

$$Y(n)(I-P_0)\left[Y^{-1}(0)u(0)+\sum_{s=0}^{\infty}Y^{-1}(s+1)f(s,u(s))\right],\quad n\in N,\ \text{ is bounded.}$$

Now, from Lemma 2.2, if $1\leqslant p<\infty$, or from i) if $p=\infty$, we get

$$(I-P_0)\left[Y^{-1}(0)u(0)+\sum_{s=0}^{\infty}Y^{-1}(s+1)f(s,u(s))\right]=0.$$

Proposition 3.2. *Assume that the conditions i) and ii) of the Proposition 3.1. hold. If for $p=1$ ($q=\infty$) there exist $\gamma\in R_+$ and a function $\lambda(n)\in B$ with $\limsup\limits_{n\to\infty}\sup\limits_{s\geqslant n}\lambda(s)=0$ such that*

$$\omega(n,|u|)\leqslant\gamma|u|+\lambda(n),\ n\in N,\ u\in R^m,\ 2K\gamma<1; \tag{3.7}$$

and for $p=\infty$ ($q=1$)

$$|Y(n)P_0|\to 0,\ as\ n\to\infty, \tag{3.8}$$

then every B-solution of (3.2) belongs to B_0.

Proof. Let $u\in B$ ($|u|\leqslant c$) be a solution of (3.2). By Proposition 3.1, u is a solution of (3.5). For a fixed $n_1\in N$, from (3.5) we have

$$|u(n)|\leqslant|Y(n)P_0Y^{-1}(0)u(0)|+\sum_{s=0}^{n_1-1}|Y(n)P_0Y^{-1}(s+1)f(s,u(s))|+ \tag{3.9}$$
$$+\sum_{s=n_1}^{n-1}|Y(n)P_0Y^{-1}(s+1)f(s,u(s))|+\sum_{s=n}^{\infty}|Y(n)(I-P_0)Y^{-1}(s+1)$$
$$f(s,u(s))|,\ n\geqslant 1.$$

If $p=\infty$, from (3.8) we have

$$|u(n)|\leqslant|Y(n)P_0|\left[|Y^{-1}(0)u(0)|+\sum_{s=0}^{n_1-1}|Y^{-1}(s+1)f(s,u(s))|\right]+K\sum_{s=n_1}^{\infty}\omega(n,c),$$

$0\leqslant n_1\leqslant n$ and from (3.8), choosing $n_1\in N$ sufficiently large, the statement follows.

If $1<p<\infty$, from (3.9) we have

$$|u(n)|\leqslant|Y(n)P_0|\left[|Y^{-1}(0)u(0)|+\sum_{s=0}^{n_1-1}|Y^{-1}(s+1)f(s,u(s))|\right]+$$
$$+\left[\sum_{s=n_1}^{n-1}|Y(n)P_0Y^{-1}(s+1)|^p\right]^{1/p}\left[\sum_{s=n_1}^{n-1}\omega^q(s,c)\right]^{1/q}+\left[\sum_{s=n}^{\infty}\omega^q(s,c)\right]^{1/q}\cdot$$
$$\cdot\left[\sum_{s=n}^{\infty}|Y(n)(I-P_0)Y^{-1}(s+1)|^p\right]^{1/p}\leqslant|Y(n)P_0|\left[|Y^{-1}(0)u(0)|+\right.$$
$$\left.+\sum_{s=0}^{n_1-1}|Y^{-1}(s+1)f(s,u(s))|\right]+K\left\{\left[\sum_{s=n_1}^{n-1}\omega^p(s,c)\right]^{1/p}+\left[\sum_{s=n}^{\infty}\omega^q(s,c)\right]^{1/q}\right\}.$$

Choosing $n_1\in N$ sufficiently large, the statement follows. When $p=1$, we shall follow similar arguments to those of W. A. Coppel ([2],, Th. 8, p. 68 and Th. 10, p. 7)) with slight modifications.

From (3.7) and (3.9) we have

$$|u(n)|\leqslant|Y(n)P_0|\,|Y^{-1}(0)u(0)|+\left(\sum_{s=0}^{n-1}|Y(n)P_0Y^{-1}(s+1)|\right)\left(\gamma\sup_{0\leqslant s\leqslant n-1}|u(s)|+\right.$$
$$\left.+\lambda(s)\right)+\sum_{s=n}^{\infty}|Y(n)(I-P_0)Y^{-1}(s+1)|\,(\gamma\sup_{s\geqslant n}|u(s)|+\lambda(s)).$$

The l^1-dichotomy and Lemma 2.1 (Remark 2.2) imply $|Y(n)P_0|\leqslant M$, for $n\in N$, and therefore

$$|u(n)|\leqslant M|Y^{-1}(0)u(0)|+2\gamma K\sup_{s\in N}|u(s)|+2K\wedge,\ \wedge=\sup_{n\in N}\lambda(n).$$

From here we have

$$\sup_{s\in N}|u(s)|\leqslant M|Y^{-1}(0)u(0)|+2\gamma K\sup_{s\in N}|u(s)|+2K\wedge,$$

which implies

$$\sup_{s\in N}|u(s)|\leqslant(1-2\gamma K)^{-1}[M|Y^{-1}(0)u(0)|+2K\wedge],$$

an so

$$|u(n)|\leqslant(1-2\gamma K)^{-1}[M|Y^{-1}(0)u(0)|+2K\wedge],\ n\in N.$$

Therefore it follows the existence of $\mu=\overline{\lim_{n\to\infty}}|u(n)|$. Now we choose θ so that $2\gamma K<\theta<1$. If $\mu>0$, then $|u(n)|\leqslant\theta^{-1}\mu$, for $n\geqslant n_1$, say. From (3.5) we obtain that for $n\geqslant n_1$

$$|u(n)|\leqslant|Y(n)P_0|\,|Y^{-1}(0)u(0)|+|Y(n)P_0|\left|\sum_{s=0}^{n_1-1}Y^{-1}(s+1)f(s,u(s))\right|+$$
$$+\sum_{s=n_1}^{n}|Y(n)P_0Y^{-1}(s+1)|[\gamma|u(s)|+\lambda(s)]+\sum_{s=n}^{\infty}|Y(n)(I-P_0)Y^{-1}(s+1)|[\gamma|u(s)|+$$
$$+\lambda(s)]\leqslant|Y(n)P_0|\,|Y^{-1}(0)u(0)|+|Y(n)P_0|\left|\sum_{s=0}^{n_1-1}Y^{-1}(s+1)f(s,u(s))\right|+\gamma K\theta^{-1}\mu+$$
$$+K\sup_{n_1\leqslant s\leqslant n}\lambda(s)+\gamma K\theta^{-1}\mu+K\sup_{s\geqslant n}\lambda(s).$$

If $n_1\in N$ is in addition such that $\sup_{n_1\leqslant s\leqslant n}\lambda(s)\leqslant\sup_{n_1\leqslant s}\lambda(s)<\varepsilon$ for any $\varepsilon>0$, then we have, letting $n\to\infty$, $\mu(1-2\gamma K\theta^{-1})<\varepsilon$, $\forall\varepsilon>0$, which is impossible and hence $\mu=0$.

Let us now consider and study an operator associated to system (3.2), below given.

Theorem 3.1. *Assume that the conditions i) and ii) from Proposition 3.1 hold for $1<p\leqslant\infty$ and that $|Y(n)P_0|\to 0$ as $n\to\infty$. Then the operator $F: B_0\to B$,*

$$(Fx)(n)=\sum_{s=0}^{n-1}Y(n)P_0Y^{-1}(s+1)f(s,x(s))-\sum_{s=n}^{\infty}Y(n)(I-P_0)Y^{-1}(s+1)f((s,x(s))$$

maps B_0 into B_0 and it is completely continuous.

Proof. If $x\in B_0$, then, from the proof of Proposition 3.2, it follows $Fx\in B_0$. To establish that the mapping F is continuous, let $\{x_i\}\subset B_0$, $x_i\to\to x\in B_0$, as $i\to\infty$. Then we have

$$|(Fx_i)(n)-(Fx)(n)|^q\leqslant\left\{\sum_{s=0}^{n-1}|Y(n)P_0Y^{-1}(s+1)|\,|f(s,x_i(s))-f(s,x(s))|+\right.$$
$$\left.+\sum_{s=n}^{\infty}|Y(n)(I-P_0)Y^{-1}(s+1)|\,|f(s,x_i(s))-f(s,x(s))|\right\}^q\leqslant$$
$$\leqslant K^p2^q\sum_{s=0}^{\infty}|f(s,x_i(s))-f(s,x(s))|^q.$$

Taking into account that $\lim_{i\to\infty}\|f(\cdot,x_i(\cdot))-f(\cdot,x(\cdot))\|^q=0$, and

$$|f(s,x_i(s))-f(s,x(s))|^q\leqslant 2^q[|f(s,x_i(s))|^q+|f(s,x(s))|^q]\leqslant$$
$$\leqslant 2^q[\omega^q(s,\|x_i\|)+\omega^q(s,\|x\|)]\in l^p,$$

it follows that $\lim_{i\to\infty}\|Fx_i-Fx\|=0$, and therefore F is continuous. A slight different argument to prove the continuity of F is given in [10] where the considered topology is that of the uniform convergence on every set $N_k(n_0)=\{n_0,n_0+1,\ldots,n_0+k\}$.

Let Ω be a bounded subset of B_0. To prove that $F(\Omega)$ is equibounded, we consider $x(n)\in\Omega$ for which $\|x\|\equiv\sup_{n\in N}|x(n)|\leqslant b$, $\forall x\in\Omega$. Then, as above,

$$|(Fx)(n)|^q\leqslant K^q\left\{\left[\sum_{s=0}^{n-1}|f(s,x(s))|^q\right]^{1/q}+\left[\sum_{s=n}^{\infty}|f(s,x(s))|^q\right]^{1/q}\right\}^q\leqslant$$
$$\leqslant K^q2^q\sum_{s=0}^{\infty}|f(s,x(s))|^q\leqslant K^q2^q\sum_{s=0}^{\infty}\omega^q(s,b).$$

The equicontinuity of the family $F(\Omega)$ follows immediately because of the functions in $F(\Omega)$ are defined for a discrete variable n.

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OPTIMAL FEEDBACK CONTROL FOR A PROBLEM GOVERNED BY A MULTIVALUED DIFFERENTIAL EQUATION

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This article is concerning with the study of the existence of optimal feedback controls for a control problem governed by a multivalued differential equation. Will be shown that, under some specified assumptions, every optimal control for that problem is an optimal feedback control.

The problem taken into consideration, denoted by (P), has the following statement :

Minimize :

$$\int_0^T (g(y(t))+h(v(t)))dt+\varphi_0(y(T)), \qquad 0<T<\infty,$$

over the set of all functions $y\in W^{1,2}(0, T; R^n)$ which satisfy the multivalued differential equation

$$(\mathrm{E}_0) \qquad y'(t)+(F+\beta)(y(t))\ni B(u(t)) \text{ a.e. } t\in]0, T[,$$

with the initial condition

$$(\mathrm{IC}_0) \qquad y(0)=y_0\in D(\beta),$$

where $u\in W^{1,2}(0, T; R^m)$ is the unique solution to the differential equation

$$(\mathrm{ED}_0) \qquad u'(t)+G(u(t))=v(t) \text{ a.e. } t\in]0, T[,$$

with the initial condition

$$(\mathrm{ICED}_0) \qquad u(0)=u_0\in R^m,$$

when $v\in L^2(0, T; R^m)$ and u_0 is an arbitrarily fixed point.

The elements which appear have the following properties :

(i) $F: R^n\to R^n$ is a local Lipschitz function, which satisfies the condition: there exist $y_1\in D(\beta)$ and the constants $D_1, D_2\in R^*_+$ such that

$$(1) \qquad (Fy, y-y_1)\geq -D_1\|y-y_1\|^2-D_2 \text{ for every } y\in R^n;$$

(ii) $G: R^m\to R^m$ is a local Lipschitz function, which satisfies the condition : there exist the constants $A_1, A_2\in R^*_+$ such that

$$(2) \qquad (Gu, u)\geq -A_1\|u\|^2_m-A_2 \text{ for every } u\in R^m;$$