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STABILITY TESTS FOR THE SOLUTIONS OF CERTAIN LINEAR EQUATIONS IN PARTIAL DIFFERENCE EQUATION

BY

H. ATTIA HUSSEIN

1.Introduction. In a previous work [1] we obtained a solution of a difference boundary problem

$$\Delta_1^{p_1}\Delta_2^{p_2}\dots\Delta_n^{p_n} - \sum A_{q_1\dots q_n}\Delta_1^{q_1}\Delta_2^{q_2}\dots\Delta_n^{q_n} y = f(t_1, t_2, \dots, t_n) \tag{1.1}$$

$$y|_{0\le t_j<p_j\delta_j} = \theta \ (j = 1, 2, \dots, n) \tag{1.2}$$

in the region $0 \le t_j < \infty$, in the form

$$\begin{aligned} y =&(-\frac{1}{2\pi i})^n \oint_{c_1}\oint_{c_2}\dots\oint_{c_n} \Pi_{i=1}^n (I-\lambda_i S_i)^{-1} S_i^{p_i}\cdot \\ &\cdot\left[I - \sum \frac{A_{q_1\dots q_n}}{\lambda_1^{p_1-q_1}\dots\lambda_n^{p_n-q_n}}\right]^{-1} f\frac{d\lambda_1}{\lambda_1}\frac{d\lambda_2}{\lambda_2}\dots\frac{d\lambda_n}{\lambda_n}\end{aligned} \tag{1.3}$$

$$\begin{aligned} y =&(-\frac{1}{2\pi i})^n \oint_{c_1}\oint_{c_2}\dots\oint_{c_n} \Pi_{i=1}^n \exp\frac{t_i}{\delta_i}\ln(1+\delta_i\lambda_i)\cdot \\ &\cdot\frac{\delta_i}{1+\delta_i\lambda_i}(E_i - I)^{-1}\exp -\frac{t_i}{\delta_i}\ln(1+\delta_i\lambda_i)(\delta_i(E_i-I)^{-1})^{p_i-1}\cdot \\ &\cdot\left[I - \sum \frac{A_{q_1\dots q_n}}{\lambda_1^{p_1-q_1}\dots\lambda_n^{p_n-q_n}}\right]^{-1} f\frac{d\lambda_1}{\lambda_1}\frac{d\lambda_2}{\lambda_2}\dots\frac{d\lambda_n}{\lambda_n}\end{aligned} \tag{1.4}$$

Here

$$\Delta_j y = \frac{y(t_1, \dots, t_{j-1}, t_j+\delta_j, t_{j+1}, \dots, t_n) - y(t_1, \dots, t_n)}{j}, \ \delta_j > 0 \quad j = 1, 2, \dots, n.$$

$p_j \geq q_j$ but $\sum p_j > \sum q_j$ and $A_{q_1,\dots,q_n}$ denotes families of constant and bounded linear operators which act in a complex Banach space. The contours $c_1, c_2, \dots, c_n$ must be chosen so that for any $\lambda_i \in c_i$ the point $(\lambda_1, \lambda_2, \dots, \lambda_n)$ is the regular point of the operator - function

$$\Gamma(\lambda_1, \lambda_2, \dots, \lambda_n) = [\lambda_1^{p_1} \dots \lambda_n^{p_n} - \sum A_{q_1 \dots q_n} \lambda_1^{q_1} \dots \lambda_n^{q_n}]^{-1}$$

The operator-function Γ is uniformly bounded for every $\lambda_i \in c_i$ $(i = 1, 2, \dots, n)$ i.e. $\|\Gamma(\lambda_1, \dots, \lambda_n)\| \leq D < \infty$.

The functions $y = y(t_1, \dots, t_n)$ and $f = f(t_1, \dots, t_n)$ are elements of the space of continuous functions whose values are in a Banach space B. In the present work we establish necessary and sufficient criteria for the exponential growth (in particular, boundedness) of the solutions y of the difference boundary problem (1.1) - (1.2) for all right hand side f satisfying this same condition. The result which obtained in this work is more accuracy from the result in [2].

2.Definitions and notations.

(i) Let N be a set of points $\{0, \delta, 2\delta, \dots\}$ then for all $t_1, t_2 \in N \times N$, we define

$$\Delta_1 \Delta_2 y = \frac{y(t_1 + \delta, t_2 + \delta_2) - y(t_1, t_2 + \delta_2) - y(t_1 + \delta_1, t_2) + y(t_1, t_2)}{\delta_1 \delta_2}$$

we have the following

$$(I - \lambda_1 S_1)^{-1} S_1 (I - \lambda_2 S_2)^{-1} S_2 f =$$

$$= \delta_1 \delta_2 \sum_{k_1=0}^{m-1} \sum_{k_2=0}^{n-1} \exp \gamma_1 (m - k_1 - 1) + \gamma_2 (n - k_2 - 1) f(k_1 \delta_1, k_2 \delta_2) \tag{2.1}$$

where $t_1 = m\delta_1, t_2 = n\delta_2$ and $\gamma_i = \frac{1}{\delta_i} \ln(1 + \delta_i \lambda_i)$ is a solution of the difference equation

$$\Delta_1 \Delta_2 y - \lambda_1 \Delta_2 y - \lambda_2 \Delta_1 y + \lambda_1 \lambda_2 y = f$$
$$y(t_1, 0) = y(0, t_2) = 0.$$

(ii) Let E be a shift operator such that $Ef(t) = f(t + \delta)(\delta > 0)$, then if we define the operator

$$\delta(E - I)^{-1} f(t) = Sf, \ then$$

$$Sf = \delta \sum_{k=0}^{n-1} f(t - (k+1)\delta) \tag{2.2}$$

where $t = [n\delta]$, $[a]$ denote the greatest positive integer $\leq a$.

(iii) $(k)^{(p)}$ is the usual factorial notation i.e. $(k)^{(p)} = k(k-1)\dots(k-p+1)$, then we can prove the following

$$S^p f(t) = \frac{\delta^p}{(p-1)!} \sum_{k=p-1}^{n-1} (k)^{(p-1)} f(t - (k+1)\delta) \tag{2.3}$$

The proof is by mathematical induction, since the identity (2.2) is true for $p = 1$, that is (2.1). Then for $p = 2$ we have $S^2 f = \delta^2 \sum_{k=1}^{n-1} k E^{-(k+1)} f(t)$.
Suppose the identity is true for $p = r$, $S^r f = \frac{\delta^r}{(r-1)!} \sum_{k=r-1}^{n-1} (k)^{(r-1)} E^{-(k+1)} f(t)$ then for $p = r + 1$ we obtain

$$S^{r+1} f = SS^r f = \delta(\sum_{i=1} E^{-i})(\frac{\delta^r}{(r-1)!} \sum_{k=r-1}^{n-1} (k)^{(r-1)} E^{-(k+1)} f(t)) =$$

$$= \frac{\delta^{r+1}}{r!} \sum_{k=r}^{n-1} (k+1)^{(r)} E^{-(k+1)} f(t)$$

(considering the summation $(r-1)^{(r-1)} + r^{(r-1)} + \dots + (r+l)^{(r-1)} = \frac{(r+l+1)^{(r)}}{r}$).

(iv) Let $\| \cdot \|$ be a norm in B. If we select a subspace from B, denote by $B_{\alpha_1, \alpha_2, \dots, \alpha_n}$ which consists of the functions satisfying the condition

$$\|f\|_{\alpha_1, \dots, \alpha_n} = \sup \|f(t_1, \dots, t_n)\| \exp(-\alpha_1 t_1 \dots - \alpha_n t_n) < \infty.$$

Then $B_{\alpha_1, \dots, \alpha_2}$ will be complete in this norm i.e. Banach space.

(v) Let f belong to the linear space $B_{\alpha_1, \dots, \alpha_n}$, then f has an exponential growth with order $\alpha_1, \alpha_2, \dots$ and α_n, that is for every $\varepsilon > 0$.

$$\lim_{t_1, t_2, \dots, t_n \to \infty} \|f(t_1, \dots, t_n)\| \exp(-\sum_{i=1}^{n} (\alpha_i + \varepsilon) t_i) = 0$$

$$\lim_{t_1, \dots, t_n \to \infty} \|f(t_1, \dots, t_n)\| \exp(-\sum_{i=1}^{n} (\alpha_i - \varepsilon) t_i) = \infty.$$

3.Some Lemmas

Lemma 3.1. *If $f(t)$ is an exponential function, $\|f\| \leq Ce^{\alpha t}$, then $\|S^p f\| \leq De^{\alpha t}$ $(p \geq 1, \alpha > 0$ and S^p defined in (2.3)).*

Proof. If $p = 1$ then from (2.3), we have $Sf = \delta \sum_{k=0}^{n-1} f(t - (k+1)\delta)$. Then

$$\|Sf\| \leq \delta \sum_{k=0}^{n-1} \|f(t - (k+1)\delta)\| \leq C \sum_{k=0}^{n-1} e^{\alpha(t-(k+1)\delta)} = C \sum_{r=0}^{n-1} e^{\alpha r \delta} =$$

$$= \frac{C}{e^{\alpha\delta} - 1}(e^{\alpha n \delta} - 1) < C^1 e^{\alpha t}$$

then the Lemma is proved when $p = 1$.
We now calculate $\|S^p f\|$. Using (2.3) we get

$$\|S^p f\| \le C \sum_{k=p-1}^{n-1} (k)^{(p-1)} \|f(t-(k+1)\delta)\| \le$$
$$\le C_1 \sum_{k=p-1}^{n-1} (k)^{(p-1)} e^{\alpha(n-k-1)\delta};$$

put $\xi = n-k-1$, then $\|S^p f\| \le C_1 \sum_{\xi=0}^{n-p} (n-\xi-1)^{(p-1)} e^{\alpha\xi\delta}$ (using the relation $\sum_{\xi=0}^{n-p} u_\xi \Delta v_\xi = u_\xi v_\xi |_0^{n-p+1} - \sum v_{\xi+1}\Delta u_\xi$) finally we have $\|S^p f\| \le C_2 e^{\alpha t}$.

Lemma 3.2.[3]. *If the function $f \in B_\alpha$ and the function $K \in B_{\alpha_1}$, then the function*

$$z(t) = \sum_{k=0}^{n-1} K(t-k\delta) f(k\delta)$$

belongs to the space B_γ where $\gamma = \alpha$ if $\alpha > \alpha_1$ and $\gamma > \alpha_1$ if $\alpha \le \alpha_1$.
[Since $(I-\lambda S)^{-1} Sf = \frac{\delta}{1+\delta\lambda} \exp \frac{t}{\delta} \ln(1+\delta\lambda)$.

$$(E-I)^{-1} \exp -\frac{t}{\delta} \ln(1+\delta\lambda) f(t) = \delta \sum_{k=0}^{n-1} \exp(n-k-1)\ln(1+\delta\lambda) f(k\delta) =$$
$$= \sum_{k=0}^{n-1} K(t-k\delta) f(k\delta)$$

then $K(t-k\delta) = \frac{\delta}{1+\delta\lambda} e^{(n-k)\ln(1+\delta\lambda)}$ has an exponential growth $\alpha_1 = \frac{1}{\delta}\ln(1+\delta\lambda)$].

Lemma 3.3.[3]. *If the function $f \in B_{\alpha_1,\alpha_2}$ and $K \in B_{\beta_1,\beta_2}$, then the function*

$$g(t_1,t_2) = \sum_{k_1=0}^{n-1} \sum_{k_2=0}^{m-1} K(t_1-k_1\delta_1, t_2-k_2\delta_2) f(k_1\delta_1, k_2\delta_2)$$

belongs to B_{γ_1,γ_2} where $\gamma_i = \max(\beta_i,\alpha_i)$ if $\alpha_i \ne \beta_i$ and $\gamma_i = \beta_i + \varepsilon$ if $\alpha_i = \beta_i$.

Lemma 3.4. *If $f \in B_\alpha$, then*

$$(I-\lambda S)^{-1} S^p \quad f \in B_\gamma \quad (p \ge 1).$$

Proof. The series $f + \lambda S f + \ldots + \lambda^m S^m f + \ldots$ is convergent for every $f \in B_\alpha$ and the sum of this series is $(I-\lambda S)^{-1} f$ then

$$(I-\lambda S)^{-1} S f = [S + \lambda S^2 + \ldots + \lambda^{p-2} S^{p-1}] f + \lambda^{p-1} (I-\lambda S)^{-1} S^p f.$$

By using lemma (3.1) and lemma (3.2) we have $(I-\lambda S)^{-1} Sf \in B_\gamma$ where $\gamma = \max(\alpha,\beta)$ if $\alpha \ne \beta$ and $(I-\lambda S)^{-1} S^p f \in B_{\alpha+\varepsilon}$ for any small $\varepsilon > 0$ and $\alpha = \beta = \frac{1}{\delta}\ln(1+\delta\lambda)$.

Lemma 3.5. *If x_0 is an element of a Banach space with $\|x_0\| = 1$ and let $(\lambda_{1,0},\lambda_{2,0})$ be an eigenvalue of the operator - function $\Gamma(\lambda_1,\lambda_2)$*

$$i.e.\ (\lambda_{1,0}\lambda_{2,0} I - \dot{A}\lambda_{2,0} - B\lambda_{1,0} - C) x_0 = \theta$$

where θ is a zero element in a Banach space, then

$$(d) \quad \Gamma(\lambda_1,\lambda_2) x_0 = \frac{x_0}{(\lambda_1-\lambda_{1,0})(\lambda_2-\lambda_{2,0})} + \Gamma(\lambda_1,\lambda_2)\left[\frac{A-\lambda_{1,0}}{\lambda_1-\lambda_{1,0}} + \frac{B-\lambda_{2,0}}{\lambda_2-\lambda_{2,0}}\right] x_0$$

Proof. Since

$$\Gamma(\lambda_1,\lambda_2)[\lambda_1\lambda_2 I - A\lambda_2 - B\lambda_1 - C] x_0 = x_0$$

$(\lambda_{1,0},\lambda_{2,0})$ be an eigenvalue of the operator - function $\Gamma(\lambda_1,\lambda_2)$, then

$$\Gamma(\lambda_1,\lambda_2)[(\lambda_1\lambda_2 - \lambda_{1,0}\lambda_{2,0}) I - A(\lambda_2-\lambda_{2,0}) - B(\lambda_1-\lambda_{1,0})] x_0 = x_0,$$

consequently

$$\Gamma(\lambda_1,\lambda_2)[(\lambda_1-\lambda_{1,0})(\lambda_2-\lambda_{2,0}) + \lambda_{1,0}(\lambda_2-\lambda_{2,0}) +$$
$$+ \lambda_{2,0}(\lambda_1-\lambda_{1,0}) - A(\lambda_2-\lambda_{2,0}) - B(\lambda_1-\lambda_{1,0})] x_0 = x_0.$$

The last equation leads to (d).

4.Result. The operator - function

$$\Gamma_1(\lambda_1,\ldots,\lambda_n) = \left[I - \sum \frac{A_{q_1\ldots q_n}}{\lambda_1^{p_1-q_1}\ldots\lambda_n^{p_n-q_n}}\right]^{-1}$$

is uniformly bounded for every $\lambda_i \in C_i$ (the point $(\lambda_1,\ldots,\lambda_n)$ is a regular point of $\Gamma_1(\lambda_1,\ldots,\lambda_n)$), then we have

$$(4.1) \qquad \|\Gamma_1(\lambda_1,\ldots,\lambda_n) f\| \le D\|f\|$$

consequently we have the following.

Theorem. *Let the right hand side f for the boundary problem (1.1) (1.2) runs all space $B_{\alpha_1,\alpha_2,\ldots,\alpha_n}$, then the totaly solution $\{y\}$ is covered by space $B_{\beta_1,\beta_2,\ldots,\beta_n}$ provided that the following conditions are satisfied*

$$\beta_i = \alpha_i \quad if \quad \alpha_i > \alpha_i^{(0)} \ and \ \beta_i > \alpha_i^{(0)} \quad if \quad \alpha_i \le \alpha_i^{(0)}$$

where $\alpha_i^{(0)} = \frac{1}{\delta_i}\ln|1+\delta_i\lambda_{i,0}|$. $(\lambda_{1,0},\lambda_{2,0},...,\lambda_{n,0})$ *be an eigenvalue of the operator - function* $\Gamma(\lambda,...,\lambda_n)$.

Proof. We split up the proof of the theorem into several parts

(i) proof of sufficiency using (1.4) we have

$$\|y\| \le L\ \Pi_{i=1}^n \exp\frac{t_i}{\delta_i}\ln|1+\delta_i\lambda_i|\cdot\frac{\delta_i}{1+\delta_i\lambda_i}$$
$$(E_i - I)^{-1}\exp -\frac{t_i}{\delta_i}\ln|1+\delta_i\lambda_i|[\delta_i(E_i-I)^{-1}]^{p_i-1}\|f\|$$

where $L = \max\|\Gamma_1(\lambda_1,\lambda_2,...,\lambda_n)\|$, lengthes $c_1,...,c_2(\frac{1}{2\pi})^n$.
If $\|f\| \le Me^{\alpha_1t_1+\alpha_2t_2+...+\alpha_nt_n}$, using lemma (3.4), we have

$$\Pi_{i=1}^n \exp\frac{t_i}{\delta_i}\ln|1+\delta_i\lambda_i|\cdot\frac{\delta_i}{1+\delta_i\lambda_i}(E_i-I)^{-1}.$$
$$\exp -\frac{t_i}{\delta_i}\ln|1+\delta_i\lambda_i|[\delta_i(E_i-I)^{-1}]^{p_i-1}e^{\alpha_1t_1+...+\alpha_nt_n}\in B_{\gamma_1,...,\gamma_n}$$

where $\gamma_i = \max(\alpha_i,\alpha_i^{(0)})$ $\alpha_i \ne \alpha_i^{(0)}$

$$(\alpha_i^{(0)} = \frac{1}{\delta_i}\ln|1+\delta_i\lambda_i| \text{ and } \lambda_i\in C_i),$$

and $\gamma_i > \alpha_i$ when $\alpha_i = \alpha_i^{(0)}$ from the above result we proved the sufficiency.

Since lemma (3.4) plays important rule in our proof, then for simpicity and without loss of generality we consider the boundary value problem

$$\Delta_1\Delta_2y - A\Delta_2y - B\Delta_1y - Cy = f \tag{4.2}$$

$$y|_{0\le t_i<\delta_i} = \theta \quad (\delta_i>0, i=1,2) \tag{4.3}$$

The solution of the above equation can be put in the form

$$y = (-\frac{1}{2\pi i})^2\oint_{c_1}\oint_{c_2}(I-\lambda_1S_1)^{-1}S_1(I-\lambda_2S_2)^{-1}S_2\cdot$$
$$\cdot(I-\frac{A}{\lambda_1}-\frac{B}{\lambda_2}-\frac{C}{\lambda_1\lambda_2})^{-1}f\frac{d\lambda_1}{\lambda_1}\frac{d\lambda_2}{\lambda_2} \tag{4.4}$$

Taking $f(t_1,t_2) = e^{\alpha_1t_1+\alpha_2t_2}x_0$, where x_0 is an element of Banach space, using lemma (3.5) in equation (4.4) leads to

$$y = (-\frac{1}{2\pi i})^2\oint_{c_1}\oint_{c_2}(I-\lambda_1S_1)^{-1}S_1(I-\lambda_2S_2)^{-1}S_2\cdot$$
$$\cdot\left\{\frac{\lambda_1\lambda_2}{(\lambda_1-\lambda_{1,0})(\lambda_2-\lambda_{2,0})}+\Gamma(\lambda_1,\lambda_2)(\frac{A-\lambda_{1,0}}{\lambda_1-\lambda_{1,0}}+\right. \tag{4.5}$$
$$\left.+\frac{B-\lambda_{2,0}}{\lambda_2-\lambda_{2,0}})\right\}e^{\alpha_1t_1+\alpha_2t_2}x_0\frac{d\lambda_1}{\lambda_1}\frac{d\lambda_2}{\lambda_2} =$$
$$= y_1 + (A-\lambda_{1,0})y_2 + (B-\lambda_{2,0})y_3$$

where

$$y_1 = \lambda_{1,0}\lambda_{2,0}(I-\lambda_{1,0}S_1)^{-1}S_1(I-\lambda_{2,0}S_2)^{-1}S_2e^{\alpha_1t_1+\alpha_2t_2}x_0 \tag{4.5$_a$}$$

$$y_2 = (I-\lambda_{1,0}S_1)^{-1}S_1y \tag{4.5$_b$}$$

$$y_3 = (I-\lambda_{2,0}S_2)^{-1}S_2y \tag{4.5$_c$}$$

If f runs through all of B_{α_1,α_2} the totality of solutions is covered by B_{β_1,β_2}, for β_1,β_2, sufficiently large. We shall determine the smallest β_i to be the solution is stable.

If $f = e^{\alpha_1t_1+\alpha_2t_2}x_0$ we consider the following cases

(1) If $\alpha_i > \alpha_i^{(0)} = \frac{1}{\delta_i}\ln|1+\delta_i\lambda_{i,0}|$ $i=1,2$ then from equation (4.5) we get

(i) y_1 belongs to the class B_{α_1,α_2},

(ii) y_2 belongs to the class B_{γ_1,β_2} where $\gamma_1 = \max(\beta_1,\alpha_1^{(0)})$ when $\beta_1\ne\alpha_1^{(0)}$ and $\gamma_1>\beta_1$ when $\beta_1 = \alpha_1^{(0)}$,

(iii) y_3 belongs to the class B_{β_1,γ_2} where $\gamma_2 = \max(\beta_2,\alpha_2^{(0)})$ when $\beta_2\ne\alpha_2$ and $\gamma_2>\beta_2$ when $\beta_2=\alpha_2^{(0)}$.

(2) If $\alpha_i = \alpha_i^{(0)}$ then from equation (4.5) we get

$$y_1 \in B_{\alpha_1+\epsilon,\alpha_2+\epsilon} \tag{$i.2$}$$

(3) If $\alpha_i < \alpha_i^{(0)}$ then from equation (4.5) we get

$$y_1 \in B_{\alpha_1^{(0)},\alpha_2^{(0)}}. \tag{$i.3$}$$

Now we can prove the necessity, the proof will be completed in the following steps.

(1) If $f\in B_{\alpha_1,\alpha_2}$ and $\alpha_i>\alpha_i^{(0)}$, the solution y belongs the space B_{β_1,β_2}.

From the equation (4.5) we have the right hand side belongs to space B_{ν_1,ν_2} where $\nu_i = \max(\alpha_i,\beta_i)$ (since $\alpha_i>\alpha_i^{(0)}$).

If $\alpha_i>\beta_i$, then equation (4.5) leads to a contradiction. It is sufficient to choice $\beta_i=\alpha_i$ in our case to obtain the solution $y\in B_{\alpha_1,\alpha_2}$ and the right hand side of equation (4.5) belongs to the same space.

(2) If $f\in B_{\alpha_1,\alpha_2},\alpha_i=\alpha_i^{(0)}$ and if $\beta_i=\alpha_i$, then from equation (4.5) we have a contradiction.

But if $\beta_i>\alpha_i$ we have both sides of equation (4.5) belongs to the same space.

(3) If $f\in B_{\alpha_1,\alpha_2},\alpha_i<\alpha_i^{(0)}$ and if $\beta_i>\alpha_i^{(0)}$, then from equation (4.5) we have both sides of equation (4.5) belongs to the same space. This completes the proof.

Remark. If $\delta_i \to 0$ $(i = 1, 2, ..., n)$ in theorem (1) we obtain the following result

$$(i)\quad \beta_i = \alpha_i \qquad if \quad \alpha_i > \mathrm{Re}\lambda_{i,0}$$
$$(ii)\quad \beta_i > \mathrm{Re}\lambda_{i,0} \quad if \quad \alpha_i \leq \mathrm{Re}\lambda_{i,0}$$

this result is more accuracy from the result in [2], because the relation $\beta_i = \alpha_i > \mathrm{Re}\lambda_{i,0}$ is new result which it disappear in theorem (1) in [2].

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ON A SOLUTION OF OPERATOR DIFFERENTIAL EQUATION AND APPLICATION TO PARTIAL DIFFERENTIAL EQUATION

BY

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1.Introduction. The field M of Mikusinski operators is an extension of the integral domain C, [1]. As it is known that the quotient field M of Mikusinski contains the differential operator s, the integral operator l and translation operator. The differential equation in M covers some classes of partial differential equations, integral equations and difference equations, which have quite wide range of applications.

In this paper we shall deal with the differential equation

$$(1) \quad L_n x(\lambda) = x^{(n)}(\lambda) + a_{n-1}(\lambda)x^{(n-1)}(\lambda) + ... + a_0(\lambda)x(\lambda) = b(\lambda), \ 0 \leq \lambda \leq \Lambda$$

where $x(\lambda), b(\lambda)$ and $a_i(\lambda)$, $i = 0, 1, .., n-1$ are functions which map the interval $[0, \Lambda]$ into the field M of operators.

2.Preliminaries. The class C of continuous complex-valued functions of non-negative real variable forms a commutative algebra without zero divisors, where the product is defined as the finite convolution, the sums and scalar products are defined in the usual way. The quotient field of this algebra is the operator field M of Mikusinski.

$K(\lambda)$ is the class of all numerical bounded function of the form of finite sum $\Sigma_\nu A_\nu e^{i\nu\lambda}$, where A_ν is a numerical constant, $i = \sqrt{-1}$ and $\lambda \in [0, \Lambda]$. We shall let $f = \{f(t)\}$ denote the representation of $f(t)$ in $C[0, \infty)$, while $x(\lambda) = q\{x(\lambda, t)\}$ stands for an operation function, where $q \in M$ and $\{x(\lambda, t)\}$ is a parametric function.

$M(\lambda)$ is the set of all mappings of $[0, \Lambda]$ into $M, \Lambda < \infty$

$D(T) : 0 \leq \lambda \leq \Lambda,\ 0 \leq t \leq T < \infty$ is the domain $\mathbf{R}^2$.

$J(D)$: is the set of all continuous complex-valud functions $f(\lambda, t)$, such that (i) for every $T < \infty, f(\lambda, t)$ is defined, bounded and measurable over $D(T)$, (ii) for every λ in $[0, \Lambda], f(\lambda, t)$ is in C.

Finally $J(\lambda)$ is the algebra which lies in $M(\lambda)$, its elements are of the form: $f(\lambda) = q\{f(\lambda, t)\}, q \in M$ and $f(\lambda, t) \in J(D)$. In $M(\lambda)$ the limit, derivative and integral are defined [1].