Remark. If $\delta_i \to 0$ $(i = 1, 2, ..., n)$ in theorem (1) we obtain the following result

$$(i)\ \beta_i = \alpha_i \quad if \quad \alpha_i > \mathrm{Re}\lambda_{i,0}$$

$$(ii)\ \beta_i > \mathrm{Re}\lambda_{i,0} \quad if \quad \alpha_i \le \mathrm{Re}\lambda_{i,0}$$

this result is more accuracy from the result in [2], because the relation $\beta_i = \alpha_i > \mathrm{Re}\lambda_{i,0}$ is new result which it disappear in theorem (1) in [2].

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ON A SOLUTION OF OPERATOR DIFFERENTIAL EQUATION AND APPLICATION TO PARTIAL DIFFERENTIAL EQUATION

BY

I.E. SHARKAWI

1.Introduction. The field M of Mikusinski operators is an extension of the integral domain C, [1]. As it is known that the quotient field M of Mikusinski contains the differential operator s, the integral operator l and translation operator. The differential equation in M covers some classes of partial differential equations, integral equations and difference equations, which have quite wide range of applications.

In this paper we shall deal with the differential equation

$$(1) \quad L_n x(\lambda) = x^{(n)}(\lambda) + a_{n-1}(\lambda)x^{(n-1)}(\lambda) + ... + a_0(\lambda)x(\lambda) = b(\lambda), \ 0 \le \lambda \le \Lambda$$

where $x(\lambda), b(\lambda)$ and $a_i(\lambda)$, $i = 0, 1, .., n-1$ are functions which map the interval $[0, \Lambda]$ into the field M of operators.

2.Preliminaries. The class C of continuous complex-valued functions of non-negative real variable forms a commutative algebra without zero divisors, where the product is defined as the finite convolution, the sums and scalar products are defined in the usual way. The quotient field of this algebra is the operator field M of Mikusinski.

$K(\lambda)$ is the class of all numerical bounded function of the form of finite sum $\Sigma_\nu A_\nu e^{i\nu\lambda}$, where A_ν is a numerical constant, $i = \sqrt{-1}$ and $\lambda \in [0, \Lambda]$. We shall let $f = \{f(t)\}$ denote the representation of $f(t)$ in $C[0, \infty)$, while $x(\lambda) = q\{x(\lambda, t)\}$ stands for an operation function, where $q \in M$ and $\{x(\lambda, t)\}$ is a parametric function.

$M(\lambda)$ is the set of all mappings of $[0, \Lambda]$ into $M, \Lambda < \infty$

$D(T) : 0 \le \lambda \le \Lambda,\ 0 \le t \le T < \infty$ is the domain $\mathbf{R}^2$.

$J(D)$: is the set of all continuous complex-valud functions $f(\lambda, t)$, such that (i) for every $T < \infty, f(\lambda, t)$ is defined, bounded and measurable over $D(T)$, (ii) for every λ in $[0, \Lambda], f(\lambda, t)$ is in C.

Finally $J(\lambda)$ is the algebra which lies in $M(\lambda)$, its elements are of the form: $f(\lambda) = q\{f(\lambda, t)\}, q \in M$ and $f(\lambda, t) \in J(D)$. In $M(\lambda)$ the limit, derivative and integral are defined [1].

3.Operator spaces, $C_s(\lambda), \bar{C}_s(\lambda)$ and $A_s(\lambda)$. In [4] S t a n k o v i c has defined and investigated the spaces $C_s(\lambda)$ and $\bar{C}_s(\lambda)$. We present here only the essentials, and define a space $A_s(\lambda) C_s(\lambda)$: is a commutative algebra which lies in $M(\lambda)$, its element are of the form $f(\lambda) = s^{\beta}\{f(\lambda,t)\}$, $\beta > 0$ and $f(\lambda,t) \in J(D)$.

$\bar{C}_s(\lambda)$: is a vector space that contains $C_s(\lambda)$.

$A_s(\lambda)$: is the set of all members of the form: $a + s^{-\beta} f(\lambda)$

where, a is numerical constant, $s^{-\beta} = l^{\beta} = \{\frac{t^{\beta-1}}{\Gamma(\beta)}\}, \beta > 0,\ t \in [0,T]$ and $f(\lambda) \in K(\lambda)$.

4.The system of differential equations. Equation (1) can be reduced to the equivalent system of differential equations:

$$x'_i(\lambda) = \sum_{j=1}^{n} a_{i,j}(\lambda) x_j(\lambda) + b_i(\lambda), \quad i = 1,2,...,n \tag{2}$$

where, $a_{i,j}(\lambda), b_i(\lambda)$ and $x_i(\lambda)$ are functions which map the interval $[0,\Lambda]$ into the field M. M i k u s i n s k i [1] proved the unicity of the solution of the system (2) when $a_i(\lambda)$ are independent of λ. He solved equation (1) when $a_i(\lambda)$ are polynomials of s with numerical coefficients [1]. In the case when $a_{i,j}(\lambda) = s^{\beta_{i,j}}\{w_{i,j}(\lambda,t)\} \in C_s(\lambda)$ and $b_i(\lambda) \in \bar{C}_s(\lambda)$, and under some conditions on $\beta_{i,j}$, S t a n k o v i c [5] proved the unicity of the solution of the system (2) with the initial conditions:

$$x_i(0) = x_i^0 \in M, \quad i = 1,2,...,n \tag{3}$$

we shall use the results of Stankovic unicity theorem to prove the unicity of the system (2) when $a_{i,j}(\lambda) \in A_s(\lambda)$, with initial conditions, (3). We need not mention Stankovic conditions in the following theorem:

Theorem 1. *Let $a_{i,j}(\lambda) \in A_s(\lambda)$ and $b_i(\lambda) \in \bar{C}_s(\lambda)$, then the system (2), with the initial conditions (3) has a unique solution in $\bar{C}_s$.*

Proof. Since $a_{i,j}(\lambda) \in A_s(\lambda)$, then $a_{i,j}(\lambda)$ can be written as $a_{i,j}(\lambda) = a^o_{i,j} + s^{-\beta} f_{i,j}(\lambda)$ where $a^o_{i,j}$ is a number, $\beta > 0$ and $f_{i,j} \in K$. For any $\alpha_{i,j} > 0$, we have

$$a_{i,j}(\lambda) = s^{\alpha_{i,j}} s^{-\alpha_{i,j}} a_{ij}(\lambda) = s^{\alpha_{i,j}}\{a^o_{i,j}\frac{t^{\alpha_{i,j}-1}}{\Gamma(\alpha_{i,j}}+$$
$$+\frac{t^{\alpha_{i,j}+\beta-1}}{\Gamma(\alpha_{i,j+\beta})} f_{i,j}(\lambda)\} \in C_s(\lambda)$$

(i.e. $A_s(\lambda) \in C_s(\lambda)$), but $\alpha_{i,j}$, $1 \le i,\ j \le n$ are arbitrary +ve numbers hence we can choose $\alpha_{i,j}$ such that Stankovic conditions are satisfied. Thus by applying Stankovic theorem of unicity we complete the proof of this theorem.

5.Homogeneous logarithmic differential equation with coefficients in $A_s(\lambda)$. Here we shall consider the equation (1) when $b(\lambda) = 0$, i.e.

$$L_n x(\lambda) = \frac{d^n}{d\lambda^n} x(\lambda) + a_{n-1}(\lambda)\frac{d^{n-1}}{d\lambda^{n-1}} x(\lambda) + ... + a_0(\lambda) x(\lambda) = 0 \tag{4}$$

where $a_i(\lambda) \in A_s(\lambda)$, $i = 0,1,...,n-1$.

Definition. *Equation (4) is said to be logarithmic if it has n lineraly independent solutions (logarithmic solutions).*

Given the coefficients $a_i(\lambda) = a_i^o + s^{-\beta} f_i(\lambda) \in A_s(\lambda)$ we can interpret equation (4) in the form of nonhomogeneous differential equation

$$F(\frac{d}{d\lambda}) x(\lambda) = (\frac{d^n}{d\lambda^n} + a^o_{n-1}\frac{d^{n-1}}{d\lambda^{n-1}} + ... + a^0_1 \frac{d}{d\lambda} + a^0_0) x(\lambda) = \sum_{k=0}^{n-1} f_k \frac{d^k}{d\lambda^k} x(\lambda), \tag{5}$$

let the characteristic equation of the differential equation $F(\frac{d}{d\lambda})x(\lambda) = 0$ denoted by $F(r) = 0$ which has n roots. In earlier papers [2] and [3] we investigated the formal solution of equation (5) and gave it in the form:

$$x_k(\lambda) = [1 + \sum_{j=1}^{\infty} s^{-\beta j} y_{kj}(\lambda)] \exp[r_k + \sum_{j=1}^{\infty} s^{-\beta j} \rho_{kj}]\lambda = Y_k(\lambda) e^{R_k \lambda} \tag{6}$$

where r_k is one of the numerical roots of the equation $F(r) = 0, \rho_{kj}$, $j = 1,2,...,$ are complex-numbers, $y_{kj} \in K(\lambda)$ and

$$Y_k = 1 + \sum_{j=1}^{\infty} s^{-\beta j} Y_{kj}(\lambda),\ R_k = r_k + \sum_{j=1}^{\infty} s^{-\beta j} \rho_{kj} \tag{7}$$

Theorem 2. *The number of linearly independent (logarithmic) solutions of equations (4) is less than or equal to the order of this equations.*

Proof. Let $x_1(\lambda), x_2(\lambda), ..., x_{n+1}(\lambda)$ be solutions of equation (4). It is always possible to choose $n+1$ constants $C_1, C_2, ..., C_{n+1}$ not all null such that : $\Sigma_{j=1}^{n+1} C_j x_j^{(i)}(0) = 0$ for $i = 0,1,...,n-1$ then the function

$$x(\lambda) = \sum_{j=1}^{n+1} C_j x_j(\lambda)$$

also satisfies the equation (4) and the initial conditions:

$$x^{(i)}(0) = 0 \quad for \quad i = 0,1,...,n-1.$$

By unicity theorem (1) we have $x(\lambda) = 0$ identity in $D(T)$. Thus $x_1(\lambda), x_2(\lambda), ...,$ $x_{n+1}(\lambda)$ are linearly dependent, which proves the theorem.

6.Homogeneous logarithmic equation with coefficients in $C_s(\lambda)$. In this section we shall consider the differential equation (1) with $a_i(\lambda) \in C_s(\lambda)$ and $b(\lambda)$ is a given operational function $\overline{C}_s(\lambda)$, then every solution of (1) can be respresented as the sum

$$x(\lambda) = u(\lambda) + v(\lambda),$$

where, $v(\lambda)$ is any solution equation (1) and $u(\lambda)$ is the solution of the homogeneous equation $L_n x(\lambda) = 0$. Thus if we know all logarithmic solutions $x_k(\lambda)$, $k = 1, 2, ..., n$ of the homogeneous equation $L_n x(\lambda) = 0$, we shall know all the solutions of equation (1).

Theorem 3. *The particular solution of equation (1) when $a_i(\lambda) \in C_s(\lambda)$ and $b(\lambda) \in \overline{C}_s(\lambda)$, always exist if the equation $L_n x(\lambda) = 0$ has n logarithmic solutions $x_i(\lambda) = 1, 2, ..., n$.*

Proof. Write $u_o(\lambda) = C_1 x_1(\lambda) + C_2 x_2(\lambda) + ... + C_n x_n(\lambda)$ and choosen the constant operators $C_1, C_2, ..., C_n$ so as to have

(8) $$u_o(0) = u'_o(0) = ... = u_o^{(n-2)}(0) = 0 \text{ and } u_o^{(n-1)}(0) = 1.$$

In order to determine $C_1, C_2, ..., C_n$ we have to solve the system of n equations. This is always possible. In fact, suppose the converse. Then the determinant of the considered system vanishes, and we can find a set of C_i such that $u_o^{(i)}(0) = 0$ for $i = 1, 2, ..., n-1$. By Stankovic theorem of unicity we then have identically $u_o(\lambda) = 0$, which contradicts the hypothisis that $L_n x(\lambda) = 0$ has n logarithmic solutions. Thus we have proved that there is a solution $u_o(\lambda)$ of $L_n x(\lambda) = 0$, satisfying conditions (8). Now we can easily verify that:

$$v(\lambda) = \int_0^\lambda u_o(\lambda, \mu) b(\mu) d\mu$$

is the required particular solution of (1) when $a_i(\lambda) \in C_s(\lambda)$ and $b(\lambda) \in \overline{C}_s(\lambda)$.

7.Application to partial differential equations. We shall retain our restriction to the partial differential equation of the form:

(9) $$\frac{\partial^m}{\partial t^m} \sum_{i=0}^{n} a_i^o \frac{\partial^i}{\partial \lambda^i} x(\lambda, t) = \sum_{i=0}^{n} f_i(\lambda) x_{\lambda^i}(\lambda, t) + g(\lambda, t)$$

for which our results can be applied when $a_i^o, i = 0, 1, ..., n$ are numerical constants, $f_i(\lambda) \in K(\lambda)$ and $g(\lambda, t)$ is a given function in $J(D)$. By using the general function given by M i k u s i n s k i [1]

(10) $$y_{t^m\lambda^i}(\lambda, t) = s^m \{y_{\lambda^i}(\lambda, t)\} - \sum_{j=0}^{m-1} s^{m-j-1} Y_{tj\lambda^i}(\lambda, 0)$$

we can write equation (9) in the operational form:

(11) $$a_n(\lambda) x^{(n)}(\lambda) + a_{n-1}(\lambda) x^{(n-1)}(\lambda) + ... + a_o(\lambda) x(\lambda) = \\ = h(\lambda) + s^{-m}\{g(\lambda, t)\} = \phi(\lambda)$$

where $x^{(i)}(\lambda) = \{x_{\lambda^i}(\lambda, t)\}$ *for* $i = 0, 1, 2, ..., n$.

$$\phi(\lambda) = h(\lambda) + s^{-m}\{g(\lambda, t)\}$$

(12) $$h(\lambda) = \sum_{i=0}^{n} \sum_{j=0}^{m-1} a_i^o s^{-j-1} x_{tj\lambda^i}(\lambda, 0)$$

and $a_i(\lambda) = (a_i^o - s^{-m} f_i(\lambda)) \in A_s(\lambda)$.

Note. Condition (12) may be replaced by Cauchy conditions

$$\frac{\partial^i}{\partial t^j} x(\lambda, 0) = h_j(\lambda) \quad (j = 0, 1, ..., m-1 \text{ see [1]}).$$

Thus the partial differential equation (9) with addition of initial condition (12) is equivalent to the nonhomogeneous operational equation (11).Similarly, the homogeneous problem is represented operationally by the equation

(13) $$a_n(\lambda) x^{(n)}(\lambda) + a_{n-1}(\lambda) x^{(n-1)}(\lambda) + ... + a_o(\lambda) x(\lambda) = 0$$

when $h(\lambda)$ are given, (12) are initial conditions on λ.

Consider the homogeneous problem

(14) $$\frac{\partial^m}{\partial t^m} \sum_{i=0}^{n} a_i^o \frac{\partial^i}{\partial \lambda^i} x(\lambda, t) - \sum_{i=0}^{n} f_i(\lambda) x(\lambda, t) = 0$$

with initial conditions on the λ axis

(15) $$h(\lambda) = \sum_{i}^{n} \sum_{j}^{m-1} a_i^o s^{-j-1} x(\lambda, 0)$$

obtained by putting $\phi(\lambda, t) = 0$ and $h(\lambda, t) = 0$ in the nonhomogeneous problem consisting of equation (9) and conditions (12). According to the theorem (3) the particular solution always exists if equation (13) has n logarithmic solutions. In other cases it is possible to find particular solutions of (11), but we are not aware of any general method. It may happen that no solution of (11) exsits.On the other hand, we can find the formal solution of (13) see [2] and [3]. Hence we can always find the general solution of (13).

Remark. Every solution of the partial equation (9) is a solution of the corresponding operational equation (11).But not conversly. For a solution of the operational problem is in general an operational function, whose values have no sense in classical analysis.Thus, the general operational solution of equation (1) contains all the solutions of partial equations of type (9) and all the solutions of partial equation with numaerical coefficient [1].Without any restrictions we can take $a_n(\lambda) = 1$ in (11).Thus our results can be useful in establishing the form of solution of the partial equation (9) in $D(T)$.

Let, $x_k(\lambda) = Y_k e^{R_k \lambda}, k = 1,2,...,n$ as defined in (6) be the set of n logarithimic solutions of equation (13) and the general solution of (13) is in the form:

$$x(\lambda) = C_1 x_1(\lambda) + C_2 x_2(\lambda) + ... + C_n x_n(\lambda) \tag{16}$$

where $C_i, i = 1,2,...,n$ are arbitary operators.
In order that (16) to be a solution of (14) and (15) we have to impose conditions on the coefficients $C_i, i = 1,2,...,n$ similar to the conditions given by M i k u s i n s k i in the case of partial differential equations with numerical coefficients [1]. Those conditions should be sufficient to ensure that (16) be a parametric function, i.e. $x(\lambda) \in J(D)$. We can establish such conditions by the following two lemmas.

Lemma 1. *If (16) is a solution of the partial equation (14), each of the terms $x_i(\lambda) = C_i Y_i e^{R_i \lambda}$ $(i = 1,2,...,n)$ on the r.h.s. of (16) must be a parametric function.*

Proof. By assumption $Y_i e^{R_i \lambda}$ $i = 1,2,...,n$ are logarithmic solutions of equation (13) then $Y_j^{(i)}$ and R_i are operationally convergent.Thus, $Y_i, Y_i^{(j)}$ and $(e^{R_i \lambda})^{(j)}$ can be written as

$$\begin{aligned} &Y_i = (1 + G_i),\ Y_i^{(j)} = G_i^{(j)}, \quad i = 1,2,...,n;\ j = 0,1,...,n-1 \\ &(e^{R_i \lambda})^{(j)} = e^{r_i \lambda}(1 + H_i) \end{aligned} \tag{17}$$

where $G_i, G_i^{(j)}$ and H_i are in $J(D)$ and r_k satisfy the characteristic equation $F(r) = 0$.

Finally, the expression: $x_i^{(j)}(\lambda) = (Y_i e^{R_i \lambda})^{(j)}$. In connection with (17) can be written in the form

$$x_i^{(j)}(\lambda) = [b_{i,j} + \phi_{i,j}]e^{R_i \lambda} = \sigma_{i,j} e^{R_i \lambda} \tag{18}$$

where $\sigma_{i,j} = b_{i,j} + \phi_{i,j}, b_{i,j} \neq 0,\ i = 1,2,...,n,\ j = 0,1,...,n-1$ are numbers and $\phi_{i,j} \in J(D)$. Thus equation (16) can be rewritten as

$$x^{(j)}(\lambda) = \sigma_{i,j} C_1 e^{R_1 \lambda} + \sigma_{i,j} C_2 e^{R_2 \lambda} + ... + \sigma_{n,j} C_n e^{R_n \lambda}, \quad j = 0,1,...,n-1.$$

Let σ be the determinant of the above system. As it is known from theorem (2) the determinant $\sigma \neq 0$.

It follows that

$$C_i e^{R_i \lambda} = \frac{A_{1i}}{\sigma} x(\lambda) + \frac{A_{2i}}{\sigma} x'(\lambda) + ... + \frac{A_{ni}}{\sigma} x^{(n-1)}(\lambda) \tag{19}$$

where A_{ij} is the minor of σ, corresponding to the element that lies in the j *th* column and i *th* row.

Since, all elements of σ are of type $\sigma_{j,i}$ then, we can write $\sigma = B(1 + \phi)$, where B is a number, $(B \neq 0)$ and $\phi \in J(D)$. Similarly, each of the minor A_{ji} can be written in the form:

$A_{ji} = B_{ji}(1 + \psi_{ji})$, where B_{ji} are number and $\psi_{ji} \in J(D)$. Hence,

$$\frac{A_{ji}}{\sigma} = \frac{B_{ji}}{B} \frac{(1 + \psi_{ji})}{(1 + \phi)} = \frac{B_{ji}}{B}(1 + Q_{ji})$$

where Q_{ji} also in $J(D)$.

If $x^{(j)}(\lambda),\ i = 0,1,..,n-1$ are supposed to be in $J(D)$, each of the terms on the r.h.s. of (19) must be in $J(D)$, i.e. $C_i e^{R_i \lambda} \in J(D)$. Hence $C_i Y_i e^{R_i \lambda} \in J(D)$.

Lemma 2. *Consider the expression*

$$w(\lambda) = pYe^{R\lambda} \tag{20}$$

where

$$\begin{aligned} &Y = 1 + \sum_{k=1}^{\infty} Y_k(\lambda) s^{-mk},\ Y^{(j)} = \sum_{k=1}^{\infty} Y_k^{(j)} s^{-mk}, \quad j = 1,2,...,n \\ &R = r_o + \sum_{k=1}^{\infty} r_k s^{-mk} \end{aligned} \tag{21}$$

are operationally convergent, $Y_k, Y_k^{(j)} \in A_s(\lambda), r_k, (k = 0,1,...)$ are numbers and $m > 0$. If $w(\lambda) \in J(D)$, then $p \in C$. If $p \in C$ then $w(\lambda)$ and all its derivatives $w^{(j)}(\lambda), j = 1,2,...,n$ are in $J(D)$.

Proof. From (21) we can write

$$Ye^{R\lambda} = e^{r_o \lambda}(1 + E(\lambda)) \tag{22}$$

where , $E(\lambda) \in J(D)$.
Hence,

$$\frac{1}{Ye^R} = e^{-r_o \lambda} \frac{1}{1 + E(\lambda)} = e^{-r_o \lambda}(1 + G(\lambda)) \tag{23}$$

where $G(\lambda)$ is again in $J(D)$, from (20) and (22) we find

$$p = e^{-r_o \lambda}(1 + G(\lambda))w(\lambda).$$

Hence it follows that if $w(\lambda)$ is parametric function, the coefficient p must be a function of the class C. On the other hand, if $p \in C$, the equality (22) implies that $w(\lambda) \in J(D)$. Since in this case

$$Ye^{R\lambda} = e^{r_o \lambda}(1 + E(\lambda))$$

where $E(\lambda)$ is parametric. As before we find

$$p = e^{-r_o\lambda}(1 + G(\lambda))w(\lambda)$$

where $G(\lambda)$ is parametric.The conclusion is the same: $w(\lambda)$ is parametric if and only if $p \in C$. Moreover if $p \in C$, all the derivatives $w^{(j)}(\lambda)$ are parametric in $D(T)$.

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TOPOLOGICAL PROPERTIES OF PROJECTIVELY PARALLELIZABLE MANIFOLDS

BY

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1.Introduction. Let M be a differentiable manifold, and let $LF(M) \to M$ be the bundle of linear frames of M. The bundle $PF(M) = LF(M)/\mathbf{R}^* \to M$ is called the bundle of projective frames of the manifold M. M is said to projectively parallelizable if the projective frame bundle admits a global section $s_P : M \to PF(M)$.

This paper has two purposes.The first one is to find necessary and/or sufficient cónditions for M to be projectively parallelizable; the second one is to study the relation between the notions of projectively parallelizable manifolds and linearly parallelizable manifolds. As for the first purpose, we have found that a neccesary and suffiecient condition for M to be projectively parallelizable is the existence of a certain decomposition of the tangent bundle of M into a direct sum of line bundles (theorem 1). This decomposition allows us to find obstruction for M to be projectively parallelizable in function of the Stieffel-Whitney classes of M (corollary 2), the orientability of M (corallary 3) and cobordism properties (corollaries 8 and 9). Also we found a necessary condition in the annulation of the Euler-Poincaré characteristic of M (corollary 6).

Regarding to the relations between the notions of projectively parallelizable manifolds and linearly parallelizable manifolds there are two questions that we have tried to solve. The first one is to know when a projectively parallelizable manifold is linearly parallelizable (since it is proved in proposition 3 that every linearly parallelizable manifold is projectively parallelizable). The second question is to know when every projective parallelism is the projection of a linear parallelism. The first question is partially solved since it is proved that if the dimension is 1,2,4 or 8, then every projectively parallelizable manifold is linearly parallelizable (corollary 10), and in the other dimensions we have found examples of projectively parallelizable manifolds that are not linearly parallelizable (examples 2 and 3).

The second question is solved since we have found a two-covering manifold associated to each projective parallelism with the property of being trivial if and