After routine calculation, we get:

$$\frac{dRe\lambda(a)}{da}\Big|_{a=a_0} = \frac{-2\gamma w^2 a_0^3}{\frac{\varepsilon^4}{b}[(\frac{a_0^2\gamma b}{\varepsilon^2} - \frac{6a_0w^2}{\varepsilon})^2 + (\frac{2a_0^2 w}{\varepsilon^2} - 4w^3)^2]} < 0.$$

The facts established about the behaviour of the eigenvalues mean that the conditions of Andronov - Hopf bifurcation theorem hold and the theorem is proved.

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Received:25.VII.1990

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ANALELE STIINTIFICE ALE UNIVERSITĂTII "AL.I.CUZA" IASI
Tomul XXXVIII, s.I.a.,Matematica, 1992,f.2.

GREEN'S FUNCTIONS FOR HIGH ORDER BOUNDARY VALUE COUPLED DIFFERENTIAL SYSTEMS

BY

R.J. VILLANUEVA and L. JODAR

1.Introduction. Two - point boundary value problems for coupled systems of differential equations of the type

$$\text{(1.1)}\qquad \begin{aligned} &X^{(p)} + A_{p-1}X^{(p-1)} + \ldots + A_1X^{(1)} + A_0X = f(t); \quad 0 \le t \le b \\ &\sum_{j=1}^{p}\{E_{ij}X^{(j-1)}(0) + F_{ij}X^{(j-1)}(b)\} = r_i; \quad 1 \le i \le p. \end{aligned}$$

where $f(t), X(t), A_k, E_{ij}, F_{ij}, r_i$ are complex matrices in $C^{n\times n}$ for $0 \le k \le p-1, 1 \le i, j \le p$, appear in many different physical problems, see [1, chap.1.]

The standard approach to study such problems is based on the consideration of an extended first order problem

$$\text{(1.2)}\qquad Y'(t) = CY(t) + F(t); B_aY(a) + B_bY(b) = R.$$

where $Y = (X, X', \ldots, X^{(p-1)})^T, F = (0, \ldots, f)^T, B_a$ and B_b are appropriate matrices in $C^{np\times pn}$,, R is a matrix in $C^{np\times n}$, and C is the companion matrix defined by

$$\text{(1.3)}\qquad C = \begin{bmatrix} 0 & I & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ \vdots & & & I \\ -A_0 & -A_i & \cdots & -A_{p-1} \end{bmatrix}$$

See [1][2][9]

This classical approach has the inconvenience of the computational cost due to the increases of the problem dimension. In particular it needs the computation of the matrix exponential $\exp(tC)$ and it is known that it is a difficult task [8].

These inconveniences motivates the study of some alternative approach that avoids the increase of the problem dimension. In [3], a solution for a very particular second order problem of the type (1.1) is proposed avoiding the increase of the problem dimension, however, the explicit solution of the problem is not computable. In recent papers [6][10] a method for solving problems of the type (1.1) for the case $p=2$, without considering the extended system (1.2) have been proposed. Such papers are based on the existance of solutions of the characteristic algebraic matrix equation

$$Z^2 + A_1 Z + A_0 = 0. \tag{1.4}$$

Unfortunately, equation (1.4) may be unsolvable [5] and in this case, the method given in [6][10] is not available.

The aim of this paper is to study a uniqueness condition for the solution of problem (1.1) as well as an explicit expression of the solution of the problem in terms of a Green's matrix function $G(t,s)$, taking the idea of [6][10] but without the restriction of the existence of solutions of the associated albegraic matrix equation

$$Z^p + A_{p-1}Z^{p-1} + \dots + A_1 Z + A_0 = 0. \tag{1.5}$$

The paper is organized as follows. In section 2 we introduce the concept of rectangular co-solution for the equation (1.5) and we state some results recently given [7], that will be used in the following sections. In section 3, we construct the Green matrix function of problem (1.1) by using an appropriate set of co-solutions of equation (1.5). Finally, in section 4 an explicit closed form solution of problem (1.1) in terms of the Green matrix function is given.

2.Rectangular co-solutions of polynomial matrix equations and applications. The results of this section are given in [7]. We begin by introducing the concept of rectangular co-solution of equation (1.5)

Definition 2.1. *We say that (X,T) is a (n,q) co-solution of equation (1.5) if $X \in \mathbb{C}^{n\times q}, T \in \mathbb{C}^{q\times q}, X \neq 0$ and*

$$XT^p + A_{p-1}XT^{p-1} + \dots + A_0 X = 0. \tag{2.1}$$

Definition 2.2. *Let (X_i,T_i) be a (n,m_i) co-solution for $1 \le i \le k$. We say that $\{(X_i,T_i), 1 \le i \le k\}$ is a k -complete set of co-solutions of (1.5) if the block matrix $W = (W_{ij})$, with $W_{ij} = X_j T_j^{i-1}$ for $1 \le i \le p, 1 \le j \le k$, is invertible.*

Theorem 1.([7]) *Let us suppose that the companion matrix C has k linearly independent eigenvectors. Then equation (1.5) admits a k-complete set of co-solutions. If $M = (M_{ij})$ with $M_{ij} \in \mathbb{C}^{n\times m_j}$, is a nonsingular matrix in $\mathbb{C}^{np\times pn}, 1 \le i \le p, 1 \le j \le k$, and if the canonical form of C takes the form $J = \operatorname{diag}(J_1,\dots,J_k)$, with $J_j \in \mathbb{C}^{m_j\times m_j}, m_1 + \dots + m_k = np$, such that*

$$M\operatorname{diag}(J_1,\dots,J_k) = CM \tag{2.2}$$

then $\{(M_{1s},J_s), 1 \le s \le k\}$ is a k-complete set of co-solutions of (1.5).

Corollary 1.([7]) *Let us suppose that C has k linearly independent eigenvectors, and let $\{(M_{ij},J_s), 1 \le s \le k\}$ the k-complete set of solutions provided by the theorem 1. Then, the general solution of the matrix differential equation (1.1) is given by*

$$X(t) = \sum_{s=1}^{k} M_{1s}\exp(tJ_s)D_s \tag{2.3}$$

where D_s, is an arbitary matrix in $\mathbb{C}^{m_s\times n}$. If W is the block partitioned matrix associated to the set $\{(M_{1s},J_s), 1 \le s \le k\}$ by definition 2.2, the only solution of (1.1) the Cauchy conditions $X^{(j)}(0) = C_j, 0 \le j \le p-1$, is given by (2.3), where the matrices D_s, for $1 \le i \le k$, are uniquely determined by the expression

$$\begin{bmatrix} D_1 \\ \vdots \\ D_k \end{bmatrix} = W^{-1}\begin{bmatrix} C_0 \\ \vdots \\ C_{p-1} \end{bmatrix} \tag{2.4}$$

3.Construction of the Green matrix function. Let us consider the homogeneous problem

$$X^{(p)} + A_{p-1}X^{(p-1)} + \dots + A_1 X^{(1)} + A_0 X = 0; \quad 0 \le t \le b \tag{3.1}$$

$$\sum_{j=1}^{p}\{E_{ij}X^{(j-1)}(0) + F_{ij}X^{(j-1)}(b)\} = 0; \quad 1 \le i \le p. \tag{3.2}$$

and let $\{(M_{1j},J_i), 1 \le i \le k\}$ be the k-complete set of co-solutions of equation (1.5) provided by corollary 1. Then, the general solution of equation (3.1) is given by

$$X(t) = \sum_{i=1}^{k} U_i(t)D_i \tag{3.3}$$

where D_i is an arbitrary matrix in $\mathbb{C}^{m_j\times n}$ and

$$U_i(t) = M_{1i}\exp(tJ_i). \qquad i = 1,\dots,k. \tag{3.4}$$

Let us consider the matrix function $G(t,s)$ defined by

$$G(t,s) = \begin{cases} \sum_{i=1}^{k} U_i(t)P_i(s), & 0 \le t \le s \\ \sum_{i=1}^{k} U_i(t)Q_i(s), & s \le t \le b \end{cases} \tag{3.5}$$

where the $\mathbb{C}^{m_i \times n}$ valued matrix functions $P_i(s), Q(s)$ have to be determined so that

1.- $G(t,s)$ is a continuos matrix function in $[0,b] \times [0,b]$ and moreover, $\partial^{(j)}G/\partial t^{(j)}$ is a continuous function in (t,s), for (t,s) in the triangles $0 \le t < s \le b$ and $0 \le s < t \le b$ for $j = 1, ..., p-2$.

2.- If I is the identity matrix in $\mathbb{C}^{n\times n}$, one gets the jump discontinuity

$$\partial^{(p-1)}G/\partial t^{(p-1)}(s+0,s) - \partial^{(p-1)}G/\partial t^{p-1)}(s-0,s) = I \tag{3.6}$$

3.- As a function of $t, G(t,s)$ satisfies (3.1) and (3.2) in $[0,b]$, if $t \neq s$.

From (3.5) the continuity condition at $t = s$ of Green's function gives us that

$$\sum_{i=1}^{k} U_i(t)P_i(s) = \sum_{i=1}^{k} U_i(t)Q_i(s),$$

and

$$\sum_{i=1}^{k} U_i(t)(P_i(s) - Q_i(s)) = 0. \tag{3.7}$$

On the other hand, by the continuity condition of the partial derivatives of the Green's function until order $p-2$ at $t = s$, we obtain

$$\sum_{i=1}^{k} U_i^{(j)}(t)P_i(s) = \sum_{i=1}^{k} U_i^{(j)}(t)Q_i(s), \ j = 1, ..., p-2$$

and then

$$\sum_{i=1}^{k} U_i^{(j)}(t)(P_i(s) - Q_i(s)) = 0, \ j = 1, ..., p-2. \tag{3.8}$$

From (3.5) and (3.6) it follows that

$$\sum_{i=1}^{k} U_i^{(p-1)}(t)(P_i(s) - Q_i(s)) = I. \tag{3.9}$$

Let us write

$$R_i(s) = P_i(s) - Q_i(s), \ i = 1, ..., k \tag{3.10}$$

then, conditions (3.7) - (3.10) may be written in the compact form

$$U(s)\begin{bmatrix} R_1(s) \\ \vdots \\ R_k(s) \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ I \end{bmatrix} \tag{3.11}$$

where

$$U(s) = \begin{bmatrix} U_1(s) & \cdots & U_k(s) \\ \vdots & & \vdots \\ U_1^{(p-1)}(s) & & U_k^{(p-1)}(s) \end{bmatrix} \tag{3.12}$$

Note that the matrix function $U(s)$ defined by (3.12) is invertible for all s, because we can decompose $U(s)$ in the form

$$U(s) = W \text{ diag } \{\exp(sJ_j), 1 \le i \le k\} \tag{3.13}$$

where

$$W = \begin{bmatrix} M_{11} & \cdots & M_{1k} \\ \vdots & & \vdots \\ M_{11}J_1^{p-1} & \cdots & M_{1k}J_k^{p-1} \end{bmatrix} \tag{3.14}$$

is invertible since $\{(M_{1i,J}, 1 \le i \le k\}$ is a k-complete set of co-solution of equation (1.5).

Let us denote $Y = [Y_{ij}]_{1\le i\le p, 1\le j\le k}$ with $Y_{ij} \in \mathbb{C}^{m_i\times n}$ the inverse of the matrix W. Then, from (3.11) and (3.13), it follows that

$$R_i(s) = \exp(-sT_i)Y_{ik} \quad 1 \le i \le k \tag{3.15}$$

If we impose that $G(t,s)$ defined by (3.5) satisfies the initial conditions (3.2), we obtain condition

$$\sum_{j=1}^{p}\left\{E_{ij}\sum_{m=1}^{k} U_m^{(j-1)}(O)P_m(s) + F_{ij}\sum_{m=1}^{k} U_m^{(j-1)}(b)Q_m(s)\right\} = 0, \tag{3.16}$$
$$i = 1, ..., p.$$

From (3.10) we have

$$Q_i(s) = P_i(s) - R_i(s), \quad i = 1, ..., p, \tag{3.17}$$

Substituting (3.17) into (3.16), it follows that

$$\sum_{j=1}^{p}\left\{E_{ij}\sum_{m=1}^{k} U_m^{(j-1)}(O)P_m(s) + F_{ij}\sum_{m=1}^{k} U_m^{(j-1)}(b)\Big[P_m(s) - R_m(s)\Big]\right\} = 0,$$
$$i = 1, ..., p.$$

and

$$(3.18)\qquad \begin{aligned} &\sum_{m=1}^{k}\sum_{j=1}^{p}\left(E_{ij}U_m^{(j-1)}(O)P_m(s)+F_{ij}U_m^{(j-1)}(b)\right)P_m(s)=\\ &\sum_{m=1}^{k}\sum_{j=1}^{p}U_m^{(j-1)}(b)R_m(s)\\ &i=1,\dots,p. \end{aligned}$$

Let S be the block matrix

$$(3.19)\qquad S=\left[\sum_{j=1}^{p}\left(E_{ij}U_m^{(j-1)}(O)+F_{ij}U_m^{(j-1)}(b)\right)\right]_{1\le i\le p,1\le m\le k}$$

and let us suppose that S is a nonsingular matrix, and let S^{-1} be the inverse of S

$$(3.20)\qquad S^{-1}=[T_{mi}]_{1\le m\le k,1\le i\le p},\ \textit{with } T_{mi}\in \mathbb{C}^{m_i\times n}.$$

Note that the conditions (3.18) may be written in the form

$$(3.21)\qquad S\begin{bmatrix}P_1(s)\\ \vdots\\ P_k(s)\end{bmatrix}=\begin{bmatrix}\sum_{m=1}^{k}\sum_{j=1}^{p}F_{1j}\quad U_m^{(j-1)}(b)\quad R_m(s)\\ \vdots\\ \sum_{m=1}^{k}\sum_{j=1}^{p}F_{pj}\quad U_m^{(j-1)}(b)\quad R_m(s)\end{bmatrix}$$

From (3.20) and (3.21), it follows that:

$$\begin{bmatrix}P_1(s)\\ \vdots\\ P_k(s)\end{bmatrix}=\begin{bmatrix}T_{11}&\cdots&T_{1p}\\ \vdots&&\vdots\\ T_{k1}&\cdots&T_{kp}\end{bmatrix}\begin{bmatrix}\sum_{j=1}^{p}\sum_{m=1}^{k}F_{1j}\quad U_m^{(j-1)}(b)\quad \exp(-sT_m)\quad Y_{mp}\\ \vdots\\ \sum_{j=1}^{p}\sum_{m=1}^{k}F_{pj}\quad U_m^{(j-1)}(b)\quad \exp(-sT_m)\quad Y_{mp}\end{bmatrix}$$

and then

$$(3.22)\qquad \begin{aligned} &P_s(s)=\sum_{l=1}^{p}\sum_{j=1}^{p}\sum_{m=1}^{k}T_{sl}F_{lj}U_m^{(j-1)}(b)\exp(-sT_m)Y_{mp}\\ &s=1,\dots,k. \end{aligned}$$

Hence and from (3.15), (3.17) it follows that

$$(3.23)\qquad \begin{aligned} &Q_s(s)=P_s(s)-R_s(s)=\\ &=P_s(s)=\left(\sum_{l=1}^{p}\sum_{j=1}^{p}\sum_{m=1}^{k}T_{sl}F_{lj}U_m^{(j-1)}(b)\exp(-sT_m)Y_{mp}\right)-\\ &-\exp(-sT_s)Y_{sp}\\ &s=1,\dots,k. \end{aligned}$$

Thus the following result has been established.

Theorem 2. *Let $\{(M_{1i},J_i),1\le i\le k\}$ be the k-complete set of co-solutions of equation (1.5) given by theorem 1 and let $\{U_i(t),1\le i\le k\}$ be defined by (3.4). If the matrix S defined by (3.19) is nonsingular, then the boundary value matrix problem (3.1) - (3.2) has a unique Green's matrix function defined by (3.5), where $P_i(s)$ and $Q_i(s)$ are given by (3.22) and (3.23).*

4.Solution of the non-homogeneous boundary problem. Let us consider the intermediate boundary value problem,

$$(4.1)\qquad \begin{aligned} &X^{(p)}+A_{p-1}X^{(p-1)}+\dots+A_1X^{(1)}+A_0X=f(t)\\ &\sum_{j=1}^{p}E_{ij}X^{(j-1)}(0)+F_{ij}X^{(j-1)}(b)=0\quad i=1,\dots,p \end{aligned}$$

where $f(t)$ is a continuous matrix function in $[0,b]$.

Let $X(t)$ be defined by

$$(4.2)\qquad \begin{aligned} X(t)&=\int_0^b G(t,s)f(s)ds=\\ &=\int_0^t G(t,s)f(s)ds+\int_t^b G(t,s)f(s)ds. \end{aligned}$$

Taking derivatives and using the Leibniz' rule, we have

$$\begin{aligned} X'(t)&=\int_0^t\frac{\partial G(t,s)}{\partial t}f(s)ds+G(t,t)f(t)+\int_t^b\frac{\partial G(t,s)}{\partial t}f(s)ds-\\ &-G(t,t)f(t)=\int_0^b\frac{\partial G(t,s)}{\partial t}f(s)ds.\\ X''(t)&=\int_0^t\frac{\partial^2 G(t,s)}{\partial t^2}f(s)ds+\frac{\partial G(t,t)}{\partial t}f(t)+\int_t^b\frac{\partial^2 G(t,s)}{\partial t^2}f(s)ds-\\ &-\frac{\partial G(t,t)}{\partial t}f(t)=\int_0^b\frac{\partial^2 G(t,s)}{\partial t^2}f(s)ds. \end{aligned}$$

$\vdots$

$$X^{(p-1)}(t)=\int_0^b \frac{\partial^{(p-1)}G(t,s)}{\partial t^{(p-1)}}f(s)ds$$

$$X^{p}(t)=\int_0^b \frac{\partial^{p}G(t,s)}{\partial t^{p}}f(s)ds+\frac{\partial^{(p-1)}G(t,t-0)}{\partial t^{(p-1)}}f(t)+$$
$$+\int_t^b \frac{\partial^{p}G(t,s)}{\partial t^{p}}f(s)ds-\frac{\partial^{(p-1)}G(t,t+0)}{\partial t^{(p-1)}}f(t)=$$
$$=\int_0^b \frac{\partial^{p}G(t,s)}{\partial t^{p}}f(s)ds+\left[\frac{\partial^{(p-1)}G(t,t-0)}{\partial t^{(p-1)}}-\frac{\partial^{(p-1)}G(t,t+0)}{\partial t^{(p-1)}}\right]f(t)=$$
$$=\int_0^b \frac{\partial^{p}G(t,s)}{\partial t^{p}}f(s)ds+f(t).$$

Hence and from the properties of $G(t,s)$ it follows that

$$X_{(p)}+A_{p-1}X^{(p-1)}+\ldots+A_iX^{(1)}+A_0X=$$
$$=\int_0^b \frac{\partial^{(p)}G(t,s)}{\partial t^{(p)}}f(s)ds+f(t)$$
$$+A_{p-1}\int_0^b \frac{\partial^{(p-1)}G(t,s)}{\partial t^{(p-1)}}f(s)ds+\ldots+A_0\int_0^b G(t,s)f(s)ds=$$
$$=\int_0^b\left[\frac{\partial^{(p)}G(t,s)}{\partial t^{(p)}}+A_{p-1}\frac{\partial^{(p-1)}G(t,s)}{\partial t^{(p-1)}}+\ldots+A_0G(t,s)\right]f(s)ds+f(t)=f(t)$$

and

$$E_iX^{(i-1)}(0)-F_iX^{(i-1)}(b)=\int_0^b\left[E_i\frac{\partial^{(i-1)}G(0,s)}{\partial t^{(i-1)}}+\right.$$
$$\left.+F_i\frac{\partial^{(i-1)}G(b,s)}{\partial t^{(i-1)}}\right]f(s)ds=0\quad i=1,\ldots,p$$

Let us solve now, the auxiliary problem

$$X^{(p)}+A_{p-1}X^{(p-1)}+\ldots+A_1X^{(1)}+A_0X=0$$
$$\sum_{j=1}^{p}E_{mj}X^{(j-1)}(0)+F_{mj}X^{(j-1)}(b)=0,\ m\neq i \tag{4.3}$$
$$\sum_{j=1}^{p}E_{ij}X^{(j-1)}(0)+F_{ij}X^{(j-1)}(b)=I$$

Then, it is known that the form of the solutions of (4.3) is

$$X(t)=\sum_{m=1}^{k}U_m(t)Q_m.$$

The initial conditions of (4.3) give us the next expression

$$\sum_{j=1}^{p}\left\{E_{lj}\sum_{m=1}^{k}U_m^{(j-1)}(0)Q_m+F_{lj}\sum_{m=1}^{k}U_m^{(j-1)}(b)Q_m\right\}=\begin{cases}0 & l\neq i\\ 1 & l=i\end{cases}$$

and

$$\sum_{j=1}^{p}\sum_{m=1}^{k}\left\{E_{lj}\sum_{m=1}^{k}U_m^{(j-1)}(0)Q_m+F_{lj}U_m^{(j-1)}(b)\right\}Q_m=\begin{cases}0 & l\neq i\\ 1 & l=i\end{cases}$$

If we set the last expression in matricial form

$$S\begin{bmatrix}Q_1\\ \vdots\\ \vdots\\ \vdots\\ Q_k\end{bmatrix}=\begin{bmatrix}0\\ \vdots\\ I\\ \vdots\\ 0\end{bmatrix}$$

and remember that S is a nonsingular matrix, then from (3.20)

$$\begin{bmatrix}Q_1\\ \vdots\\ \vdots\\ \vdots\\ Q_k\end{bmatrix}=\begin{bmatrix}T_{11} & \cdots\cdots & T_{1p}\\ \vdots & & \vdots\\ \vdots & & \vdots\\ T_{k1} & \cdots\cdots & T_{kp}\end{bmatrix}\begin{bmatrix}0\\ \vdots\\ I\\ \vdots\\ 0\end{bmatrix}=\begin{bmatrix}T_{1i}\\ \vdots\\ \vdots\\ \vdots\\ T_{ki}\end{bmatrix}$$

Thus $Q_j=T_{ji}$ and the solutions, which we call $G_i(t)$, are

$$G_i(t)=\sum_{j=1}^{k}U_j(t)T_{ji}\qquad i=1,\ldots,p \tag{4.4}$$

Thus, the following result has been proved:

Theorem 3. *Let us assume that the algebraic equation (1.5) has a k - complete set of co-solutions $\{(M_{1i},J_i),1\leq i\leq k\}$ and let $\{U_i(t),1\leq i\leq k\}$ be defined by (3.4). If the matrix S defined by (3.19) is nonsingular, $S^{-1}=[T_{mi}]_{1\leq m\leq k,1\leq i\leq p}$ and $f(t)$ is continuous, then the boundary value problem (1.1) has a unique solution given by*

$$X(t)=\int_0^b G(t,s)f(s)ds+\sum_{i=1}^{p}G_i(t)r_i$$

where $G_i(t), 1 \leq i \leq p$, is given by (4.4) and $G(t,s)$ is the Green's matrix function constructed by theorem 2.

ACKNOWLEDGEMENTS. - This paper has been suported by the NATO grant CRG 900040.

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Received:8.11.1991

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ANALELE STIINTIFICE AL UNIVERSITATII "AL.I.CUZA" IASI
Tomul XXXVIII , s.I.a Matematica 1992, f.2

NOUVELLES CLASSES DE VARIETES DE KENMOTSU

PAR

T.TSHIKUNA-MATAMBA

0.Introduction. Soit (M', g', J') une variété presque hermitienne. Utilisant la classification de G r a y - H e r v e l l a [6] des variétés presque hermitiennes, J.A. O u b i ñ a [10,11] a pu obtenir quelques classes remarquables des structures métriques presque de contact sur le produit $M' \times \mathbf{R}$; c'est le cas notamment des structures trans - sasakiennes, nearly - trans - sasakiennes, presque trans - sasakiennes , G_1 et G_2 - sasakiennes , pour ne citer que celles - lá.

Il nous a semblé intéressant d'étudier, de manière analogue, le produit déformé de la droite réelle par une variété presque hermitienne. En effet,lorsque M' est une variété kählerienne, K e n m o t s u [9] a construit une structure métrique presque de contact sur ce produit ; J a n s s e n s - V a n h e c k e [8] comme B l a i r - O u b i ñ a [2] appellent cette structure: "*structure de Kenmotsu* ".

Dans ce papier nous nous proposons de caractériser quelques classes remarquables des structures métriques presque de contact sur ce produit déformé. En faisant parcourir la variété M' dans la classification de G r a y - H e r v e l l a , nous obtenons huit nouvelles classes apparentées aux variétés de Kenmotsu que nous appelons: variétés *nearly de Kenmotsu* , *semi-de Kenmotsu normale* , *quasi de Kenmotsu* , G_1 *-de Kenmotsu* , G_1 *-semi-de Kenmotsu* , G_2 *- de Kenmotsu*, G_2 *-semi-de Kenmotsu* et *presque trans-de Kenmotsu*. Pour y parvenir, il nous a fallu déterminer la 2-forme fondamentale ainsi que sa différentielle , sa co différentielle et la connexion riemannienne de ce produit déformé. Les formules que nous avons obtenues permettent de retrouver les définitions des variétés de Kenmotsu et presque de Kenmotsu comme déjà formulées par [8]. Après avoir rappelé quelque notions fondamentales sur les variétés presque hermitiennes et les variétés métriques presque de contact, nous avons consacré le Sec.2 à la classification de ces nouvelles variétés. Au Sec.3 nous produisons quelques exemples afin d'illustrer les classes obtenues.

Nous terminons par l'étude des relations d'inclusion entre les diverses classes obtenues.

Les variétés presque hermitiennes qui interviennent dans ce papier sont supposées de dimensions $2m$ et lisses. Nous noterons par $\mathcal{X}(M)$ l'algèbre de Lie des champs de vecteurs sur M.

1.Préliminaires. Une variété *presque hermitienne* est la donnée d'un triplet (M, g, J) soumis aux conditions suivantes:

(i) (M, g) est une variété riemannienne;