Observations. i) The Test 3 ,with itmax = 900 has been executed on the same computer without math coprocessor. The CPU-time spent was about 1h 15min ...!.

ii) nr conv.comb from tables represents the number of iterations of the Step 3 at each **(iter)** .

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ON CAUCHY'S PROBLEM FOR AN EQUATION OF THIRD ORDER

BY

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1.Introduction. In the domain

$$\mathcal{R} = \{(x,t) \in R^2 : -\infty < x < +\infty,\ t > 0\} \tag{1}$$

we consider the following partial differential equation

$$Lu \equiv D_x^3 u - D_t^2 u = 0, \tag{2}$$

where $D_x^3 = \frac{\partial^3}{\partial x^3}$, $D_t^2 = \frac{\partial^2}{\partial t^2}$.

In papers [6],[7] and [8] it is proved that equation (2) is one of the canonical forms of third-order partial differential equations and it is called the equation with characteristics multiple.

The problem consisting in finding solutions of equations with characteristics multiple by the method of the Laplace transformation has been intensively examined in papers [2],[3] and [10]. The relevant fundamental solutions of (2) were found by L. C a t t a b r i g a ([4]). he also constructed special potentials related to this equation and studied their properties. The papers [1] and [5] were devoted to solve some boundary-value problems for this equation by the method of the Green's function.

In this paper we deal with the Cauchy's problem for equation (2). The initial conditions (cf. (25),(26)) are prescribed on the straight line $t = 0$ (in papers [1],[5] one of the conditions is posed on the segment of the straight line $t = 0$ and the second on the segment of $t = T$).

Let us also observe that, to the best of our knowledge, the Cauchy's problem for equation (2) has not been examined so far.

2. Fundamental solution. Consider the function

$$U(x,t;y,\tau) = (t-\tau)^{-\frac{2}{3}}\Phi\left[(x-y)(t-\tau)^{-\frac{2}{3}}\right], \tag{3}$$

where Φ satisfies the differential equation

$$\Phi''(z) - \frac{4}{9}z^2\Phi'(z) - \frac{10}{9}z\Phi(z) = 0.$$

It can be verified directly that function (3) satisfies equation (2) and has the following properties (see [4], p.14 and p.25)

$$\int_0^{+\infty}\Phi(z)dz = \frac{\pi}{3}, \quad \int_{-\infty}^{0}\Phi(z)dz = \frac{2\pi}{3}, \tag{4}$$

$$|\Phi^{(k)}(z)| \le \begin{cases} Cz^{-k-\frac{5}{2}} & \text{when } z \to +\infty \\ C\exp[-c|z|^3] & \text{when } z < K \end{cases} \tag{5}$$

where $k = 0,1,\ldots, K = const. > 0$; C and c are positive constants depending on K and k.

Let us introduce the function

$$V(x,t;y,\tau) = (t-\tau)^{\frac{1}{3}}\Psi\left[(x-y)(t-\tau)^{-\frac{2}{3}}\right], \tag{6}$$

where Ψ satisfies the equation

$$-\frac{2}{3}z\Psi'(z) + \frac{1}{3}\Psi(z) = \Phi(z).$$

Since

$$D_tV = -D_\tau V = U, \tag{7}$$

the function V satisfies equation (2). The function V will be called the fundamental solution of the said equation.

It is easy to see that the function Ψ is of the form

$$\Psi(z) = \begin{cases} \frac{3}{2}z^{\frac{1}{2}}\int_z^{+\infty}\xi^{-\frac{3}{2}}\Phi(\xi)d\xi & \text{if } z > 0 \\ \frac{3}{2}|z|^{\frac{1}{2}}\int_{-\infty}^{z}|\xi|^{-\frac{3}{2}}\Phi(\xi)d\xi & \text{if } z < 0 \end{cases} \tag{8}$$

It follows from (5),(6) and (8) that the function V and its derivatives satisfy the following inequalities

$$|D_x^kV(x,t;y,\tau)| \le C(t-\tau)^{\frac{1}{3}-\frac{2}{3}k}\left|(x-y)(t-\tau)^{-\frac{2}{3}}\right|^{-\frac{5}{2}-k}, \tag{9}$$

when $(x-y)(t-\tau)^{-\frac{2}{3}} \to +\infty$,

$$|D_x^kV(x,t;y,\tau)| \le C(t-\tau)^{\frac{1}{3}-\frac{2}{3}k}\exp\left[-c\left|(x-y)(t-\tau)^{-\frac{2}{3}}\right|^3\right], \tag{10}$$

when $(x-y)(t-\tau)^{-\frac{2}{3}} < K$, where $(x,t),(y,\tau) \in \mathcal{R}$, $\tau < t$, $k = 0,1,\ldots$; $K = const. > 0$, C and c are positive constants depending on K, k.

3. Properties of some integrals. Now, we consider the integral

$$I(x,t;\nu) = \frac{1}{\pi}\int_{-\infty}^{+\infty}V(x,t;y,0)\nu(y)dy, \tag{11}$$

where V is given by formula (6) and ν is defined and bounded in $(-\infty,+\infty)$ and integrable in every bounded closed interval.

The importance of integral (11) in the theory of equation (2) is connected with the behaviour of this integral and its first derivative when the point $P(x,t)$ tends to a point $P_0(x_0,0)$ of the characteristic $t = 0$. We shall prove the following

Lemma 1. *The integral (11) satisfies equation (2) for $(x,t) \in \mathcal{R}$. Moreover, if ν is continuous at x_0, then*

$$\lim_{P\to P_0} I(x,t;\nu) = 0, \tag{12}$$

$$\lim_{P\to P_0} D_tI(x,t;\nu) = \nu(x_0). \tag{13}$$

Proof. To show that the function I satisfies equation (2) observe that in accordance with (9) and (10) we have

$$D_x^3I = \frac{1}{\pi}\int_{-\infty}^{+\infty}D_x^3V(x,t;y,0)\nu(y)dy,$$

$$D_t^2I = \frac{1}{\pi}\int_{-\infty}^{+\infty}D_t^2V(x,t;y,0)\nu(y)dy,$$

and hence

$$D_x^3I - D_t^2I = 0.$$

Now, we proceed to prove the relation (12). In view of (6) we obtain

$$|I(x,t;\nu)| \le M_\nu \int_{-\infty}^{+\infty} t^{\frac{1}{3}} \left|\Psi\left(\frac{x-y}{t^{\frac{2}{3}}}\right)\right| dy,$$

where $M_\nu = \sup_x |\nu(x)|$.

Changing the integration variable $x - y = zt^{\frac{2}{3}}$ we have

$$|I(x,t;\nu)| \le 2M_\nu Ct \int_0^{+\infty} |\Psi(z)|dz =$$

$$= 2M_\nu Ct \left[\int_0^K |\Psi(z)|dz + \int_K^{+\infty} |\Psi(z)|dz\right].$$

By virtue of (5) and (8) we obtain

$$|I(x,t;\nu)| \le 2M_\nu Ct \left[\int_0^K \exp[-cz^3]dz + \int_K^{+\infty} z^{-\frac{5}{2}}dz\right],$$

hence, we get the following relation

$$|I(x,t;\nu)| \le 2M_\nu Ct \left[\frac{1}{3\sqrt[3]{c}}\Gamma\left(\frac{1}{3}\right) + \frac{2}{3}K^{-\frac{3}{2}}\right],$$

which proves (12).

In accordance with (7), $D_t I$ can be represented as follows

$$D_t I(x,t;\nu) = \frac{\nu(x_0)}{\pi} \int_{-\infty}^{+\infty} U(x,t;y,0)dy + \tag{14}$$
$$+ \frac{1}{\pi}\int_{-\infty}^{+\infty} U(x,t;y,0)[\nu(y) - \nu(x_0)]dy.$$

Denote the terms of the sum on the right-hand side of (14) by $I_1(x,t)$ and $I_2(x,t)$ respectively. Using (3) and introducing the new integration variable $x - y = zt^{\frac{2}{3}}$ we get

$$I_1(x,t) = \frac{\nu(x_0)}{\pi} \int_{-\infty}^{+\infty} \Phi(z)dz,$$

hence, by virtue of (4), we obtain $I_1(x,t) = \nu(x_0)$.

With an arbitrary number $\epsilon > 0$ we can associate a number $\delta > 0$, such that for $|x - x_0| < \delta$, the inequality $|\nu(x) - \nu(x_0)| < \epsilon$ holds.

We represent the second integral of (14) as the sum of three integrals I_{21}, I_{22}, I_{23} over the intervals $(-\infty, x_0 - \delta)$, $[x_0 - \delta, x_0 + \delta]$, $(x_0 + \delta, +\infty)$ respectively.

Now, we consider the integral I_{22}

$$I_{22} = \frac{1}{\pi} \int_{x_0-\delta}^{x_0+\delta} U(x,t;y,0)[\nu(y) - \nu(x_0)]dy.$$

In view of (7) and (10), we obtain

$$|I_{22}| \le \frac{\epsilon}{\pi} \int_{x_0-\delta}^{x_0+\delta} t^{-\frac{2}{3}} \exp\left[-c|x-y|t^{\frac{2}{3}}\right] dy < const.\epsilon,$$

which implies that $I_{22} \to 0$, when $\delta \to 0$ $(x \to x_0)$.

For $|x - x_0| < \frac{\delta}{2}$ we get

$$|I_{23}| \le \frac{2M_\nu}{\pi} \int_{-\infty}^{(x-x_0-\delta)t^{-\frac{2}{3}}} |\Phi(z)|dz,$$

hence

$$|I_{23}| \le \frac{2M_\nu}{\pi} \int_{-\infty}^{-\frac{1}{2}\delta t^{-\frac{2}{3}}} |\Phi(z)|dz.$$

If $P \to P_0$, then $-\frac{1}{2}\delta t^{-\frac{2}{3}} \to -\infty$. Thus $I_{23} \to 0$.

In a similar way we examine the integral over the interval $(-\infty, x_0 - \delta)$. Finally we find that $I_2(x,t) \to 0$, which leads via (14) to the validity of the relation (13).

Now, we consider the integral

$$J(x,t;\mu) = \frac{1}{\pi}\int_{-\infty}^{+\infty} U(x,t;y,0)\mu(y)dy, \tag{15}$$

where U is given by formula (3) and μ is defined and bounded in $(-\infty, +\infty)$ and integrable in every interval $a \le y \le b$, where a and b are arbitrary real numbers.

Now we shall give a lemma connected with the properties of the integral J and its first derivative when the point $P(x,t)$ tends to a point $P_0(x_0, 0)$.

Lemma 2. *The integral (15) satisfies equation (2) for $(x,t) \in \mathcal{R}$. Furthermore, if the function μ possesses continuous derivatives up to the third order in $(-\infty,+\infty)$ and the following inequality is valid*

$$|\mu^{(k)}(x)| \le M_\mu, \tag{16}$$

where $M_\mu = const. > 0$, $k = 0,1,2,3$, $x \in (-\infty,+\infty)$, then

$$\lim_{P\to P_0} J(x,t;\mu) = \mu(x_0). \tag{17}$$

$$\lim_{P\to P_0} D_t J(x,t;\mu) = 0. \tag{18}$$

Proof. The first part of the lemma can be proved analogously to that of Lemma 1. Moreover, it is sufficient to prove relation (18), since the proof of (17) is similar to that of (13). Making use of (7) we get

$$D_t J(x,t;\mu) = \frac{1}{\pi}\int_{-\infty}^{+\infty} V(x,t;y,0)\mu'''(y)dy,$$

hence, the validity of relation (18) follows from the assumption (16).

In further reasoning we will also need the following integral

$$Z(x,t;\rho) = \frac{1}{\pi}\int\int_{\mathcal{R}_t} V(x,t;y,\tau)\rho(y,\tau)dyd\tau, \tag{19}$$

where

$$\mathcal{R}_t = \{(y,\tau) \in R^2 : -\infty < y < +\infty,\ 0 < \tau < t\}$$

and the function ρ is continuous in $\mathcal{R}$.

The following lemma is valid

Lemma 3. *If the function ρ is continuous in $\mathcal{R}$ and satisfies the Hölder condition*

$$|\rho(y,\tau) - \rho(\bar{y},\tau)| \le M_\rho |y-\bar{y}|^\alpha, \tag{20}$$

where $M_\rho = const. > 0$, $(y,\tau),(\bar{y},\tau) \in \mathcal{R}_t$, $0 < \alpha \le 1$, then the equality

$$LZ(x,t;\rho) = -\rho(x,t) \tag{21}$$

holds.

Proof. In view of (6)-(9) we get

$$D_t Z(x,t;\rho) = \frac{1}{\pi}\int\int_{\mathcal{R}_t} D_t V(x,t;y,\tau)\rho(y,\tau)dyd\tau,$$

hence

$$\begin{aligned} D_t^2 Z(x,t;\rho) = {} & \frac{1}{\pi}\lim_{\tau\to t}\int_{-\infty}^{+\infty} U(x,t;y,\tau)\rho(y,\tau)dy + \\ & + \frac{1}{\pi}\int\int_{\mathcal{R}_t} D_t^2 V(x,t;y,\tau)\rho(y,\tau)dyd\tau. \end{aligned} \tag{22}$$

Denoting the first integral on the right-hand side of (22) by $\mathcal{I}(x,t;\tau)$ we can write

$$\begin{aligned} \mathcal{I}(x,t;\tau) = {} & \rho(x,\tau)\int_{-\infty}^{+\infty} U(x,t;y,\tau)\rho(y,\tau)dy + \\ & + \int_{-\infty}^{+\infty} U(x,t;y,\tau)\left[\rho(y,\tau) - \rho(x,\tau)\right] dy. \end{aligned} \tag{23}$$

Basing on (3) and (5), we get the relation

$$\int_{-\infty}^{+\infty} U(x,t;y,\tau)dy = \pi.$$

It follows from the condition (20) that the second integral of (23) tends to zero when $\tau \to t$.

On joining above results we obtain

$$D_t^2 Z(x,t;\rho) = \rho(x,t) + \frac{1}{\pi}\int\int_{\mathcal{R}_t} D_x^2 V(x,t;y,\tau)\rho(y,\tau)dyd\tau;$$

moreover, it can be calculated by differentiating the integrand

$$D_x^3 Z(x,t;\rho) = \frac{1}{\pi}\int\int_{\mathcal{R}_t} D_x^3 V(x,t;y,\tau)\rho(y,\tau)dyd\tau.$$

By virtue of these formulae we arrive at the equality (21).

Basing on the properties of the function V, one can prove the following remark

Remark 1. The integral Z has the following properties

$$\lim_{t\to 0} D_t^k Z(x,t;\rho) = 0,$$

where $k = 0, 1$, $x \in (-\infty, +\infty)$.

4. The linear problem. In this section we examine the linear Cauchy's problem $(\mathcal{C} - \mathcal{L})$:

Find a solution u of the partial differential equation

$$Lu(x,t) = f(x,t) \tag{24}$$

in $\mathcal{R}$ satisfying the initial conditions

$$u(x,0) = g(x), \tag{25}$$

$$D_t u(x,0) = h(x), \tag{26}$$

where $x \in (-\infty, +\infty)$, f, g, h are given functions.

We make the following assumptions

(A.1) The function f is defined and continuous for $(x,t) \in \mathcal{R}$ and satisfies the conditions

$$|f(x,t)| \le M_f,$$
$$|f(x,t) - f(y,t)| \le M_f |x-y|^\alpha,$$

where $M_f = const. > 0$, $(x,t), (y,t) \in \mathcal{R}$, $0 < \alpha \le 1$.

(A.2) The function g possesses continuous derivatives up to the third order in $(-\infty, +\infty)$ and satisfies the inequality

$$|g^{(k)}(x)| \le M_g,$$

where $M_g = const. > 0$, $k = 0, 1, 2, 3$, $x \in (-\infty, +\infty)$.

(A.3) The function h is continuous in $(-\infty, +\infty)$ and satisfies the inequality

$$|h(x)| \le M_h,$$

where $M_h = const. > 0$, $x \in (-\infty, +\infty)$.

We now prove

Theorem 1. *If assumptions (A.1)-(A.3) are satisfied, then there exists a solution u of the $(\mathcal{C} - \mathcal{L})$ problem; moreover, the following inequality holds*

$$\|u\|_0 \le const.(\|f\|_0 + |g|_0 + |h|_0), \tag{27}$$

where $\|u\|_0 \equiv \sup_{x,t} |v(x,t)|$, $|w|_0 \equiv \sup_x |w(x)|$.

Proof. A solution of the $(\mathcal{C} - \mathcal{L})$ problem is of the form

$$u(t,x) = J(x,t;g) + I(x,t;h) - Z(x,t;f). \tag{28}$$

In fact, by virtue of Lemmas 1-3 and Remark 1 the function u, given by formula (28), satisfies the equation (24) and the initial conditions (25)-(26).

Now, we proceed to the estimation (27).

Basing on (A.3) and (11), and repeating the argument used in Lemma 1, we get the inequality

$$|I(x,t;h)| \le const.|h|_0. \tag{29}$$

The integral $J(x,t;g)$ may be estimated in a similar way, using (A.2), (15) and consequently

$$|J(x,t;g)| \le const.|g|_0. \tag{30}$$

It also follows from (7) that the integral $Z(x,t;f)$, given by formula (19), can be written in the form

$$Z(x,t;f) = \frac{1}{\pi}\int_0^t \int_{-\infty}^{+\infty} \int_\tau^t U(x,s;y,\tau) f(y,\tau)\, ds\, dy\, d\tau.$$

Equalities (4) and the assumption (A.1) applied to the above integral yields the inequality

$$|Z(x,t;f)| \le const.\|f\|_0. \tag{31}$$

On joining (29),(30) and (31), we obtain the inequality (27), as required.

5. The nonlinear problem. Let us consider the following problem $(\mathcal{C} - \mathcal{N})$:

Find a solution u of the partial differerential equation

$$Lu(x,t) = F[x,t;u(x,t)] \tag{32}$$

in $\mathcal{R}$ satisfying the initial conditions

$$u(x,0) = g(x), \tag{33}$$

$$D_t u(x,0) = h(x), \tag{34}$$

where $x \in (-\infty, +\infty)$, F, g, h are given functions.

We make one more assumption

(A.4) the function F is defined and continuous for $(x,t) \in \mathcal{R}$, $|u| < \infty$ and satisfies the conditions

$$|F[x,t;u]| \leq M_F(1+|u|^{k_F}),$$
$$|F[x,t;u] - F[\bar{x},t;\bar{u}]| \leq m_F(|x-\bar{x}|^\alpha + |u-\bar{u}|^{h_F}),$$

where $M_F, m_F = const. > 0$, $k_F \in (0,1)$, $\alpha \in (0,1]$, $h_F \in (0,1]$.

Theorem 2. *If assumptions (A.2)-(A.4) are satisfied, then there exists a solution u of the $(\mathcal{C}-\mathcal{N})$ problem.*

Proof. We seek a solution of the $(\mathcal{C}-\mathcal{N})$ problem in the following form

$$u(x,t) = J(x,t;g) + I(x,t;h) - Z(x,t;F[u]), \tag{35}$$

where

$$Z(x,t;F[u]) = \frac{1}{\pi}\int\int_{\mathcal{R}_t} V(x,t;y,\tau)F[y,\tau;u(y,\tau)]dyd\tau.$$

We shall prove the existence of a solution of the equation (35) using a topological method based on the Schauder's fixed point theorem.

Let us consider the set B of all functions u, defined and continuous in $\overline{\mathcal{R}}$, such that $\|u\|_0 < \infty$. If we define in B in the usual manner linear operations and the distance of two points $u, v \in B$ by the formula $\delta(u,v) \equiv \|u-v\|_0$, then it can be shown that B is Banach space (comp. [9], p.144).

Let Λ denote the set of all points of B satisfying the condition

$$\|u\|_0 \leq \lambda, \tag{36}$$

where λ is a positive parameter. It is easily seen that λ is non-empty, closed and convex set.

In view of the equation (35), we map Λ by the following transformation

$$Tu(x,t) = J(x,t;g) + I(x,t;h) - Z(x,t;F[u]). \tag{37}$$

We shall denote by v the image of an arbitrary chosen point u of Λ, i.e. $v = Tu$ and by Θ the image of the whole set Λ, i.e. $\Theta = T\Lambda$. Now we are going to obtain a condition implying the inclusion $\Theta \subset \Lambda$.

By Theorem 1 the function v satisfies the relation

$$\|v\|_0 \leq const.\ (\|\mathrm{F}\|_0 + |g|_0 + |h|_0),$$

whence by assumptions (A.2) - (A.4) we get

$$\|v\|_0 \leq const.\ [M_F(1+|u|^{k_F}) + M_g + M_h].$$

Thus, the inclusion $\Theta \subset \Lambda$ holds if

$$const.\ \left[M_F(1+\lambda^{k_F}) + M_g + M_h\right] \leq \lambda. \tag{38}$$

Since, in view of the assumption (A.4) $k_F < 1$, then we can always choose λ so that the condition (38) holds true.

By an argument analogous to that in [9] one can prove that the transformation T is continuous and Θ is a compact set.

Taking into account the results obtained above we can assert that all the assumptions of Schauder's theorem are satisfied and therefore there exists in Λ a fixed point $\tilde{u}$ of the trasnformation T. Hence, the function $\tilde{u}$ satisfies the equation (35). In virtue of Lemmas 1-3 and Remark 1, this function fulfils the equation (32) and the conditions (33)-(34). Thus, the function $\tilde{u}$ is the required solution of the $(\mathcal{C}-\mathcal{N})$ problem.

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TENSOR PRODUCT OF STANDARD H-CONES

BY

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Introduction. R.Cairoli seems to be the first to consider the product of two (transition) semi-groups, with application to the tensor product of Markov processes [6,7]. Independently, in 1978, a tensor product of harmonic structures,was constructed by: U.S c h i r m e i e r [16], L.S t o i c a [17,18] and E.P o p a [14,15]. The main result was essentially the same: the tensor product of two semi-groups, one of which is strongly Feller, has a strongly Feller associated resolvent. Hence, the cone of excessive functions (with respect to the product semi-group), is a standard H-cone of functions. Results concerning the product of spaces of balayage are included in the monograph by B l i e d t n e r-H a n s e n [2]. H a n s e n [10] used the fact that the harmonic structure associated with the heat equation is a product one. W.M e i e r [13] considered also the product of harmonic structures,especially from the probabilistic viewpoint. In 1980, B e r t i n [1],constructed a tensor product of (symmetric) Dirichlet spaces,using an essentially different procedure.

The main result of this paper shows that the product of two semi-groups, one of which is absolutely continuous, has the associated resolvent absolutely continuous; hence,the cone of excessive functions with respect to the product semi-group is still a standard H-cone. Moreover, the absolutely continuous semi-groups are characterised as those semi-groups, whose tensor product with the translation semi-group on **R**, has an absolutely continuous associated resolvent.

The paper also contains a sufficient condition in order that an ab-

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