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ON TOTALLY UMBILICAL CR - SUBMANIFOLDS IN A KAEHLERIAN MANIFOLD

BY

IZUMI HASEGAWA

1.Introduction. S. D e s h m u k h and S.I. H u s a i n [2] proved a classification theorem for totally umbilical CR - submanifold in a Kaehlerian manifold. From their theorem we have the following.

Problem. *Let M be a 3- dimensional totally umbilical proper CR - submanifold in a Kaehlerian manifold $\tilde{M}$. Then, is the mean curvature vector μ parallel ?*

We arrange their theorem in section 3 and give an affirmative answer in section 4.

2.Preliminaries. Let $\tilde{M}$ be a real 2 m - dimensional Kaehlerian manifold and M an n -dimensional submanifold in $\tilde{M}$. We denote by $<,>$ the Riemannian metric of $\tilde{M}$ as well as that induced on M, and by J the almost complex structure on $\tilde{M}$. The Riemannian connection $\tilde{\nabla}$ induces the Riemannian connection ∇ on M and the normal connection $\nabla^{\perp}$ in $T^{\perp}M$ obeying the Gauss and Weingarten formulas

$$\tilde{\nabla}_X Y = \nabla_X Y + \sigma(X,Y), \tag{2.1}$$

$$\tilde{\nabla}_X \alpha = -A_\alpha X + \nabla^{\perp}_X \alpha \tag{2.2}$$

for tangent vector fields X, Y and normal vector field α on M, where σ is the second fundamental form on M and A_α the Weingarten map with respect to α related as

$$< \sigma(X,Y), \alpha >=< A_\alpha X, Y > . \tag{2.3}$$

An n -dimensional submanifold M is said to be *totally umbilical* if

$$\sigma(X,Y) =< X,Y > \mu, \tag{2.4}$$

where $\mu := \frac{1}{n}$ (trace σ), called the *mean curvature vector* of M in $\tilde{M}$. For totally umbilical submanifold M, the equations (2.1) and (2.2) take the form

$$\tilde{\nabla}_X Y = \nabla_X Y + < X, Y > \mu, \tag{2.5}$$

$$\tilde{\nabla}_X \alpha = - < \alpha, \mu > X + \nabla^{\perp}_X \alpha. \tag{2.6}$$

The equation of Codazzi for totally umbilical submanifold M is given by

$$< \tilde{R}(X,Y)Z, \alpha > = < Y, Z > < \nabla^{\perp}_X \mu, \alpha > - < X, Z > < \nabla^{\perp}_Y \mu, \alpha > \tag{2.7}$$

for tangent vector fields X, Y, Z and normal vector field α on M, where $\tilde{R}$ denotes the Riemannian curvature tensor of $\tilde{M}$.

A submanifold M in $\tilde{M}$ is said to be a $CR-submanifold$ if there exist two orthogonal complementary distributions D and $D^{\perp}$ such that $JD = D$ and $JD^{\perp} \subset T^{\perp}M$. If $D = \{0\}$ (resp. $D^{\perp} = \{0\}$), then M is said to be a *totally real* (resp. *holomorphic*) submanifold. It follows that dim D = even and the normal bundle $T^{\perp}M$ splits as $T^{\perp}M = JD^{\perp} \oplus E$, where E is the invariant subbundle of $T^{\perp}M$ under J. A CR- submanifold is said to be *proper* if $D \neq \{0\}$ and $D^{\perp} \neq \{0\}$.

Lemma 1. *Let M be a totally umbilical CR -submanifold in a Kaehlerian manifold $\tilde{M}$. If M is not totally real, then $\mu \in JD^{\perp}$.*

Proof. We have

$$0 = (\tilde{\nabla}_X J)JX = -\tilde{\nabla}_X X - J\tilde{\nabla}_X JX = -\nabla_X X - \|X\|^2 \mu - J\nabla_X JX \tag{2.8}$$

for any $X \in D$. Taking inner product with $\alpha \in E$, we have

$$\|X\|^2 < \mu, \alpha > = 0. \tag{2.9}$$

Therefore we have $\mu \in JD^{\perp}$.

Lemma 2. *Let M be a totally umbilical proper CR -submanifold in a Kaehlerian manifold $\tilde{M}$. If dim $D^{\perp} = 1$ and $V \in D^{\perp}$, then $\nabla^{\perp}_V \mu = 0$.*

Proof. For any $X \in D$ and $V \in D^{\perp}$, we have

$$\begin{aligned} 0 &= < \tilde{R}(V,X)JX, J\alpha > = < \tilde{R}(V,X)X, \alpha > = \\ &= \|X\|^2 < \nabla^{\perp}_V \mu, \alpha > \quad (\forall \alpha \in E), \end{aligned}$$

from which

$$\nabla^{\perp}_V \mu \in JD^{\perp}. \tag{2.10}$$

Since $< \tilde{R}(V,X)X, V > = < \tilde{R}(V,X)JX, JV > = 0$, we have

$$\begin{aligned} 0 &= < \tilde{R}(V,X)JX, V > = - < \tilde{R}(V,X)X, JV > = \\ &= -\|X\|^2 < \nabla^{\perp}_V \mu, JV >, \end{aligned}$$

from which

$$\nabla^{\perp}_V \mu \in E. \tag{2.11}$$

From (2.10) and (2.11), we have the conclusion.

3. n (≥ 4)– dimensional totally umbilical CR -submanifolds.

Lemma 3.[2] *Let M be a totally umbilical CR -submanifold in a Kaehlerian manifold $\tilde{M}$. If dim $D^{\perp} \geq 2$, then $\mu \in E$. Furthermore, if M is proper, then M is totally geodesic.*

Proof. Using (2.5) and (2.6) and $J\tilde{\nabla}_V W = \tilde{\nabla}_V JW$ for $V, W \in D^{\perp}$, we have

$$J\nabla_V W + < V, W > J\mu = - < \mu, JW > V + \nabla^{\perp}_V JW.$$

Taking inner product with V, we obtain

$$< \mu, JW > \|V\|^2 = < \mu, JV > < V, W > . \tag{3.1}$$

Interchanging the role of V and W in (3.1) , we have

$$< \mu, JV > \|W\|^2 = < \mu, JW > < V, W > . \tag{3.2}$$

Using above two equations (3.1) and (3.2), we obtain

$$< \mu, JV > (\|V\|^2 \|W\|^2) - < V, W >^2) = 0,$$

from which

$$\mu \in E \tag{3.3}$$

because of dim $D^{\perp} \geq 2$.

Furthermore, assume that M is proper. Then, from Lemma 1 and (3.3), we have $\mu = 0$, that is, M is totally geodesic.

Theorem 4. (cf. [2]). *Let M be an $n(\geq 4)$– dimensional totally umbilical proper CR - submanifold in a Kaehlerian manifold $\tilde{M}$. Then the mean curvature vector μ of M in $\tilde{M}$ is parallel.*

Proof. Assume that M is not totally geodesic. Then $\dim D^{\perp} = 1$. In fact, if $\dim D^{\perp} \geq 2$, then M is totally geodesic by virtue of Lemma 3. In this case we have $\dim D \geq 4$ because of $\dim M \geq 4$ and $\dim D =$ even. For any $X \in D$ there exists nonzero $Y \in D$ such that $< X, Y >=< X, JY >= 0$. Using (2.7), we have

$$0 =< \tilde{R}(Y, JX)JY, \alpha >=< \tilde{R}(X, JY)JY, \alpha >=$$
$$= \|Y\| < \nabla^{\perp}_{X}\mu, \alpha >,$$

for any normal vector field α on M, from which

$$\nabla^{\perp}_{X}\mu = 0. \tag{3.4}$$

Therefore, from Lemma 2 and (3.4), the mean curvature vector μ is (nonzero) parallel.

Theorem 5.(cf.[2], [5]). *Let M, be an $n(\geq 4)$ - dimensional totally umbilical proper CR -submanifold in a Kaehlarian manifold $\tilde{M}$. Then M is one of the following:*

(1) M is an extrinsic sphere admitting homothetic Sasakian structure.

(2) M is totally geodesic and locally the Riemannian product of a holomorphic submanifold and a totally real submanifold in $\tilde{M}$.

Proof. If M is totally geodesic, then D and $D^{\perp}$ are integrable and their integral manifolds are totally geodesic in M. Therefore we have the case (2).

Assume that M is not totally geodesic. Then, from Theorem 4, the mean curvature vector μ is (nonzero) parallel, i.e. M is an extrinsic sphere. In this case, we put as follows:

$$\xi := \frac{-1}{\|\mu\|}J\mu,$$
$$\eta(X) :=< X, \xi >,$$
$$\varphi X := JX - \eta(X)J\xi \quad (\forall X \in TM).$$

Then it is easy to see that

$$\eta(\xi) = -1,$$
$$\varphi^2 X = -X + \eta(X)\xi,$$
$$< \varphi X, \varphi Y >=< X, Y > -\eta(X)\eta(Y),$$
$$(\nabla_X\varphi)Y = \|\mu\|\{\eta(Y)X- < X, Y > \xi\},$$

that is, $(\varphi, \xi, \eta <, >)$ is a homothetic Sasakian structure with constant $\|\mu\|$ on M.

4. 3 -dimensional totally umbilical CR - submanifolds with parallel Ricci tensor. Let M be a 3 -dimensional totally umbilical proper CR -submanifold in a Kaehlerian manifold $\tilde{M}$. Then $\dim D = 2$ and $\dim D^{\perp} = 1$.

Theorem 6. *Let M be a 3 -dimensional totally umbilical proper CR -submanifold in a Kaehlerian manifold $\tilde{M}$. If the Ricci tensor of M, is parallel, then the mean curvature vector μ is parallel.*

Proof. Let $\{X, JX, V\}$ be an orthonormal frame field on M satisfying $X \in D$ and $V \in D^{\perp}$. Since $\mu \in JD^{\perp}$ and $\dim D^{\perp} = 1$, we can put $\mu = hJV$, where $h :=< \mu, JV >$. Therefore we have

$$A_{JV}X = hX, \quad A_{JV}JX = hJX, \quad A_{JV}V = hV. \tag{4.1}$$

Using (4.1), we can easily see that

$$\nabla_X V = hJX, \quad \nabla_{JX}V = -hX, \quad \nabla_V V = 0, \tag{4.2}$$

and

$$\nabla^{\perp}_{X}JV = \nabla^{\perp}_{JX}JV = \nabla^{\perp}_{V}JV = 0. \tag{4.3}$$

Using Lemma 2 and (4.3), we have

$$0 = \nabla^{\perp}_{V}\mu = \nabla^{\perp}_{V}(hJV) = (Vh)JV + h\nabla^{\perp}_{V}JV = (Vh)JV,$$

from which

$$Vh = 0. \tag{4.4}$$

Then, by definition of the Ricci tensor S of M we have

$$S(V, V) =< R(V, X)X, V > + < R(V, JX)JX, V > \tag{4.5}$$

Here, using Gauss equation, we obtain

$$< R(V, X)X, V > =< \tilde{R}(V, X)X, V > + < \sigma(X, X), \sigma(V, V) >=$$
$$=< \tilde{R}(V, X)JX, JV > +h^2 = h^2,$$

and

$$< R(V, JX)JX, V >= h^2.$$

Therefore we get

$$S(V,V) = 2h^2, \tag{4.6}$$

where $h := \langle \mu, JV \rangle$. Since S is parallel, we have

$$0 = (\nabla_X S)(V,V) = \nabla_X S(V,V) - 2S(\nabla_X V, V) =$$
$$= 2h\{2Xh - S(JX,V)\} = 2h\{2Xh - \langle R(JX,X)X, V \rangle\} =$$
$$= 2h\{2Xh - \langle \tilde{R}(JX,X)JX, JV \rangle\} =$$
$$= 2h\{2Xh + \langle \nabla^{\perp}_X \mu, JV \rangle\} = 6h(Xh).$$

and

$$h((JX)h) = 0. \tag{4.8}$$

From (4.4), (4.7) and (4.8), h is constant. Therefore the mean curvature μ is parallel by virtue of (4.3).

The following corollary is proved by the same way as Theorem 5 in Sec.3.

Corollary 7. *Let M be a 3 -diemnsional totally umbilical proper CR - submanifold in a Kaehlerian manifold $\tilde{M}$. If the Ricci tensor of M is parallel, then M is one of the following:*

(1) M is an extrinsic sphere admitting homothetic Sasakian structure.

(2) M is totally geodesic and locally the Riemannian product of a holomorphic surface and a totally real curvature in $\tilde{M}$.

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ZUR FINITE UND RANDELEMENTE - APPROXIMATION ZWEIDIMENSIONALER TRANSSONISCHER TRAGFLÜGELUMSTRÖMUNGEN

VON

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1.Problemdarstellung. 1.1.Das Randwertproblem. Sei $\Omega \subset \mathbb{R}^2$ ein beschränktes Gebiet, das die einfach zusammenhängende Sektion Ω_P eines Tragflügels umfaßt. Der Rand von Ω besteht aus drei Teilen:

$$\partial\Omega = \Gamma_\infty \cup \Gamma_P \cup \Sigma,$$

wobei:

- die Kurve Γ_∞ ist ein C^∞ künstlicher äußerer Rand von Ω, welches eingeführt wird um ein beschränktes Rechengebiet zu erhalten;
- die Kurve Γ_P ist der gemeinsame Rand zwischen Ω und Ω_P; er besitzt ein Eckpunkt (Trailing Edge; Hinterpunkt) und ist eine C^∞ - Kurve sonst;
- Σ bezeichnet einen Schlitz in Ω, der den Hinterpunkt mit Γ_∞ verbindet.

Mit Σ^c bezeichnen wir die Fortsetzung des Schlitzes Σ in Ω^c. Ohne Beschränkung der Allgemeinheit, nehmen wir an, daß die