$\|x\| \leq r$ alors $df(x)(S(0,1)) \supseteq S(0,1)$. Dans ce cas, f est une application ouverte de X sur Y. ■

L'auteur desire mentionner les discussions utiles qu'il a eu avec C.Ursescu.

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Received:28.IV.1995

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ANALELE ŞTIINŢIFICE ALE UNIVERSITĂŢII "AL. I. CUZA" IAŞI
Tomul XLI, s.I a, Matematică, 1995, f.2

BARRELLEDNESS OF CLASS $\aleph_0$ OF THE SPACE $l_0^\infty(X, \mathcal{A})$ AND MEASURE THEORY

BY

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1. Introduction and background. Let us recall that a Hausdorff locally convex space E is Baire-like [5] if given an increasing sequence of closed absolutely convex subsets of E covering E, there is one of them that is a neighbourhood of the origin; and E is suprabarrelled [6] if given an increasing sequence of subspaces of E covering E, there is one of them which is dense and barrelled. Following ideas of [4], we will call barrelled of class 1 to the suprabarrelled spaces and, for each $n \geq 2$, we will say that the space E is barrelled of class n if given an increasing sequence of subspaces of E covering E, there is one of them which is barrelled of class $n-1$, this also being true for $n = 1$ if we consider Baire- like spaces as barrelled spaces of class 0. If E is barrelled of class n for every $n \in \boldsymbol{N}$, then E is said to be barrelled of class $\aleph_0$ [2].

On the other hand, following V a l d i v i a [7], let us denote by $l_0^\infty(X, \mathcal{A})$ the real or complex space generated by the characteristic functions $e(A)$ defined on the elements of a σ-algebra $\mathcal{A}$ on a set X, and endowed with the norm $\|z\| = sup\{z(j) : j \in X\}$. In that paper, using duality theory arguments and *sliding hump* methods, Valdivia proved that $l_0^\infty(X, \mathcal{A})$ is suprabarrelled and used that fact to obtain some measure theory results. R o d r i g u e z S a l i n a s [4] stated that $l_0^\infty(X, \mathcal{A})$ was even barrelled of class 2. Indeed, this space is barrelled of class $\aleph_0$, [1], and hence you may reinforce the measure results obtained by Valdivia. In [3] we gave a proof of the suprabarrelledness of $l_0^\infty(X, \mathcal{A})$ using measure theory techniques; the aim of this paper is to show that these techniques are also valid in order to prove that $l_0^\infty(X, \mathcal{A})$ is barrelled of class $\aleph_0$. Throughout all the paper $\mathcal{A}$ will stand for a α-algebra on a set X.

We finish recalling that given $s \in \boldsymbol{N}$, a subset $P \subset \boldsymbol{N}^s$ is said to be of class s [1], if there are infinitely many $n_1 \in \boldsymbol{N}$ such that there are some

$(n_1,\dots,n_s) \in P$, and given $(n_1,\dots,n_s) \in P$, and any $i \in \{1,\dots,s-1\}$, there are infinitely many $m_{i+1} \in \mathbf{N}$ such that there are some $(n_1,\dots,n_i)$, $(m_{i+1},\dots,m_s) \in P$.

2. The space $l_0^\infty(X,\mathcal{A})$ is barrelled of class $\aleph_0$. We will consider the set $\mathbf{N}^s$ provided with the total ordering settled by $(n_1,\dots,n_s) < (m_1,\dots,m_s)$ if $n_1+\dots+n_s < m_1+\dots+m_s$, or if being $n_1+\dots+n_s = m_1+\dots+m_s$, the former element is the one to have a component r, $1 \le r \le s$ so that $n_r < m_r$, being $n_1+\dots+n_i = m_1+\dots+m_i$, $1 \le i < r$. This enables us to write the elements of $\mathbf{N}^s$ as a sequence $\{(n_1(i),\dots,n_s(i)) : i \in \mathbf{N}\}$. Given $s \in \mathbf{N}$, a subset $P \subset \mathbf{N}^s$ is said to be of class s [1], if there are infinitely many $n_1 \in \mathbf{N}$ such that there are some $(n_1,\dots,n_s) \in P$, and given $(n_1,\dots,n_s) \in P$, and any $i \in \{1,\dots,s-1\}$, there are inifinitely many $m_{i+1} \in \mathbf{N}$ such that there are some $(n_1,\dots,n_i,m_{i+1},\dots,m_s) \in P$. The following definition will help us to smooth the forthcoming results.

Definition 1. *A subset $P \subset \mathbf{N}^s$ is said to be of type (k), $k \in \mathbf{N}$, if there are some sets $S_i^{(s-r)}$ of class $s-r$, $1 \le r \le s-1$, $1 \le i \le k$, and a set S of class s, such that P is formed by the union of S, $\{(n_1(i),\dots,n_s(i)) : 1 \le i \le k\}$, and $\{\{(n_1(i),\dots,n_r(i))\} \times S_i^{(s-r)} : 1 \le i \le k\}$.*

Denoting by $ba(\mathcal{A})$ the space formed by the bounded finitely additive scalar valued measures defined on $\mathcal{A}$, the following definition was already used to show that $l_0^\infty(X,\mathcal{A})$ was suprabarrelled [3].

Definition 2. *Given $A \in \mathcal{A}$ and $M \in ba(\mathcal{A})$, we shall say that M is quasi-bounded with regard to A if there are some $B_1, B_2, \dots, B_n \in \mathcal{A}$ such that for each $A_o \subset A$, $A_o \in \mathcal{A}$,*

$$\sup\{|u(A_o)| : u \in M, \sum_{j=1}^{n} |u(B_j)| \le 1\} < \infty.$$

Following the reasoning of [3, Propositions 2.3 and 4], and similarly to [1, Lemmas 2,3 and 4] is easy to obtain the following results.

Proposition 1. *Let $A \in \mathcal{A}$ and let $P \subset \mathbf{N}^s$ be a set of class s of type (k), $k \in \mathbf{N}$. If $\{M_m : m \in P\}$ is a subset of $ba(\mathcal{A})$ such that no M_m is quasi-bounded with regard to A, then for each real number $\alpha > 0$ and $B_1,\dots,B_r \in \mathcal{A}$, there exists a family of pairwise disjoint subsets of A belonging to $\mathcal{A}$, $A_1,\dots,A_k$ and $u_1,\dots,u_k \in M_{n_1(i),\dots,n_s(i)}$, $1 \le i \le k$, such that*

$$|u_i(A_i)| > \alpha, \sum_{j=1}^{r} |u_i(B_j)| \ge 1, \ 1 \le i \le k,$$

and no M_m is quasi-bounded with regard to $A \sim \cup_{i=1}^k A_i$ for m taking values in some set $P^* \subset \mathbf{N}^s$ of class s and type (k).

Proposition 2. *Let $\{M_{n_1(i),\dots,n_s(i)} : \in \mathbf{N}\}$ be a sequence in $ba(\mathcal{A})$ such that no of its elements is quasi-bounded with regard to X. Then there is a subsequence $\{M_{n_1'(i),\dots,n_s'(i)} : i \in \mathbf{N}\}$, whose subindex set $P = \{(n_1'(i),\dots,n_s'(i)) : i \in \mathbf{N}\}$ forms a set of class s in $\mathbf{N}^s$, and a family of pairwise disjoint elements of $\mathcal{A}$, $\{A_{ij} : i,j \in \mathbf{N}\}$, and $\{u_{ij} : i,j \in \mathbf{N}\} \subset M_m$, $m \in P$, such that*

$$|u_{ij}(A_{ij})| > i+j, \sum_{g+h<i+j} |u_{ij}(A_{gh})|, \ i,j \in \mathbf{N}.$$

Proposition 3. *Let $\{A_{n_1} : n_1 \in \mathbf{N}\}$ be an increasing sequence of subsets of $\mathcal{A}$ covering $\mathcal{A}$, and for each $2 \le r \le s$, $(n_1,\dots,n_{r-1}) \in \mathbf{N}^{r-1}$, let $\{\mathcal{A}_{n_1,\dots,n_r} : n_r \in \mathbf{N}\}$ be an increasing sequence of subsets of $\mathcal{A}_{n_1,\dots,n_{r-1}}$ covering $\mathcal{A}_{n_1,\dots,n_{r-1}}$. Then there exists some $k_0 \in \mathbf{N}$ such that if $M \subset ba(\mathcal{A})$ is quasi-bounded with regard to $\mathcal{A}_{n_1(k_0),\dots,n_s(k_0)}$, then M is quasi-bounded with regard to X.*

With these results we are able to prove the desired barrelledness property of $l_0^\infty(X,\mathcal{A})$.

Theorem. *The space $l_0^\infty(X,\mathcal{A})$ is barrelled of class s for each $s \in \mathbf{N}$.*

Proof. With the techniques we are using here we already showed in [3] that $l_0^\infty(X,\mathcal{A})$ is barrelled of class 1. Suppose that $l_0^\infty(X,\mathcal{A})$ is barrelled of class $s-1$, $s \ge 2$, but not barrelled of class s. Then there is an increasing sequence of subspaces $\{E_{n_1} : n_1 \in \mathbf{N}\}$ of $l_0^\infty(X,\mathcal{A})$ covering $l_0^\infty(X,\mathcal{A})$, formed by dense subspaces of $l_0^\infty(X,\mathcal{A})$ which are barrelled of class $s-2$ but not barrelled of class $s-1$. And for each $n_1 \in \mathbf{N}$, there is some increasing sequence $\{E_{n_1,n_2} : n_2 \in \mathbf{N}\}$ of subspaces of E_{n_1} covering it which are dense in $l_0^\infty(X,\mathcal{A})$ and barrelled of class $s-3$ but not barrelled of class $s-2$. Going on, we may get a family $\{E_{n_1,\dots,n_{s-1}} : (n_1,\dots,n_{s-1}) \in \mathbf{N}^{s-1}\}$ of dense subspaces of $l_0^\infty(X,\mathcal{A})$ which are barrelled of class 1 but not barrelled of class 0. Hence, for each $(n_1,\dots,n_{s-1}) \in \mathbf{N}^{s-1}$, there must be an increasing sequence $\{E_{n_1,\dots,n_{s-1},n_s} : n_s \in \mathbf{N}\}$ of dense subspaces of $l_0^\infty(X,\mathcal{A})$ covering $E_{n_1,\dots,n_{s-1}}$ none of which is barrelled. By [3, Proposition 5], for each $m \in \mathbf{N}^s$, there is some $M_m \subset ba(\mathcal{A})$, such that $sup_{u \in M_m}\{|<u,f>|\} < +\infty$ for each $f \in E_m$ but M_m is not quasi-bounded with regard to X. If for each $m \in \mathbf{N}^s$ we settle $\mathcal{A} := \{B \in \mathcal{A} : e(B) \in E_m\}$, then M_m is quasi-bounded with regard to $\mathcal{A}$. This is not possible by Proposition 3. ∎

Corollary. *The space $l_0^\infty(X,\mathcal{A})$ is barrelled of class $\aleph_0$*

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Received:11.V.1995

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ANALELE ŞTIINŢIFICE ALE UNIVERSITĂŢII "AL. I. CUZA" IAŞI
Tomul XLI, s.I.a, Matematică, 1995, f.2

A H_∞-CONTROL PROBLEM WITH UNBOUNDED DISTURBANCE AND IMPULSIVE PARAMETER; THE PARABOLIC CASE

BY

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1. Introduction. Consider the following input-output system with impulsive parameter

$$(1.1)\qquad \begin{cases} x'(t) = Ax(t) + \sum_{i=0}^\infty B_2 u_i \otimes \delta(iT) + B_1 w(t), & t \in \boldsymbol{R}^+ \\ x(0) = x_0 \end{cases}$$

$$(1.2)\qquad z(t) = Cx(t) + D\sum_{i=0}^{\infty} u_i \otimes \delta(iT), \quad t \in \boldsymbol{R}^+,$$

where A is the infinitesimal generator of an analytic semigroup e^{At} on X, $B_2 \in L(U,X)$, $C \in L(X,Z)$ and $D \in L(U,Z)$. Here X,U,W,Z are real Hilbert spaces and we denote by $(\cdot,\cdot),(\cdot,\cdot)_U,(\cdot,\cdot)_W,(\cdot,\cdot)_Z$ the scalar products and by $|\cdot|,|\cdot|_U,|\cdot|_W,|\cdot|_Z$ the corresponding norms.

In system (1.1)-(1.2), $T>0$ is a given positive number, $\delta(iT), i \in \boldsymbol{N}$ is the Dirac measure supported in iT; $x(t) \in X$ is the state, $u_i \in U, i \in \boldsymbol{N}$ are the controllers, $w(t) \in W$ is the disturbance and $z(t) \in Z$ is the controlled output.

The following hypotheses will be maintained throughout the paper :

(i) There exists $\lambda_0 > 0, \lambda_0 \in \rho(A)$ such that $A - \lambda_0$ is the generator of an exponentially stable semigroup;

(ii) $B_1 = (\lambda_0 - A)H_1$, with $H_1 \in L(W; D((\lambda_0 - A)^\alpha))$, where $\alpha \in (\frac{1}{2}, 1)$,