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A H_∞-CONTROL PROBLEM WITH UNBOUNDED DISTURBANCE AND IMPULSIVE PARAMETER; THE PARABOLIC CASE

BY

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1. Introduction. Consider the following input-output system with impulsive parameter

$$\begin{cases} x'(t) = Ax(t) + \sum_{i=0}^{\infty} B_2 u_i \otimes \delta(iT) + B_1 w(t), & t \in \mathbf{R}^+ \\ x(0) = x_0 \end{cases} \tag{1.1}$$

$$z(t) = Cx(t) + D\sum_{i=0}^{\infty} u_i \otimes \delta(iT), \quad t \in \mathbf{R}^+, \tag{1.2}$$

where A is the infinitesimal generator of an analytic semigroup e^{At} on X, $B_2 \in L(U, X)$, $C \in L(X, Z)$ and $D \in L(U, Z)$. Here X, U, W, Z are real Hilbert spaces and we denote by $(\cdot,\cdot), (\cdot,\cdot)_U, (\cdot,\cdot)_W, (\cdot,\cdot)_Z$ the scalar products and by $|\cdot|, |\cdot|_U, |\cdot|_W, |\cdot|_Z$ the corresponding norms.

In system (1.1)-(1.2), $T > 0$ is a given positive number, $\delta(iT), i \in \mathbf{N}$ is the Dirac measure supported in iT; $x(t) \in X$ is the state, $u_i \in U, i \in \mathbf{N}$ are the controllers, $w(t) \in W$ is the disturbance and $z(t) \in Z$ is the controlled output.

The following hypotheses will be maintained throughout the paper :

(i) There exists $\lambda_0 > 0, \lambda_0 \in \rho(A)$ such that $A - \lambda_0$ is the generator of an exponentially stable semigroup;

(ii) $B_1 = (\lambda_0 - A)H_1$, with $H_1 \in L(W; D((\lambda_0 - A)^\alpha))$, where $\alpha \in (\frac{1}{2}, 1)$,

(iii) The pair (C, A) is exponentially detectable, i.e., there exists $K \in L(Z, X)$ such that $A + KC$ generates an exponentially stable semigroup $e^{(A+KC)t}$;

(iv) $D^*[C \quad D] = [0 \quad I]$ (where D^* is the adjoint of D).

Example. Let Ω be a bounded open subset of $\boldsymbol{R}^N$ with a smooth boundary $\partial\Omega$. Consider the following system

$$
(1.3)\qquad \begin{cases} \dfrac{\partial x}{\partial t}(t,\xi) = \Delta_\xi x(t,\xi) + \sum\limits_{i=0}^{\infty} u_i(\xi) \otimes \delta(iT), \ (t,\xi) \in \boldsymbol{R}^+ \times \Omega \\ \dfrac{\partial x}{\partial \nu}(t,\xi) = w(t,\xi), \ (t,\xi) \in \boldsymbol{R}^+ \times \partial\Omega \\ x(0,\xi) = x_0(\xi), \ \xi \in \Omega, \end{cases}
$$

where $x_0 \in X = U = Z = L^2(\Omega)$.

We denote by A the Neumann realization of the Laplace operator, i.e.,

$$
\begin{cases} D(A) = \{x \in H^2(\Omega);\ \dfrac{\partial x}{\partial \nu} = 0 \text{ on } \partial\Omega\} \\ Ax = \Delta_\xi x, \ \forall x \in D(A), \end{cases}
$$

$W = L^2(\partial\Omega)$, $B_2 =$ the identity on $L^2(\Omega)$ and $H_1 : W \to X$ the Neumann mapping $v \mapsto H_1 v = h$, where

$$
\begin{cases} \Delta_\xi h - h = 0 \text{ in } \Omega \\ \dfrac{\partial h}{\partial \nu}(\xi) = v(\xi) \text{ in } \partial\Omega \end{cases}
$$

If we consider $B_1 = (1 - A)H_1$, C and D the identity on $L^2(\Omega)$, then system (1.3) can be written in the form (1.1) and all the hypotheses are fulfilled with $\lambda_0 = 1$ (for more details see [5], [8]).

System (1.1) is equivalent to

$$
(1.4)\qquad \begin{cases} x'(t) = Ax(t) + B_1 w(t), \ t \in [iT, (i+1)T), i \in \boldsymbol{N} \\ x(iT+) = x(iT-) + B_2 u_i, \ i \in \boldsymbol{N} \\ x(0-) = x_0 \end{cases}
$$

The mild solution of (1.1) (or equivalently (1.4)) is $x(t) = e^{At}x_0 + \int_0^t e^{A(t-s)} B_1 w(s) ds + \sum\limits_{i=0}^{\infty} \chi(t - iT) e^{A(t-iT)} B_2 u_i$, $t \in \boldsymbol{R}^+ \setminus T\boldsymbol{N}$, where χ denotes the Heaviside function (i.e., $\chi(t) = 1$ for $t \geq 0$ and 0 for $t < 0$).

The mild solution $x \in C(\boldsymbol{R}^+ \setminus T\boldsymbol{N}; X)$, $(T\boldsymbol{N} = \{0, T, 2T, 3T, \ldots\})$ and there exist $x(iT-), x(iT+)$ in X, for $i \in \boldsymbol{N}$ (see [5],[8]). Denote by $l_2(U)$ the space of sequences $(u_i)_{i \in \boldsymbol{N}}$, which satisfy $\sum_{i=0}^{\infty} |u_i|_U^2 < \infty$.

According to the theory of standard H_∞-problems (see e.g. [3],[6],[7]) we define the H_∞-control problem for system (1.1)-(1.2) as follows: Given $\gamma > 0$, find $F \in L(X, U)$ such that for every $w \in L^2(\boldsymbol{R}^+; W)$ and $x_0 \in X$:

$$
(1.5)\qquad \int_0^\infty |Cx(t)|_Z^2 dt + \sum_{i=0}^{\infty} |Fx(iT-)|_U^2 \leq \rho^2 \int_0^\infty |w(t)|_W^2 dt + \beta |x_0|^2,
$$

where x is the mild solution of

$$
(1.6)\qquad \begin{cases} x'(t) = Ax(t) + B_1 w(t), \ t \in [iT, (i+1)T), i \in \boldsymbol{N} \\ x(iT+) = x(iT-) + B_2 Fx)(iT-), \ i \in \boldsymbol{N} \\ x(0-) = x_0, \end{cases}
$$

where $0 < \rho < \gamma$ and $0 < \beta < \infty$.

Remark. For related results concerning the H_∞ problem, with impulsive controllers (but bounded disturbance) we refer to [4] (for the finite dimensional case) and to [1] (for the infinite dimensional case).

2. The main result.

Theorem 2.1. *Suppose $\gamma > 0$ and assumptions (i)-(iv) hold. If there exists $F \in L(X, U)$ such that (1.5) is satisfied, then there exist the operators $P(t) \in L(X), t \in [0, T]$ such that*

(j) $P(t) = P^*(t) \geq 0$, $\forall t \in [0, T]$

(jj) P is an integral solution of

$$
(2.1)\qquad P'(t) + A^* P(t) + P(t)A + C^*C + \gamma^{-2} P(t) B_1 B_1^* P(t) = 0, \ t \in [0, T]
$$

(jjj) $P(0)(x_0 - B_2 B_2^* P(T) x_0) = P(T) x_0$, $\forall x_0 \in X$

(jv) $|P(T)|_{L(X)} \leq \beta$

Conversely, if there exist $P(t)$ satisfying (j)-(jv), then $F = -B_2^* P(T)$ satisfies (1.5) with $\rho = \gamma$.

Remark. A discussion concerning the different types of solutions for (2.1) can be found in [5].

3. Proof of the necessity. Firstly we shall assume that the H_∞-control problem has a solution $F \in L(X, U)$

Denote $\mathcal{U} = l_2(U), \mathcal{U}_t = \{(u_i) \in \mathcal{U};\ i \in \boldsymbol{N},\ i \geq \max\{1, \frac{t}{T}\}\}$, $\mathcal{W} = L^2(\boldsymbol{R}^+; W), \mathcal{W}_t = L^2((t, +\infty); W), \boldsymbol{R}_t^+ = (t, \infty)$.

Then, on the space $\boldsymbol{R}_t^+ \times \mathcal{U}_t \times \mathcal{W}_t$ we define the function

(3.1)
$$K(t, u, w) = 2^{-1} \int_t^\infty (|Cx(s)|_Z^2 - \gamma^2 |w(s)|_W^2) ds + 2^{-1} \sum_{i=0}^{\infty} \chi(t - iT)|u_i|_U^2,$$

where x is the mild solution of the problem

$$\text{(3.2)} \quad \begin{cases} x'(s) = Ax(s) + B_1 w(s) + \sum\limits_{i \geq \max\{1, t/T\}} B_2 u_i \otimes \delta(iT), & s \in [t, +\infty) \\ x(t) = x_0 \end{cases}$$

Consider the max-min problem

(MM_t) $\sup_{w \in \mathcal{W}_t} \inf_{u \in \mathcal{U}_t} K(t, u, w) = \phi(t)$

Clearly, $\phi(t) \geq 0$. In a standard way can be proved that problem (MM_t) has a unique solution $(u_t^*, w_t^*) \in \mathcal{U}_t \times \mathcal{W}_t$.

Consider the minimization problem

(M_t) $\min_{u \in \mathcal{U}_t} K(t, u, w) = \phi_t(x_0)$,

for $w \in \mathcal{W}_t$ fixed.

In a standard manner follows the existence of a unique optimal control for (M_t).

Lemma 3.1. *The pair (u_t, x_t), which satisfies (3.2) and $u_t \in \mathcal{U}_t$, $Cx_t \in L^2(\boldsymbol{R}_t^+; Z)$ is optimal for (M_t) if and only if there exists $p_t \in C(\boldsymbol{R}_T^+; X) \cap L^2(\boldsymbol{R}_T^+; X)$ such that*

$$\text{(3.3)} \quad \begin{cases} p_t'(s) = -A^* p_t(s) + C^* C x_t(s), & s \in (t, \infty) \\ p_t(\infty) = 0 \end{cases}$$

(i.e., $p_t(s) = e^{A^*(\widetilde{T} - s)} p_t(\widetilde{T}) - \int_s^{\widetilde{T}} e^{A^*(\widetilde{T} - \tau)} C^* C x_t(\tau) d\tau, \forall \widetilde{T} \in \boldsymbol{R}, t < s < \widetilde{T}$) and

$$\text{(3.4)} \quad u_{t,i} = B_2^* p_t(iT), \ i \geq \max\{1, \frac{t}{T}\}, \ i \in \boldsymbol{N}$$

(where $u_t = (u_{t,i})$).

Proof. We shall prove only for $t \in [0, T]$ (For $t > T$ the argument is similar). Consider the minimization problem

$$(M_{n,t}) \quad Minimize\{\frac{1}{2} \int^n (|Cx(s)|_Z^2 - \gamma^2 |w(s)|_W^2) ds + \frac{1}{2} \sum^{[n/T]} \chi(t - iT)|u_i|_U^2;$$

$$x' = Ax + B_1 w + \sum_{i=1}^{[n/T]} B_2 u_i \otimes \delta(iT), x(t) = x_0\} = \phi_{n,t}(x_0)$$

In a standard way it is possible to prove the existence of a unique optimal pair $(u_{n,t}, x_{n,t})$ corresponding to $(M_{n,t})$.

From the obvious inequality $\phi_{n,t}(x_0) \leq \phi_t(x_0)$ it follows the boundedness of the sequences $\{u_{n,t}\}$ in $\mathcal{U}_t$. We infer that on a subsequence (also denoted by $\{u_{n,t}\}$) we have

$$u_{n,t} \longrightarrow \widetilde{u}_t, \ for\ n \longrightarrow \infty, \ weakly\ in\ \mathcal{U}_t \text{ and}$$

$$Cx_{n,t} \longrightarrow Cx_t^{\widetilde{u}_t}, \ for\ n \longrightarrow \infty. \ weakly\ in\ L^2(t, \widetilde{T}; Z), \forall \widetilde{T} \in (t, \infty)$$

(we have denoted by $x_t^{\widetilde{u}_t}$ the mild solution of (3.2) corresponding to $\widetilde{u}_t$).

These convergences imply $\liminf_{n \to \infty} \phi_{n,t}(x_0) \geq \phi_t(x_0)$, which allow us to conclude that $u_{n,t} \longrightarrow \widetilde{u}_t$, *for* $n \longrightarrow \infty$, *strongly* $\in \mathcal{U}_t$ $x_{n,t} \longrightarrow x_t^{\widetilde{u}_t}$, *for* $n \longrightarrow \infty$, *strongly in* $L^2(t, \widetilde{T}; X)$ $Cx_{n,t} \longrightarrow Cx_t^{\widetilde{u}_t}$, *for* $n \longrightarrow \infty$, *strongly in* $L^2(t, \widetilde{T}; Z)$.

We conclude that $\widetilde{u}_t = u_t, x_t^{\widetilde{u}_t} = x_t$, i.e., (u_t, x_t) is an optimal pair for (M_t). By standard device follows the existence of $p_{n,t} \in C([t, n]; X)$ such that

$$\text{(3.5)} \quad \begin{cases} p_{n,t}' = -A^* p_n + C^* C x_{n,t} & in\, [t, n] \\ p_{n,t}(n) = 0 \end{cases}$$

$$\text{(3.6)} \quad u_{n,t,i} = B_2^* p_{n,t}(iT)$$

$(u_{n,t} = (u_{n,t,i}))$ where (3.5) should be viewed in the "mild" sense, i.e.,

(3.7)
$$p_{n,t}(s) = e^{A^*(\widetilde{T} - s)} p_{n,t}(\widetilde{T}) - \int_s^{\widetilde{T}} e^{A^*(\tau - s)} C^* C x_{n,t}(\tau) d\tau, \ t \leq s \leq \widetilde{T} \leq n$$

The function $x_0 \to \phi_{n,t}(x_0)$ is convex and differentiable. Moreover, we have

$$p_{n,t}(s) = -\nabla \phi_{n,s}(x_{n,t}(s)), \quad \forall s \in [t, n].$$

Since $x_{n,t}(s) \longrightarrow x_t(s)$, uniformly in $\boldsymbol{R}_t^+$ we infer that

$$\limsup_{n \to \infty} |p_{n,t}(s)| \leq c(\mu - |x_t(s)|)^{-1} (\mu^2 + \sum_{i \geq \max\{1, s/T\}} |u_i|_U^2), \ \mu \in \boldsymbol{R}_+^*$$

which implies that $\{p_{n,t}\}$ is bounded in $L^\infty(\boldsymbol{R}_t^+;X)$ and so on a subsequence, also denoted by $p_{n,t}$:

$$p_{n,t} \longrightarrow p_t \textit{ weakly star in } L^\infty(\boldsymbol{R}_t^+;X)$$

Relation (3.7) implies as in [1] the conclusion of the lemma (the proof use the detectability assumption (iii) and the convergences of $\{p_{n,t}\}$).

Consider $\Gamma_t\colon \mathcal{W}_t \longrightarrow \mathcal{U}_t$ defined by $\Gamma_t w = u_t$. In the same manner as in [1], [3] follows that Γ_t is an affine function and

$$\Gamma_t w = \Gamma_{0t} w + f_{0t}, \forall w \in \mathcal{W}_t \tag{3.8}$$

where $f_{0t} = \arg\inf K(t,u,0)$ and $\Gamma_{0t} \in L(\mathcal{W}_t,\mathcal{U}_t)$.

Let $\eta_t\colon \mathcal{W}_t \longrightarrow \boldsymbol{R}$ be defined by $\eta_t(w) = -K(t,\Gamma_t(w),w)$.

By (3.8) we may reprezent η_t as $\eta_t(w) = ||Gw||^2_{\mathcal{U}_t} + ((w,f)) + \theta$, where $G \in L(\mathcal{W}_t,\mathcal{U}_t)$, $((,.,))$ is the scalar product of $\mathcal{W}_t$, $f \in \mathcal{W}_t$ and $\theta \in \boldsymbol{R}$.

Relation (1.5) implies

$$\eta_t(w) \geq a||w||^2_{\mathcal{W}_t} + b, \forall w \in \mathcal{W}_t,$$

where $a > 0$ and $b \in \boldsymbol{R}$ (see for the argument [1], [3]). In a standard way we can prove the existence of a unique $w_t^* \in \mathcal{W}_t$ such that

$$w_t^* = arg\ inf\{\eta_t(w); w \in \mathcal{W}_t\}$$

Lemma 3.2.

$$w_t^*(s) = -\gamma^{-2}B_1^* p_t(s), \text{ a.e. } s > t$$

Proof. We start with the inequality

$$\int_t^\infty (|Cx_t^*|^2_Z - \gamma^2 |w_t^*|_W)ds + |\Gamma_t w_t^*|^2_{\mathcal{U}_t} \geq \tag{3.9}$$

$$\geq \int_t^\infty (|Cx_t^{w_t^*+\varepsilon w}|^2_Z - \gamma^2|w_t^* + \varepsilon w|^2_{\mathcal{W}_t} ds + |\Gamma_t(w_t^* + \varepsilon w)|^2_{\mathcal{U}_t}$$

$\forall \varepsilon > 0$ (where x_t^* is the mild solution of (3.2) with $x_t^*(t) = x_0$, with the inputs $u := \Gamma_t w_t^*$, and $x_t^{w_t^*+\varepsilon w}$ is the mild solution of (3.2) with $x_t^{w_t^*+\varepsilon w}(t) = x_0$, corresponding to the inputs $u := \Gamma_t(w_t^* + \varepsilon w)$ and $w := w_t^* + \varepsilon w$).

From (3.9) we obtain

(3.10)
$$\int_t^\infty ((Cx_t^*(s), Cy(s))_Z - \gamma^2(w_t^*(s), w(s))_W)ds + (\Gamma_{0t}w_t^* + f_{0t}, \Gamma_{0t}w)_{\mathcal{U}_t} \leq 0,$$

$\forall w = (w_i) \in \mathcal{W}_t(w_t^* = (w_i^*))$, where y is the mild solution of

$$\begin{cases} y'(s) = Ay(s) + B_1 w(s) + \sum\limits_{\substack{i=1 \\ i \geq t/T}}^{\infty} B_2(\Gamma_t w)_i \otimes \delta(iT) \\ y(t) = 0 \end{cases}$$

and satisfies $Cy \in L^2(\boldsymbol{R}_t^+;Z)$.

By (3.3) we deduce that for all $\widetilde{T} > t$ we have

$$\int_t^{\widetilde{T}} (Cx_t^*(s), Cy(s))_Z ds = (p_t(\widetilde{T}), y(\widetilde{T})) - \int_t^{\widetilde{T}} (B_1 w(s), p_t(s))ds - \tag{3.11}$$

$$-(B_2(\Gamma_{0t}w)(s), p_t(s))_{\mathcal{U}_t}$$

Since (C,A) is exponentially detectable and $Cy \in L^2(\boldsymbol{R}_t^+;Z)$ we infer that $\lim_{s\to\infty} y(s) = 0$.

Letting $\widetilde{T} \longrightarrow \infty$ in (3.11) it follows $\int_t^\infty (Cx_t^*(s), Cy(s))_Z ds =$
$= -\int_t^\infty (w(s), B_1^* p_t(s))_W ds - (\Gamma_{0t}w_t^* + f_{0t}, \Gamma_{0t}w)_{\mathcal{U}_t}$ and so by (3.10) we get the lemma.

Thus, up to now we have proved that the problem (MM_t) has an unique optimal solution (u_t^*, w_t^*, x_t^*) characterized by the Euler-Lagrage system:

$$\begin{cases} x_t^{*\prime} = Ax_t^* + B_1 w_t^* + \sum\limits_{i \geq \max\{1,t/T\}} B_2 u_{t,i}^* \otimes \delta(iT), \ s \in [t,+\infty) \\ x_t^*(t) = x_0 \end{cases} \tag{3.12}$$

$$\begin{cases} p_t'(s) = -A^* p_t(s) + C^* C x_t^*(s), \ s \in (t,\infty) \\ p_t(\infty) = 0 \end{cases} \tag{3.13}$$

$$\begin{cases} u_{t,i}^* = B_2^* p_t(iT), \ i \geq \max\{1,t/T\} \\ w_t^*(s) = -\gamma^{-2} B_1^* p_t(s), \ a.e.\ s > t \end{cases} \tag{3.14}$$

We set $P(t)x_0 = -p_t(t)$, $t \in [0,T]$. It follows that $P(s)x_t^*(s) = -p_t(s), \forall s \geq t \Rightarrow$

$$(3.15)\qquad P(t)x_0 = -e^{A^*(T-t)}p_t(T) + \int_t^T e^{A^*(\tau-t)}C^*Cx_t^*(\tau)d\tau =$$

$$= \int_t^\infty e^{A^*(\tau-t)}C^*Cx_t^*(\tau)d\tau$$

Taking into account (3.15), for $x_0, y_0 \in X$ we have:

$$(3.16)\qquad (P(t)x_0, y_0) = \int_t^\infty [(Cx_t^*(s), Cy_t^*(s))_Z - \gamma^2(w_t^*(s), \widetilde{w}_t^*(s))_W]ds +$$

$$+\sum_{i\geq 1}(u_{t,i}^*, \widetilde{u}_i^*)_U,$$

where (u_t^*, w_t^*, x_t^*) satisfies (3.12)–(3.14) and $(\widetilde{u}_t^*, \widetilde{w}_t^*, y_t^*)$ satisfies (3.12)–(3.14), with $y_t^*(t) := y_0$.

From (3.16) we deduce

$$P(t) = P^*(t) \geq 0, \ \forall t \in [0,T]$$

As in [1] can be proved that $P(t) \in L(X)$ and that (jjj) and (jv) hold.

Using the optimality conditions we get that (w_t^*, x_t^*) is the unique optimal pair for

$(\widetilde{M}_t)$ *Maximize* $\int_t^T(|Cx(s)|_Z^2 - \gamma^2|w(s)|_U^2)ds + (P(T)x(T), x(T))$,

subject to

$$\begin{cases} x'(s) = Ax(s) + B_1w(s), \ s \in [t,T] \\ x(t) = x_0 \end{cases}$$

In the same manner as in [1] follows that P is satisfying (jj) (the proof follows the same steps as that in [5], corresponding to the LQ problem).

4. Proof of the sufficiency. For $w \in L^2(\mathbf{R}^+;W) \cap C^1(\mathbf{R}^+;W)$ we denote by x the "mild" solution of

$$\begin{cases} x'(t) = Ax(t) + B_1w(t), \ t \in [iT, (i+1)T), \ i \in \mathbf{N} \\ x(iT+) = x(iT-) - B_2B_2^*P(T)x(iT-), i \in \mathbf{N} \\ x(0-) = x_0 \end{cases}$$

and by x_n the "mild" solution of the equation

$$(4.1)\qquad \begin{cases} x_n'(t) = Ax_n(t) + B_1w(t), t \in [0,T), \\ x_n(0+) = I_n(x_0 - B_2B_2^*P(T)x_0), \end{cases}$$

where $I_n = n(n-A)^{-1}$.

Let us observe that $I_n(x_0 - B_2B_2^*P(T)x_0) \in D(A)$. So, the solution of (4.1) is a strong solution (see [2]). Differentiating $(P(t)I_nx_n(t), I_nx_n(t))$ on $(0,T)$, taking into account (jj) and passing to the limit we obtain after a little calculation

$$\int_0^T (|Cx(t)|_Z^2 - \gamma^2|w(t)|_W^2)dt + |B_2^*P(T)x_0|_U^2 \leq$$

$$\leq (P(T)x_0, x_0) - (P(T)x(T-), x(T-))$$

In the same manner we get

$$\int_{iT}^{(i+1)T} (|Cx(t)|_Z^2 - \gamma^2|w(t)|_W^2)dt + |B_2^*P(T)x(iT-)|_U^2 \leq$$

$$\leq (P(T)x(iT-), x(iT-)) - (P(T)x((i+1)T-), x((i+1)T-))$$

Since $L^2(\mathbf{R}^+;W) \cap C^1(\mathbf{R}^+;W)$ is dense in $L^2(\mathbf{R}^+;W)$ we obtain that $F = -B_2^*P(T)$ satisfies (1.5) with $\rho = \gamma$.

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CR–STRUCTURES ON THE UNIT TANGENT BUNDLE OF S^2

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Introduction. In [2], P.Matzeu and the author have obtained the expression of a Bochner type curvature tensor field for a pseudo-convex CR structure in terms of a canonical torsion free adapted connection. It appeared that the obtained expresion is much more simple than that obtained in [9]. In [3], P.Matzeu and the author studied the vanishing of this tensor in the case of the real hypersurfaces in complex space forms. On the other hand, in [12] (see also [5]) it is shown that the Bochner type tensor field the CR structure induced on the unit tangent bundle of an n-dimensional Riemannian space of constant sectional curvature, with $n > 2$, from the usual almost complex structure on the tangent bundle is zero if and only if the curvature of the base manifold is -1. In the case of the sphere S^2 the dimension is 2 and the constant sectional curvature is +1. We get that the Bochner type curvature tensor is again trivial in this case. Further, we study the properties of this Bochner type tensor field in the case of the CR–structures induced on the unit tangent bundle T_1S^2 of the 2–dimensional sphere S^2 in $\boldsymbol{R}^3$ from several lifts of the usual (integrable) almost complex structure J defined by usual vector (cross) product in $\boldsymbol{R}^3$. It is obtained that all the lifts of J to TS^2 induce, essentially, the same CR–structure on T_1S^2 and the Bochner tensor field of this CR–structure is trivial.

1. Pseudo–convex CR–structures and associated almost contact structures. A CR–structure on the differentiable manifold M is defined by a complex vector subbundle $H(M)$ in the complexification T^cM of the tangent bundle TM of M, satisfying the following conditions:

(i) $A(M) \cap H(M) = 0$ where $A(M) = \overline{H(M)}$

(ii) If Z, W are two sections in $H(M)$ then their bracket $[Z, W]$ is also a section in $H(M)$ (the complex involutivity condition for the CR–structure).