According to Theorem 2 and the method used in Application 1 (Pascal normal sequence), it follows that the sequence

$$s_1, s_2, s_3, \ldots s_n \ldots, \tag{15}$$

is V–normal with respect to the vector (13). If $g = 2$, from the sequence (15) it is obtained the sequence

0001000010...

which is a Bernoulli normal sequence with respect to the vector $V = (p_0, p_1)$, where

$$p_0 = \frac{1}{1\cdot 2} + \frac{1}{3\cdot 4} + \cdots = \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots = \ln 2,\ p_1 = 1 - \ln 2.$$

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ON CONFORMALLY FLAT PSEUDO–SYMMETRIC MANIFOLDS

BY

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Introduction. In a recent paper [2] M.C. Chaki introduced and studied a type of non–flat Riemannian manifold (M^n, g) $(n \geq 2)$ whose curvature tensor R satisfies the condition

$$\begin{aligned}(\nabla_X R)(Y,Z)W &= 2A(X)R(Y,Z)W + A(Y)R(X,Z)W + \\ &+ A(Z)R(Y,X)W + A(W)R(Y,Z)X + g(R(Y,Z)W,X)P\end{aligned} \tag{1}$$

where A is a non–Zero 1–form

$$g(X,P) = A(X) \tag{2}$$

for any vector field X and ∇ denotes the operator of covariant differentiation with respect to the metric g.

Such a manifold was called by him a pseudo–symmetric manifold, the 1–form was called its associated 1–form and an n–dimensional manifold of this kind was denoted by $(PS)_n$. The present paper deals with conformally flat $(PS)_n$ $(n \geq 3)$. For such a manifold, the associated 1–form A may or may not be closed. The nature of the vector field P given by (2) is determined in both cases. It has been shown that in such a manifold the vector field P given by (2) is a torse–forming vector field when the associated 1–form is not closed. The last section shows that in such a manifold of non–Zero constant scalar curvature the Vector field P given by (2) is a proper concircular or a proper torse forming vector field according as the associated 1–form is closed or not closed. Using Adati's [1] result, some more information about the manifold has been given in the form of a theorem, when the associated 1–form is closed.

1. Preliminaries. We now consider some formulas [2] which will be required in the study of conformally flat $(PS)_n$.

Let r denote the scalar curvature and L denote the symmetric endomorphism of the tangent space at each point corresponding to the Ricci tensor S i.e.

$$g(LX, Y) = S(X, Y) \text{ for every vector fields } X, Y. \tag{1.1}$$

Let B be the 1–form defined by

$$B(X) = A(LX). \tag{1.2}$$

From (1) we get

$$\begin{aligned}(\nabla_X S)(Y, Z) = 2A(X)S(Y, Z) + A(Y)S(X, Z)+ \\ + A(Z)S(X, Y) + A(R(X, Y)Z) + A(R(X, Z)Y).\end{aligned} \tag{1.3}$$

Contracting (1.3) we have

$$dr(X) = 2A(X)r + 4B(X). \tag{1.4}$$

We shall use these formulas later.

2. Conformally flat $(PS)_n$ $(n \geq 3)$ with non–closed associated 1–form. It is known [2] that in a conformally flat $(PS)_n$ $(n \geq 3)$

$$\begin{aligned}(n-1)A(X)S(Y, Z) - (n-1)A(Y)S(X, Z) - rA(X)g(Y, Z)+ \\ +rA(Y)g(X, Z) + B(X)g(Y, Z) - B(Y)g(X, Z) = 0.\end{aligned} \tag{2.1}$$

Putting $Z = P$ in (2.1) we get

$$A(X)B(Y) - A(Y)B(X) = 0. \tag{2.2}$$

Hence

$$B(X) = tA(X) \text{ where } t \text{ is a scalar.} \tag{2.3}$$

Using (2.3), it follows from (2.1) that

$$S(Y, Z) = \frac{r-t}{n-1}g(Y, Z) + \frac{nt-r}{(n-1)A(P)}A(Y)A(Z). \tag{2.4}$$

From (2.4) we get

$$LY = \frac{r-t}{n-1}Y + \frac{nt-r}{(n-1)A(P)}A(Y)P. \tag{2.5}$$

Since the manifold is conformally flat, we have

$$\begin{aligned}R(X, Y)Z = \frac{1}{n-2}\{S(Y, Z)X - S(X, Z)Y + g(Y, Z)LX- \\ -g(X, Z)LY\} + \frac{r}{(n-1)(n-2)}\{g(X, Z)Y - g(Y, Z)X\}.\end{aligned}$$

Hence

(2.6) $A(R(X, Y)P) = 0$ and

(2.7) $A(R(X, P)Y) = \dfrac{t}{n-1}A(X)A(Y) - \dfrac{t}{n-1}A(P)g(X, Y)$.

Hence from (1.3) we have

$$\begin{aligned}(\nabla_X S)(Y, P) = A(P)S(X, Y) + \frac{(3n-2)t}{n-1}A(X)A(Y)- \\ - \frac{t}{n-1}A(P)g(X, Y).\end{aligned} \tag{2.8}$$

Again from (2.3) we get

$$(\nabla_X B)Y = (X \cdot t)A(Y) + tg(Y, \nabla_X P). \tag{2.9}$$

Now, by (2.9), we get

$$\begin{aligned}(\nabla_X S)(Y, P) = \nabla_X S(Y, P) - S(\nabla_X Y, P) - S(Y, \nabla_X P) = \\ = (x \cdot t)A(Y) + tg(Y, \nabla_X P) - S(Y, \nabla_X P).\end{aligned} \tag{2.10}$$

From (2.8) and (2.10) it follows that

$$\begin{aligned}A(P)S(X, Y) + \frac{(3n-2)t}{n-1}A(X)A(Y) - \frac{t}{n-1}A(P)g(X, Y) = \\ = (x \cdot t)A(Y) + tg(Y, \nabla_X P) - S(Y, \nabla_X P).\end{aligned} \tag{2.11}$$

From (2.11) we get

$$A(P)LX + \frac{(3n-2)t}{n-1}A(X)P - \frac{t}{n-1}A(P)X = \tag{2.12}$$

Using (2.5) one gets from above

(2.13) $\nabla_X P = \sigma X + \omega(X)P$ where

(2.14) $\sigma = \dfrac{r-2t}{nt-r}A(P)$ and

(2.15) $\omega(X) = \dfrac{2t(2n-1)-r}{nt-r}A(X) - \dfrac{n-1}{nt-r}(X\cdot t) + \dfrac{X\cdot A(P)}{2A(P)}$.

Since by hypothesis the 1–form A is not closed, σ cannot be Zero, for otherwise from (2.14) one gets $r = 2t$. Hence, from (1.4), we get $X\cdot 2t = 8t\cdot A(X)$. But this means that A is closed. Hence $\sigma \neq 0$ and P is a torse forming vector field [6] and we state:

Theorem 1. *If in a conformally flat $(PS)_n$ $(n \geq 3)$ the associated 1–form is not closed, then the vector field P, defined by (2) is a torse forming vector field.*

3. Conformally flat $(PS)_n$ $(n \geq 3)$ with constant scalar curvature. It is known [2] that a conformally flat $(PS)_n$ $(n \geq 3)$ cannot be of Zero scalar curvature. In this section we consider a conformally flat $(PS)_n$ $(n \geq 3)$ whose scalar curvature is a non–Zero constant. Then from (1.4) we get

$$B(X) = -\frac{r}{2}A(X). \tag{3.1}$$

In this case, putting $t = -\frac{r}{2}$ in (2.14) and in (2.15) we have

$$\nabla_X P = \sigma X + \omega(X)P \quad \text{where} \tag{3.2}$$

$$\sigma = -\frac{4}{n+2}A(P) \quad \text{and} \tag{3.3}$$

$$\omega(X) = \frac{4n}{n+2}A(X) + \frac{X\cdot A(P)}{2\cdot A(P)}. \tag{3.4}$$

From (3.3) it follows that σ cannot be Zero. Again from (3.4) it follows that ω is closed if A is closed and ω is not closed if A is not closed. Thus we state

Theorem 2. *In a conformally flat $(PS)_n$ $(n \geq 3)$ with non–Zero constant scalar curvature, the Vector field P, defined by (2), is a proper torse forming vector field if the associated 1–form is not closed and P is a*

It is known [1] that if a conformally flat manifold (M^n, g) $(n \geq 3)$ admits a proper concircular vector field, then the manifold is a subprojective manifold in the sense of Kagan. Thus we state

Theorem 3. *If in a conformally flat $(PS)_n$ $(n \geq 3)$ with non–Zero constant scalar curvature, the associated 1–form is closed, then the manifold is a subprojective manifold in the sense of Kagan.*

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