ANALELE ŞTIINŢIFICE ALE UNIVERSITĂŢII "AL.I.CUZA" IAŞI
Tomul XLII, s.I.a, Matematică, 1996, f2.

A CONTROL PROBLEM DEPENDING ON COEFFICIENTS, ARISING FROM AN INDUSTRIAL PROCESS

BY

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Abstract. We consider the manner to obtain aluminium starting from a mixture (blend) which is smelted by using an electric current. In order to optimize this process, we study two types of control problems by the coefficients. The control variables are the electric and thermic conductivities, contained in a bounded domain $\mathcal{D}$ of $\boldsymbol{R}^3$. The optimal values of the considered functionals (of the "energetic" type) are obtained on the boundary of the domain $\mathcal{D}$.

1. Introduction. A blend (mixture) is submitted to a smelting process, to obtain aluminium. The receptacle (tank) Ω which contains the blend is heated by using a carbon bloc U, which in his turn is heated by using an electric current V. This current produces in the carbon bloc and is the receptacle a temperature T. To obtain a more homogeneous aluminium alloy and to avoid the shrinkage cavities, we consider two problems:

1) in the receptacle, to obtain a distribution of grad (V) as close as possible to a given value;

2) on the surface S between the carbon bloc and the receptacle, to obtain a distribution of the electric flux $(\partial V/\partial n)$ as close as possible to a given constant.

The control variables, from these above problems, are the electric and thermic conductivity constants of the blend and of the carbon bloc; in general, these constants are functions of the temperature T.

We denote by Γ the boundary of $\Omega \subset \boldsymbol{R}^3$.

The results of this paper were obtained with the financial support of the Région Rhône Alpes (programe TEMPRA), for the second author.

The process is described by the following system:

$$\partial V/\partial t - \text{div } (\gamma(T)\nabla V) = f \text{ in } \Omega,$$

$$(P) \qquad \partial T/\partial t - \text{div } (k(T)\nabla T) = C|\nabla V|^2 \text{ in } \Omega,$$

$$\partial V/\partial n = 0 \text{ on } \partial\Omega \setminus S, S = \partial\Omega \cap \partial U,\ V = 0 \text{ on } S.$$

We have the same system in the carbon bloc U.

The constants $\gamma(T)$ and $k(T)$ have different values in Ω and U; we consider that these coefficients are contained in a bounded domain $\mathcal{D}$ of $\boldsymbol{R}^3$.

It is possible to see that, if $f \in L^2(\Omega \cup U)$, we have a parabolic problem with the right hand side in L^1. A definition for the solution in this case is given in [2], for an elliptic equation.

We denote by V_γ the solution of the system (P), where γ is a function of $C^\infty(\boldsymbol{R})$.

In the following we consider the functionals:

$$J(\gamma) = \int_\Omega |\nabla V_\gamma - \theta|^2,$$

$$I(\gamma) = \int_S |\partial V_\gamma/\partial n - \xi|^2,$$

where θ is a given vector in $\boldsymbol{R}^3$ and $\xi \in \boldsymbol{R}$.

Using these functionals (denoted both by F), the problems 1) and 2) have the following formulation:

$$\text{Find } \overline{\gamma} \in C^\infty(\boldsymbol{R}) \text{ such that } F(\overline{\gamma}) = \text{Inf } \{F(\gamma),\ \text{Im } \gamma \in \overline{\mathcal{D}}\}.$$

In this way, we obtain a control problem depending on the coefficient γ and k, with "distributed" and "boundary" observation.

The mathematical model considered in this paper was obtained in [1],[3]. We prove the continuity of the considered functionals with respect to the coefficients γ and k; then the above problems have at least a solution.

We consider the case $f = 0$ and γ independent of T. The derivability of the solution of the problem (P) with respect to the coefficients is obtained, for smooth enough conditions, by using the Galerkin method [4],[5]. Then we obtain the differentiability of the considered functionals. We prove that the stationary points of the functionals J and I are not contained in the

interior of the domain $\mathcal{D}$; therefore the optimal values are obtained on the boundary of $\mathcal{D}$.

2. Distributed observation. We suppose, for the sake of simplicity, that $\Omega \subset \boldsymbol{R}^2$, but the obtained results are valid in the n–dimensional case (see the last section). We consider the following boundary value problem:

$$-\alpha(\partial^2 u/\partial x^2) - \beta(\partial^2 u/\partial y^2) = f \text{ in } \Omega, \tag{1}$$

$$u/_\Gamma = 0 \quad \text{or} \quad (\partial u/\partial n_a)/_\Gamma = 0, \tag{2}$$

where a is the positive diagonal matrix (α, β) and $\partial u/\partial n_a = \alpha \partial u/\partial x(n_x) + \beta \partial u/\partial y(n_y)$. The coefficients $(\alpha, \beta) > 0$ are contained in the bounded domain $\mathcal{D} \subset \boldsymbol{R}^2$; we use the notation $a \in \mathcal{D}$. The solution if (1)–(2) is denoted by $u_a, u_a \in H^1(\Omega)$.

The following functional and the corresponding control problem are considered:

$$J(a) = \int_\Omega |\nabla u_a - \theta|^2, \tag{3}$$

$$(P1) \qquad \text{Find } \overline{a} \in \mathcal{D} \text{ such that } J(\overline{a}) = \text{Inf}\{J(a), a \in \overline{\mathcal{D}}\},$$

where $\theta = (\theta_1, \theta_2)$ is a given vector of $\boldsymbol{R}^2$.

We have to prove that the interior of $\mathcal{D}$ not contains stationary points of $J(a)$.

Proposition 1. *The map $a \to J(a)$ is continuous.*

Proof. Let be $a_n \to a$, therefore $\alpha_n \to \alpha$ and $\beta_n \to \beta$ in $\boldsymbol{R}$. We put $|\alpha_n - \alpha| = \varepsilon_n, |\beta_n - \beta| = \eta_n$, which ε_n and $\eta_n \to 0$. The equation (1) for α_n and β_n give us the solution $u_n = u_{a_n}$ such that:

$$-(\alpha + \varepsilon_n)(\partial^2 u_n/\partial x^2) - (\beta + \beta_n)(\partial^2 u_n/\partial y^2) = f. \tag{4}$$

By multiplying (1) and (4) with a test function $\phi \in H^1(\Omega)$, we get:

$$(\alpha + \varepsilon_n) \int_\Omega \partial u_n/\partial x \cdot \partial \phi/\partial x + (\beta + \eta_n) \int_\Omega \partial u_n/\partial y \cdot \partial \phi/\partial y = \int_\Omega f\phi$$

$$\alpha \int_\Omega (\partial u_a/\partial x)\cdot(\partial\phi/\partial x) + \beta \int_\Omega (\partial u_a/\partial y)\cdot(\partial\phi/\partial y) = \int_\Omega f\phi.$$

Therefore, we obtain by substraction:

$$\alpha \int_\Omega (\partial u_n/\partial x - \partial u_a/\partial x)\partial\phi/\partial x + \beta \int_\Omega (\partial u_n/\partial y - \partial u_a/\partial y)\partial\phi/\partial y =$$

$$= -\varepsilon_n \int_\Omega (\partial u_n/\partial x)(\partial\phi/\partial x) - \eta_n \int_\Omega (\partial u_n/\partial y)(\partial\phi/\partial y).$$

The solution u_n is uniformly bounded in $H^1(\Omega)$: $||u_n||_{H^1(\Omega)} <$ constant, $\forall n$; the we take $\phi = u_n - u_a$ and get $||\nabla(u_n - u_a)||_{L^2(\Omega)} \to 0$. Therefore the expresion (3) of $J(a)$ gives us $J(a_n) \to J(a)$, q.e.d.

The functional J is continuous on the compact set $\mathcal{D}$, therefore we obtain the following:

Proposition 2. *The problem (P1) has at least a solution.*

It is possible to see that we have to minimize a two–variable real function. We obtain the stationary points by using the derivatives of J with respect to α and β. First, we compute the derivatives of u_a, by using the Galerkin method. We denote by $\{\phi_i\}_{i=1}^{\infty}$ a basis of $H^1(\Omega)$ and consider $\phi = (\phi_1, \phi_2, \ldots, \phi_N)$.

Proposition 3. *The solution u_a is differentiable with respect to α and β.*

Proof. The N–order approximation of the u_a is given by:

$$u_a^N(x,y) = \sum_{i=1}^{N} c_i(\alpha,\beta)\phi_i(x,y) = c\cdot\phi(x,y) \tag{5}$$

The variational formulation of the problem (1)–(2) is:

$$\alpha \int_\Omega (\partial u_a^N/\partial x)(\partial\phi_j/\partial x) + \beta \int_\Omega (\partial u_a^N/\partial y)(\partial\phi_j/\partial y) = \int_\Omega f\phi_j.$$

The following notations are used in the sequel:

$$A_{ij} = \int_\Omega (\partial\phi_i/\partial x)(\partial\phi_j/\partial x), \tag{6}$$

$$B_{ij} = \int_\Omega (\partial\phi_i/\partial y)(\partial\phi_j/\partial y),\ k_j = \int_\Omega f\phi_j. \tag{7}$$

Then we obtain the approximated form of the problem (1)–(2):

$$(\alpha A + \beta B)c = k \text{ or } c = Dk, \tag{8}$$

with $D = (\alpha A + \beta B)^{-1}$. The derivatives of u^N are computed using the above relation; we get:

$$(\partial c/\partial\alpha) = -DADk, \quad (\partial c/\partial\beta) = -DBDk. \tag{9}$$

The estimation of u_a^N gives us the existence of a function $u_a \in H^1(\Omega)$ such that:

$$u_a^N \to u_a,\ (\partial u_a^N/\partial\alpha) \to h,\ (\partial u_a^N/\partial\beta) \to g \text{ in } L^2(\Omega).$$

The above convergences are uniform with respect to α and β, therefore, for smooth enough u_a, we obtain $\partial u_a/\partial\alpha = h, \partial u_a/\partial\beta = g$, q.e.d.

The approximated value of J is denoted by $J^N(\alpha,\beta) = J(u_a^N)$.

Proposition 4. *The derivatives of J are given by the following expressions:*

$$\partial J^N/\partial\alpha = (\partial c/\partial\alpha)^T[(A+B)c - v] \tag{10}$$

$$\partial J^N/\partial\beta = (\partial c/\partial\beta)^T[(A+B)c - v], \tag{11}$$

where $v = \theta_1 \int_\Omega \partial\phi/\partial x + \theta_2 \int_\Omega \partial\phi/\partial y$ is a vector of $\boldsymbol{R}^N$.

Proof. Using the above notation, we have:

$$J^N(\alpha\cdot\beta) = \int_\Omega \left[\sum_{i=1}^N c_i(\partial\phi_i/\partial x) - \theta_1\right]^2 + \int_\Omega \left[\sum_{i=1}^N c_i(\partial\phi_i/\partial y) - \theta_2\right]^2.$$

We obtain the result with the relations (6)–(7), q.e.d.

Theorem 1. *The boundary of $\mathcal{D}$ contains all extremal points of $J(\alpha,\beta)$.*

Proof. The following equations are verified by the interior extremal points:

$$\partial J^N/\partial\alpha = \partial J^N/\partial\beta = 0. \tag{12}$$

Therefore, by using the relations (10)–(11), we obtain the following result: $(\partial c/\partial\alpha)$ and $(\partial c/\partial\beta)$ are orthogonal to the vector $[(A+B)c-v]$; then

$$\exists q \in \boldsymbol{R} \text{ such that } (\partial c/\partial\alpha) = q(\partial c/\partial\beta). \tag{13}$$

The last relation and the relations (8) and (9) give us:

$$DAc = qDBc, \text{ or } Ac = qBc,\ Bc = pAc,\ p = 1/q. \tag{14}$$

The definition (8) of c can be used for the following relations:

$$c = \frac{1}{q\alpha+\beta}B^{-1}k = \frac{q}{q\alpha+\beta}A^{-1}k;\ B^{-1}k = qA^{-1}k. \tag{15}$$

The last relation is not possible: B and A are functions of ϕ_i, but the right hand side k of (8) depends of f, the right hand side of (1), which can take any value. Therefore the relations (12) can not be verified in the interior of the domain $\mathcal{D}$, q.e.d.

3. Boundary observation. We consider now a particular form for the receptacle: $\Omega = [x_0, x_1] \times [y_0, y_1] \subset \boldsymbol{R}^2$. The boundary of Ω is decomposed in two parts: $\Gamma = \Gamma_1 \cup S,\ S = [x_0, x_1] \times \{y_1\}$. We have to study the following boundary value problem:

$$-\alpha(\partial^2 u/\partial x^2) - \beta(\partial^2 u/\partial y^2) = f \text{ in } \Omega, \tag{16}$$

$$(\partial u/\partial n_a)/_{\Gamma_1} = 0,\ u/_S = 0. \tag{17}$$

and denote by u_a the solution.

The following functional and the corresponding control problem are considered:

$$I(\alpha,\beta) = \int_S |\partial u_a/\partial n - \xi|^2, \tag{18}$$

$$\text{Find } \overline{a} \in \overline{\mathcal{D}} \text{ such that } I(\overline{a}) = \text{ Inf } \{I(a), a \in \overline{\mathcal{D}}\}, \tag{P2}$$

where $\xi \in \boldsymbol{R}$.

Theorem 2. *The boundary of $\mathcal{D}$ contains all extremal points of $I(\alpha, \beta)$.*

Proof. The variational formulation of (16)–(17) is given by:

$$\alpha \int_\Omega (\partial u_a/\partial x)(\partial \phi/\partial x) + \beta \int_\Omega (\partial u_a/\partial y)(\partial \phi/\partial y) = \int_\Omega f \cdot \phi, \tag{19}$$

$$\forall \phi \in H^1(\Omega),\ \phi/_S = 0.$$

By using the same notations as above, we consider a basis $\{\phi\}_i$ of $H^1(\Omega)$, but, this time, with the condition $\phi_i/_S = 0$. The N–order approximation for the solution u_a is given also by the relation (5). The "constants" c_i are defined by (8); we have also the relations (9) for the derivatives of c. Therefore we get:

$$(\partial u_a^N/\partial n)/_S = (\partial u_a^N/\partial y)/_S = \sum_{i=1}^N c_i(\alpha, \beta)(\partial \phi_i/\partial y)/_S. \tag{20}$$

The approximated value of I is denoted by $I^N(\alpha, \beta) = I(u_a^N)$; therefore we have

$$I^N(\alpha, \beta) = \int_S \left[\sum_{i=1}^N c_i(\alpha, \beta)(\partial \phi_i/\partial y) - \xi \right]^2, \tag{21}$$

and by using (20), we obtain the derivatives of $I^N(\alpha, \beta)$:

$$\partial I^N/\partial \alpha = 2 \int_S \left(\sum_{i=1}^N \partial c_i/\partial \alpha \cdot \partial \phi_i/\partial y \right) \left(\sum_{i=1}^N c_i \cdot \partial \phi_i/\partial y - \xi \right)$$

$$\partial I^N/\partial \beta = 2 \int_S \left(\sum_{i=1}^N \partial c_i/\partial \beta \cdot \partial \phi_i/\partial y \right) \left(\sum_{i=1}^N c_i \cdot \partial \phi_i/\partial y - \xi \right). \tag{22}$$

For simplicity, we put

$$F_{ij} = \int_S \partial \phi_i/\partial y \cdot \partial \phi_j/\partial y. \tag{23}$$

Then we obtain the equation for the interior stationary points:

$$(\partial c/\partial\alpha)^T(Fc-v)=0,\ (\partial c/\partial\beta)^T(Fc-v)=0. \tag{24}$$

where $v=\theta\int_S \partial\phi/\partial y$. We can see that, as before, $(\partial c/\partial\alpha)$ and $(\partial c/\partial\beta)$ are orthogonal to the vector $(Fc-v)$; as consequence, we obtain the following relation

$$\exists q\in \boldsymbol{R} \text{ such that } (\partial c/\partial\alpha)=q(\partial c/\partial\beta). \tag{25}$$

Using the same way as in the section 2, we can see that this last relation is not possible, q.e.d.

4. Non–steady problem, distributed observation. We consider now the non–steady boundary problem ($t\in[0,T]$):

$$\partial u/\partial t-\alpha(\partial^2 u/\partial x^2)-\beta(\partial^2 u/\partial y^2)=f \text{ in } \Omega\subset \boldsymbol{R}^2, \tag{26}$$

$$u/_\Gamma=0 \text{ or } (\partial u/\partial n_a)/_\Gamma=0,\ u(0,x,y)=u_{a0}(\alpha,\beta,x,y), \tag{27}$$

and denote by $u_a(t,x,y)$ the solution, θ being a given vector of $\boldsymbol{R}^2$.

The following functional and the corresponding control problem are considered:

$$J(\alpha,\beta,t)=\int_\Omega |\nabla u_a-\theta|^2$$

$(P3)$ Find $\overline{a}(t)\in\overline{\mathcal{D}}$ such that $J(\overline{a}(t),t)=$ Inf $\{J(a,t),\ \text{Im } a\in\overline{\mathcal{D}}\}$.

Theorem 3. *The solution $\overline{a}(t)$ of $(P3)$ belongs to the boundary of $\mathcal{D}$, $\forall t\in[0,T]$.*

Proof. We consider the N–order approximation of $u_a(t)$, as before:

$$u_a^N(t,x,y)=\sum_{i=1}^N c_i(\alpha,\beta,t)\cdot\phi_i(x,y), \tag{28}$$

$$u_a^N(0,x,y)=\sum_{i=1}^N c_i(\alpha,\beta,0)\cdot\phi_i(x,y), \tag{29}$$

denoting by $c(t)$ and $c(0)$ the values of the vector c at the time t and for $t = 0$. It is possible that $c(0)$ depends on α and β.

The above relations (6)–(7) and (26)–(27), after multiplication by $\phi_j, j = 1, \ldots, N$, give us:

$$Ec'(t) + (\alpha A + \beta B)c(t) = k,\ E_{ij} = \int_\Omega \phi_i \phi_j, \tag{30}$$

where $c'(t)$ is the derivative of $c(\alpha, \beta, t)$ with respect to the time t.

The following approximation holds

$$(\Delta t)c'(t) = c(t + \Delta t) - c(t) + O(\Delta t)^2, \tag{31}$$

and we get, by using the relation (30):

$$c(t + \Delta t) \approx (\Delta t)E^{-1}k + \{I - (\Delta t)E^{-1}(\alpha A + \beta B)\}c(t). \tag{32}$$

For $t = 0$ we obtain:

$$c(\Delta t) \approx (\Delta t)E^{-1}k + \{I - (\Delta t)E^{-1}(\alpha A + \beta B)\}c(0).$$

Therefore, by using the relation (32), we have the approximations:

$$\partial c(t + \Delta t)/\partial \alpha \approx -(\Delta t)E^{-1}Ac(t) + \{I - (\Delta t)E^{-1}(\alpha A + \beta B)\}(\partial c(t)/\partial \alpha),$$

$$\partial c(t + \Delta t)/\partial \beta \approx -(\Delta t)E^{-1}Bc(t) + \{I - (\Delta t)E^{-1}(\alpha A + \beta B)\}(\partial c(t)/\partial \alpha),$$

In this way, the derivatives of $c(t)$ with respect to α and β at the time t are obtained as functions of the corresponding derivatives at the preceding moment; if $c(0)$ depends of α and β, we suppose given $\partial c(0)/\partial \alpha$ and $\partial c(0)/\partial \beta$.

We use the following approximation of the functional $J : J^N(\alpha, \beta, t) = J(u_a^N(t))$; by using the relations (10)–(11) we get:

$$\partial J^N/\partial \alpha = (\partial c(t)/\partial \alpha)^T[(A + B)c(t) - v],$$

$$\partial J^N/\partial \beta = (\partial c(t)/\partial \beta)^T[(A + B)c(t) - v],$$

The stationary interior points are given also by the relations (12) therefore we obtain, for any time t, the "colinearity" of the vectors $\partial c(t)/\partial \alpha$ and $\partial c(t)/\partial \beta$. On this way, we obtain the existence of a real constant p such that:

$$-(\Delta t)E^{-1}Ac(t) + [I - (\Delta t)E^{-1}(\alpha A + \beta B)](\partial c(t)/\partial \alpha) =$$

$$= p\{-(\Delta t)E^{-1}Bc(t) + [I - (\Delta t)E^{-1}(\alpha A + \beta B)](\partial c(t)/\partial\beta)\}.$$

If $c(0)$ is independent of α and β, we have:

$$Ac(0) = p\ Bc(0).$$

The last two relations are not possible, because α and β are not depending on $\{\phi_i\}$, q.e.d.

5. The n–dimensional case. The same results are obtained in the case of Ω (and $\mathcal{D}$) bounded domain of $\boldsymbol{R}^n$ (respective of $\boldsymbol{R}^{n^2}$). Indeed, consider the Neumann or Dirichlet problem for the following equation, $t \in [0, T]$:

$$\partial u/\partial t - a_{ij}(\partial u^2/\partial x_i \partial x_j) = f \text{ in } \Omega, \tag{33}$$

$$u(0, x, y) = u_{a0}(\alpha, \beta, x, y), \tag{34}$$

and the functional

$$J(a, t) = ||\nabla u_a - \theta||^2_{L^2(\Omega)},$$

where $u_a(t)$ is the solution of (33)–(34) and a is the positive matrix a_{ij}. We use in the following the usual convention of dumb indices.

We consider for example the control problem (P3). In the following, we obtain the same result as in previous section: the optimum values of $J(a, t)$ are attained on the boundary of $\mathcal{D}, \forall t \in [0, T]$.

Let u^N be the N–order approximation like (28)–(29) for u_a, the solution of (33)–(34), with $c_i = c_i(a, t)$. We introduce the following notations:

$$E_{ki} = \int_\Omega \phi_k(x)\phi_i(x);\ g_p = \int_\Omega \phi_p(x)f(x);$$

$$B_{ksij} = \int_\Omega \partial\phi_k(x)/\partial x_i \cdot \partial\phi_s(x)/\partial x_j;\ A_{ks} = B_{ksij}a_{ij}.$$

The approximated form of the problem (33)–(34) is:

$$Ec'(a, t) + Ac(a, t) = g, \text{ or } E_{pi}c_i'(t) + B_{psij}a_{ij}c_s(a, t) = g_p,$$

$c'(a, t)$ being the derivative of c with respect to the time t. Therefore, for $k = 1, 2, \ldots, N$ we obtain:

$$E_{ki} \cdot \partial c_i'/\partial a_{pq} + B_{kspq} \cdot c_s + B_{ksij}a_{ij} \cdot (\partial c_s/\partial a_{pq}) = 0.$$

This relation and the approximation (31) give us the derivatives of $c(t+\Delta t)$ in terms of the derivatives of $c(t)$:

$$\partial c_l(t+\Delta t)/\partial a_{pq} \approx -(\Delta t)E^{-1}_{lk}B_{kspq}\cdot c_s(t)+$$

$$+\{\delta_{ls}-(\Delta t)E^{-1}_{lk}B_{ksij}a_{ij}\}\cdot \partial c_s(t)/\partial a_{pq}.$$

On this way, as before, we obtain the derivatives of $J^N(a,t)=J(a^N_a(t))$:

$$\partial J^N(a,t)/\partial a_{pq}=2\partial c_i(a,t)/\partial a_{pq}\cdot\{M_{ij}c_j(a,t)-v_i\},$$

$$M_{ij}=\sum_{r=1}^{n}\int_\Omega \partial\phi_i/\partial x_r\cdot\partial\phi_j/\partial x_r,\ v_i=\sum_{r=1}^{n}\theta_r\int_\Omega\partial\phi_i/\partial x_r.$$

The condition (12) for the interior stationary points of J^N gives us the "colinearity" of the vectors $\partial c(a,t)/\partial a_{pq}$ and $\partial c(a,t)/\partial a_{rs}, \forall p,q,r,s = = 1,2,\dots n, \forall t\in[0,T]$. By using this last property, we obtain a relation which is not possible, because α and β do not depend on $\{\phi_i\}$.

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Received: 20.XI.1996

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