ANALELE ŞTIINŢIFICE ALE UNIVERSITĂŢII "AL.I.CUZA" IAŞI
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BOOK REVIEWS *RECENZII*

TIKHONOV, A.N., GONCHARSKY, A.V., STEPANOV, V.V., YAGOLA, A.G. - *Numerical Methods for the Solution of Ill-Posed Problems*, Kluwer Academic Publishers, Dordrecht, Boston, London, 1995, IX + 253 pp., ISBN 0-7923-3583-X

This book is the translation of the Russian original monograph published in Russia in 1990. It deals with linear ill-posed problems with or without a priori constraints. Tikhonov's variational approach with choice of regularization parameter and the generalized discrepancy principle are the basic regularization methods used.

The book contains 4 chapters. The first one is dedicated to regularization methods. A detailed discussion of the means for matching the regularization parameter with the specification error of the initial information is given. A priori schemes for choosing the regularization parameter introduced by Tikhonov are also discussed. Considerable attention is payed to the problem of finite-difference approximation and to numerical methods for solving the resulting system of linear equations.

Chapter 2 deals with ill-posed problems on compact sets. The construction of regularizing algorithms is based on the use of additional information regarding the solution, more exactly regarding the membership of the required solution to a compact class.

The numerical aspects of constructing efficient regularizing algorithms on special sets are studied in Chapter 3. Algorithms for solving ill-posed problems on sets of special kinds (using information on the monotonicity of the required solution, its convexity, the existence of a finite number of inflection points, etc.) are considered.

A FORTRAN library for solving ill-posed problems is presented in Chapter 4. That library includes :

- various program versions for solving linear integral equations based on the original Tikhonov scheme;

- special programs for solving convolution-type 1D and 2D integral equations using the Fourier fast transform;

- various program versions for solving 1D Fredholm integral equations of the first kind on special sets of functions.

Test examples are given for each program. The Appendix contains the program listings. A Postscript on the main monographs on ill-posed problems that have appeard after 1990 is given. There are also 220 references.

V. Arnăutu.

DAVID SPEISER (editor) *Die Werke von Daniel Bernoulli.* Im Auftrag der Naturforschenden Gesellschaft Basel und der Otto–Spiess–Stiftung. Band 2 Analysis Wahrscheinlichkeitsrechnung Bearbeitet und Kommentiert von L.P.Bouckaert, B.L. van der Waerden unter Benützung von Vorarbeiten von H. Straub †. 403 p. 1982 Birkhäuser Verlag. Basel. Boston. Stuttgart. ISBN 3–7643–1084–7.

Iniţiativa editării operelor complete ale matematicienilor şi fizicienilor familiei Bernoulli a fost luată în 1936 de Otto Spiess. De editarea operelor lui Daniel Bernoulli (1700–1782) s-a ocupat Hans Straub (†1972) care a scris o introducere asupra lucrărilor lui D. Bernoulli din domeniul probabilităţilor şi a publicat, după lungi şi stăruitoare căutări, lista tuturor lucrărilor acestui remarcabil matematician. Editarea operelor lui D. Bernoulli a fost continuată din 1973 de David Speiser care a alcătuit şi planul general al lucrării. Scopul declarat al editării operei lui D. Bernoulli este de a o face accesibilă istoricilor, matematicienilor şi fizicienilor atât prin tipărirea lor, care urmează în linii mari uzanţele actuale, cât şi prin pertinentele comentarii ce o însoţesc. Lucrările lui D. Bernoulli au fost divizate în următoarele opt volume, însoţite fiecare, de valoroase comentarii datorate unor reputaţi specialişti pe care îi specificăm mai jos: 1. Scrieri din tinereţe şi Medicină (F. Rintelen şi L.P. Bouckaert), 2. Analiză şi Calculul probabilităţilor (L.P. Bouckaert şi B.L. van der Waerden), 3. Mecanică (D. Speiser, J.L. Pietenpol şi P. Radelet), 4 şi 5. Hidrodinamică (C. A. Truesdell), 6. Elasticitate (D. Speiser şi P. Radelet), 7. Magnetism (P. Radelet) şi Tehnologie (L. Bossy), 8. Tehnologie (L. Bossy).

Seria operelor complete ale eminentului om de ştiinţă începe cu prezentul volum 2 consacrat în proporţii egale scrierilor de analiză matematică şi celor de calculul probabilităţilor. Lucrările sunt prezentate în ordine cronologică, majoritatea lor fiind publicate în limba latină în revista "Novi Commentarii Academiae Scientiarum Imperialis Petropolitanae". Primul grup de scrieri, pregătit de L.P. Bouckaert din Löwen- Belgia, cuprinde, în principal, lucrări despre serii recurente, serii trigonometrice şi fracţii continue.

Cele referitoare la seriile trigonometrice sunt legate de cercetările sale în domeniul elasticităţii. Lista lucrărilor din această secţiune este următoarea: 1) Observaţii asupra seriilor formate prin adunarea sau scăderea termenilor succesivi şi regula de aflare a rădăcinilor ecuaţiilor algebrice. 2) Observaţii asupra ecuaţiilor care tind la infinit şi a rezolvării lor prin metoda seriilor recurente: despre un nou tip de serii (două prelegeri). 3) Extras din scrisoarea lui D. Bernoulli către Leonard Euler. 4) Extras din scrisoarea lui D. Bernoulli către dl Garcin, doctor în medicină, cu privire la elemente de algebră a d-lui Clairaut de la Academia regală de ştiinţe din Paris. 5) Despre sumarea seriilor oarecare, interpretarea şi utilizarea lor (I şi II). 6) Despre proprietăţile particulare ale seriilor infinite în sinusuri şi cosinusuri de unghiuri în progresie aritmetică şi despre sumarea şi utilizarea lor (I şi II). 7) Teoria elementară a seriilor formate în diverse moduri din sinusuri şi cosinusuri de arce în progresie aritmetică, expusă clar (I şi II). 8) Diverse note analitice asupra fracţiilor continue (I şi II). 9) Cercetări ulterioare asupra proprietăţilor fracţiilor continue (I şi II). Al doilea grup de lucrări este analizat de B.V. van der Waerden din Zürich – Elveţia şi se referă la Calculul probabilităţilor. În prima lucrare, D. Bernoulli introduce noţiunea de "speranţă morală" în vederea lămuririi "Paradoxului din Petersburg". În lucrările de mai târziu el introduce noţiunea de "valoare medie", studiază probleme de ereditate, durata vieţii omeneşti, mortalitatea cauzată de variolă şi influenţa vaccinului, durata medie a căsniciilor, raportul dintre numărul noilor născuţi în funcţie de sex etc. În ultima lucrare el caută să determine o lege universală a erorilor. Lista publicaţiilor din această parte este următoarea: 1) Eseu asupra unei noi teorii de măsurare a şansei. 2) Eseu asupra unei noi analize a mortalităţii cauzată de variolă şi despre avantajele inoculării pentru prevenire. 3) Reflexii asupra avantajelor inoculării. 4) Asupra utilizării algoritmului infinitesimal în arta combinării zarurilor. 5) Despre utilizarea algoritmului infinitezimal în arta combinării probelor. 6) Despre durata medie a căsniciilor oricare ar fi vârsta căsătoriţilor şi despre alte probleme înrudite (I şi II). 7) Cercetări analitice asupra unei noi probleme conjecturale (I şi II). 8) Măsura şansei aplicată la o succesiune întâmplătoare de lucruri contingente în mod natural (I, II şi III). 9) Concluzia cea mai probabilă a mai multor observaţii discrepante cât şi inducţia cea mai probabilă ce trebuie formată de aici. 10) Eseu filosofic cu privire la compensarea orologiilor şi măsura timpului cea mai apropiată de adevăr.

Volumul se încheie cu Indice de nume, Lista lucrărilor lui D. Bernoulli (alcătuită de H. Straub şi completată de P. Radelet–de Grave şi V. Scheuber), Lucrările lui D. Bernoulli după data lor de realizare şi Repartizarea lucrărilor lui D. Bernoulli pe volume.

Publicarea operei lui D. Bernoulli cotat ca unul din marii matemati-

cieni şi fizicieni ai secolului XVIII reprezintă, fără îndoială, un remarcabil eveniment ştiinţific şi cultural. La succesul acestui eveniment se adaugă şi calităţile grafice de excepţie pe care editura Birkhäuser le-a oferit cu generozitate acestui volum. Reproducerea impecabilă a portretului marelui savant după tabloul realizat de pictorul Johann Niclaus Grooth între anii 1750 şi 1755 este şi ea pe măsura acestui eveniment.

Gh. Bantaş şi V. Şciurevici

WALLIS, W.D., *One–Factorizations.* Mathematics and Its Applications, 390, Kluwer Academic Publishers, Dordrecht–Boston–London, 1997 (36 fig, 11 tab, XIV+242 pag), ISBN 0–7923–4323–9.

Lucrarea este o adevărată monografie – prima în acest domeniu particular al 1–factorizării grafurilor, realizând o punte de legătură între teoria grafurilor şi teoria designului combinatorial. Sunt deosebite capitolele referitoare la grafurile decompozabile, la teoria cuplajelor şi cele din teoria designului.

Adresându-se studenţilor şi deopotrivă cercetătorilor în domeniu, lucrarea conţine exerciţii propuse spre rezolvare precum şi enunţurile unor probleme nerezolvate încă.

O bibliografie aproape exhaustivă, conţinând 177 titluri de referinţă în domeniu şi un index alfabetic completează această monografie excelentă şi bine venită.

J. Moscovici

BISHOP, A.J., CLEMENTS, K., KEITEL, C., KILPATRICK, J., LABORDE, C. (editors), *International Handbook of Mathematics Education,* Parts 1-2, Kluwer International Handbooks of Education, vol.4, Kluwer Academic Publishers, Dordrecht, Boston, London, 1996, 1358 p., ISBN 0-7923-3533–3.

This book, the 4^{th} volume of "Kluwer International Handbook of Education", points out the great dimension, the growing richness, and the complexity of the international activities in Mathematics Education. Let us say that at the 8^{th} International Congress on Mathematics Education (Seville – Spain, July 1996) there were more than 3700 participants from 93 countries grouped in 26 specialist Working Groups and in 26 specialist Topic Groups.

The book is not a proceeding of this 8^{th} ICME but offers synthetic and reflective overviews on the varied directions in the field, the current problems being faced, the future possibilities for development. We appreciate as useful to present here the titles of Section of this book and their numbers of chapters and pages: Curriculum, Goals, Contents, Resources (13, 494); Teaching & Learning Mathematics (8, 316); Perspectives & Interdisciplinary Contexts (7, 272); Social Conditions & Perspectives on Professional Development (8, 230). As a Romanian reader of the book, I regret the weak implication of the Eastern Europe in the field of Mathematics Education.

Dan Brânzei

HACHENBERGER, DIRK, *Finite Fields. Normal Bases and Completely Free Elements.* Kluwer Academic Publishers, Boston, Dordrecht, London, 1997, ISBN 0-7923-9851-3.

This book focuses on topics related to the classical Normal Basis Theorem from the field theory, which states that every finite Galois extension of fields E over F has a basis consisting of the conjugates of a certain element (such an element is called free in E over F).

This kind of bases are suited for arithmetic computations in the case of finite fields, and the explicit construction of normal bases is now a major research theme in Finite Field Theory.

The first chapter presents an extension of the Normal Basis Theorem, namely that exist elements in E which are simultaneously free over every intermediate field of E over F (such elements are called completely free over F). Sec. 6 of this chapter gives an outline of the results presented in Ch. III to VI: the characterization, the enumeration and the explicit construction of completely free elements in extensions of finite fields. Chapter II is also introductory and deals with the application of the theory of modules over principal ideal domains in the investigation of the finite field extensions. The book is self–contained, prerequisites being only the Galois Theory of field extensions and some general algebraic background. It addresses the researchers in finite fields (particularly interested in constructive and algorithmic methods) or can be used (at least, some chapters can) as an advanced text on the theory of finite fields.

Claudiu Volf

R.N. BOLL, *Loop checking in logic programming;* CWI TRACT, 112, Centrum voor Wiskunde en Informatika, Amsterdam, 1995, 198 p., ISBN 90 6196 456 3.

This book is a revised version of the author's PhD. Thesis and it is fundamental in the field of logic programming theory. If one wants that a practical implementation to be a good approximation of a logic programming language, than loop checking must be taken into account. But what is a good loop checking procedure? This book provide rigurous conditions that a loop checker must fulfill in order to be integrated in a logic programming language. The notions of *sl sound* and *weakly sound* loop checking procedures are defined and then carefully analized. Applications of the loop checks are presented. They include approximations for *non-monotonic reasoning* and *partial evaluation.* Then more specific loop checks are defined: *equality checks, subsumption checks* and *context checks.* Completness of loop check for positive (definite) logic programs is studied for the classes of loop checkers defined in the book. The loop check is then extended to logic programs with default negation. This book has already became a classic and I must emphasize that it is indispensable for those who investigate (either theoretical or practical) techniques of implementation for logic programming languages or non-monotonic logics.

Cristian Papp

ZHE-XIAN WAN, SHEN-MING SHI (editors) *Group Theory in China* Mathematics and Its Applications (China series), Science Press (New York/ Beijing), Kluwer Academic Publishers, Dordrecht/Boston/London, 1996. ISBN 0–7923–3989–4.

This book, dedicated to Professor Hsio–Fu Tuan on his 82nd birthday, is a collection of essays, papers and surveys on Group Theory and some related areas. It is intended as a review of the present state of the research in Group Theory in China and it summarizes the problems that Chinese group theorists are interested in. The book is of interest to group theorists and to mathematicians specializing in algebraic K–theory, Lie algebras, graph theory, representation theory. A brief introduction to the developments in the study of Differential Geometry of homogeneous spaces is also given.

Claudiu Volf

C.D. AHLBRANDT, A.C. PETERSON, *Discrete Hamiltonian Systems. Difference Equations, Continued Fractions and Riccati Equations.* Kluwer Texts in Mathematical Sciences, vol. 16, Kluwer Academic Publishers, Dordrecht/Boston/London, 1996, XIV+375 pp. ISBN

This volume in the graduate–level book series develops topics on discrete Hamiltonian systems and provides a unique study about interconnections between symplectic systems, recesive and dominant solutions of second order difference equations, discrete variational problems, discrete Riccati equations for symmetric three term recurrences, and matrix continued fraction representations of their extreme solutions. In a somewhat modular setting, the book presents the importance of a variable step size discrete variational theory, based on a benefic unification of some major themes revealed in the current research on discrete Hamiltonian systems. Those nine chapters of the volume are dealing, in order, with the following subjects: second order scalar difference equations, continued fractions, linear symplectic systems, a discrete vriational theory, symmetric three term recurrence relations, discrete Riccati equations for three term recurrences, Green's functions for nonhomogeneous second order difference equations, disconjugacy criteria, and an introduction to Martin Bohner's approach to the discrete linear Hamiltonian systems. The book constitutes a foundation and an accessible work for mathematics and engineering students (or researchers) who have completed a course in matrix theory at least and are interested in discrete aspects of optimal control, estimation and variational problems arising in physics, chemistry and economics.

Florin Iacob

ROITBERG, Y., *Elliptic Boundary Value Problems in the Spaces of Distributions.* Mathematics and Its Applications, vol. 384, Kluwer Publishers, Dordrecht/Boston/London, 1996, XI+420pp. ISBN

Devoted to the theoretical solvability of general elliptic boundary–value problems in generalized functions, the monograph under review summarize the most part of available data on the theorems on complete collection of isomorphisms – and their applications – for the operators generated by such problems.

The book consists of an introduction and ten chapters containing, in order, the following questions: definitions and necessary properties of certain functional spaces; imbedding and interpolation theorems; an analysis

of the concepts of general elliptic boundary-value problem, normality of a set of boundary conditions, and Noetherian property of an elliptic problem; the theorem on complete collection of isomorphisms for general elliptic boundary–value problems; an investigation on weak and strong solvability of elliptic problems with normal boundary conditions; studies of the traces of generalized solutions of elliptic equations on the boundary of a domain; results on local increase in smoothness of distributional solutions of elliptic problems; the settlement of some appropriate Green's formulae; the exploration of strongly degenerate elliptic problems, elliptic problems with power singularities on the right–hand sides, elliptic problems with a parameter, and elliptic boundary–value problems for general systems of equations. At the end of the book, some useful bibliographical notes present necessary comments to the included list of references. The volume is of interest to all of those whose work involves partial differential equations, functional analysis, operator theory and mathematics of mechanics.

Florin Iacob

ZAVALISHCHIN, S.T., SESEKIN, A.N., *Dynamic Impulse Systems. Theory and Applications.* Mathematics and Its Applications. vol. 394, Kluwer Academic Publishers, Dordrecht/Boston/London, 1997, XI+260pp. ISBN

Based on up–to–date mathematical methods, the book supplies a general and systematic treatment of dynamic systems having an impulse character. It contains results obtained mainly by the authors in the following directions: necessity of the constructing dynamical models by the use of the concept of generalized function and the systematization of such models by the construction of some appropriate general form of linear and nonlinear operator equations in distributions; proper determination of solutions of equations that have nonlinear operations over generalized functions in their description; definition and investigation of the concept of stability for generalized processes; establishment of optimality conditions for optimization problems for dynamic systems in distributions. The volume suggests new approaches to investigate equations in distributions by means of a certain integral representation of generalized functions and by the use of an adequate description of the solutions of differential equations containing products of discontinuous functions and distributional derivatives of functions of bounded variation. Thus, a new theory of differential equations with nonlinear operations over impulse input actions is developed. As applications, a

topological characterization of stability of dynamic impulse systems is given, a scheme for finding discontinuous periodic motions of systems with automatic regulation is described, schemes for solving singular problems of the quadratic optimization of linear systems are presented, necessary conditions for optimality in the Lagrange problem in a Pontryagin's form are formulated, and some singular optimization problems are treated on the subject of the mechanics of space flight, the motion of transport manipulators in a viscous medium, quantum physics and economics. The book is of interest both to applied mathematicians and to electrical and mechanical engineers whose work involves ordinary differential equations, control and optimization, circuit synthesis, or dynamics and vibration.

Florin Iacob

BAUER, HEINZ, *Probability theory,* Studies in Mathematics, vol.23, Walter de Gruyter, Berlin, New York, 1996, XV+523p.

Volumul de faţă reprezintă ediţia, revizuită şi îmbunătăţită, a cărţii *Wahrscheinlichkeitstheorie,* Walter de Gruyter, Berlin, New York, 1991, a autorului H. Bauer. Conţinutul volumului este structurat în nouă capitole. În capitolul I sunt prezentate conceptele de bază ale teoriei probabilităţilor ca: spaţiul de probabilitate, eveniment, variabilă aleatoare, valoare medie, varianţă şi repartiţie. Conceptul de independenţă stochastică pentru evenimente, corpuri boreliene şi variabile aleatoare face obiectul capitolului II. Capitolul III este consacrat legii numerelor mari, insistându-se asupra variantelor Borel, Kolmogorov, Hewitt–Savage ale legii 0–1. Conceptele de medie condiţionată şi martingal, indispensabile teoriei moderne a teoriei probabilităţilor, sunt tratate în capitolul IV. În capitolul V se studiază funcţia caracteristică a unei repartiţii probabilistice, ca parte a analizei Fourier. Capitolul VI se ocupă de teorema limită centrală unidimensională şi multidimensională, repartiţii infinit divizibile şi măsuri gaussiene. În capitolul VII este prezentată legea logaritmului iterat, sub forma lui V. Strassen. Construcţia proceselor stochastice, cu referire la procesele: mişcare browniană, Poisson, Markov, Gauss, face obiectul capitolului VIII. Capitolul IX este consacrat studiului aprofundat al procesului mişcării browniene, în strânsă legătură cu teoria martingalelor. Materialul cuprins în acest volum se poate parcurge în două semestre, a câte 4 ore de curs săptămânal, de către studenţii interesaţi în studiul Teoriei probabilităţilor. Însuşirea cunoştinţelor prezentate este uşurată de numeroasele exemple şi exerciţii propuse. Pentru înţelegerea conţinutului cărţii sunt necesare cunoştinţe generale de teoria măsurii şi

integrării. Volumul se încheie cu o bibliografie şi trei indexuri.

Elena Nenciu

YÛSAKU KOMATU, *Distortion Theorems in Relation to Linear Integral Operators,* Mathematics and Its Applications, vol. 385, Kluwer Academic Publishers, Dordrecht/Boston/London, 1996, 305 p, ISBN 0–7923–4304-2.

"Distortion Theorem" refers to the precise determination of the range of a real–valued functional defined on a class of holomorphic functions. The monograph consists of two parts. The first part is devoted to the study of some linear integral operators defined on classes of holomorphic functions in the unit disk. These operators play the main roles of basic tools in constructing several functionals as the object of distortions which will be discussed in the latter part. The dependence of the integral operators of the probability measure of integration as well as the fractional calculus with these operators are considered. Applications are given to the product of operators, analytic prolongation and functional equations. The second part of the monograph, which consists of several chapters, deals with the study of some kinds of distortions under the action of integral operators for various functionals: maximum modulus, real part, range, lenght and area, angular derivative, mean etc. Distortions on the class of univalent functions and its subclasses, Carathéodory class as well as distortions by a differential operator are also considered.

Cătălin Lefter

V. VAN LOON, *Numerical methods in smog prediction,* C.W.I. Tract 120, Stichting Mathematisch Centrum, Amsterdam, 1997, 149p, ISBN 90-6196-473-3.

This is a book concerning the results reported by the author in the field of numerical methods for solution of mathematical models in smog prediction. After a description of the physical and chemical models, the author studies the grid refinement technique and a few finite volume advection schemes, wich is followed by numerical experiments. Then special purpose solvers for systems of stiff ordinary differential equations arising from chemical kinetics are discussed and tested. The last chapters provide a

comparison between model results and measurements for a winter and summer smog episode and some conclusions and recommendations for future results. The book contains a bibliography of 75 titles and an index.

C. Ilioi

ENGL, HEINZ W., HANKE, MARTIN, NEUBAUER, ANDREAS, *Regularization of Inverse Problems,* Mathematics and Its Applications, vol. 375, Kluwer Academic Publishers, Dordrecht/Boston/London, 1996, 251p, ISBN 0–7923–4157-0.

The field of inverse problems is one of the fastest growing areas in applied mathematics. This book is devoted to the mathematical theory of regularization methods for ill–posed inverse problems. For linear problems, this theory can be considered to be relatively complete and it is described in the first seven chapters of the book. The main emphasis is on a functional analytical treatment in the context of operator equations. However it is included also some informations on numerical aspects for linear problems in a special chapter. In the last two chapters the authors give account of some of the currently available results for nonlinear problems. The book contains a large bibliography of 290 titles and an index.

C. Ilioi

BRACKX, F., CONSTALES, D., *Computer Algebra with LISP and REDUCE. An Introduction to Computer–aided Pure Mathematics,* Kluwer Academic Publishers, Dordrecht/Boston/London, 1991, XI+264p, ISBN 0–7923–1441–7.

Această carte este bazată pe o serie de lucrări şi comunicări asupra limbajelor de programare LISP şi REDUCE, prezentate la Universităţile din Aveiro şi Coimbra (Portugalia) şi la Universitatea din Gent (Belgia) ca parte a Proiectului "ERASMUS" al Comunităţii Europene. Cartea se adresează în primul rând acelor persoane care doresc să vadă o altă abordare a Algebrei cu ajutorul computerelor faţă de cea bazată pe limbajele de programare clasice FORTRAN, PASCAL sau C. Se arată avantajele folosirii limbajelor LISP şi REDUCE, a versiunilor RLISP şi REDUCE 3.4, precum şi preţul care se plăteşte pentru aceste avantaje. Ea este utilă tuturor matematicienilor familiarizaţi cu Algebra şi cu limbajele de programare FORTRAN,

PASCAL sau C şi, prin aplicaţiile prezentate, invită la o cunoaştere a limbajelor LISP şi REDUCE.

V.M. Gontineac

WILLEM, M., *Minimax Theorems,* Progress in Nonlinear Differential Equations and Their Applications, 24, Birkhäuser Basel. Berlin. Boston. 1996, 176pp, ISBN 3–7643–3913–6.

Willem's book is devoted to minimax theorems and their applications to partial differential equations. Presenting basic minimax theorems in a simple and unified way, starting from a quantitative deformation lemma, the author gives many applications to problems dealing with lack of compactness, in particular, problems with critical exponents and existence of solitary waves. The material covers many recent tools and some unpublished results, such as a treatment of the generalized Kadometsev–Petviashvili equation. It assumes only a basic knowledge of Sobolev spaces, partial differential and linear functional analysis. This book may serve as a textbook for advanced graduate students in partial differential equations.

R. SERAPIONI, F. TOMARELLI (editors) *Variational Methods for Discontinuous Structures,* Applications to image segurentation, continuum mechanics, homogenization, Villa Olmo, Como, 8–10 September, 1994, Progress in Nonlinear Differential Equations and Their Applications, 25, Birkhäuser Basel Berlin Boston 1996, 204pp, ISBN 3–7643–5273–6.

In recent years many researchers in material science have focused their attention on the study of composite materials, equilibrium of crystals and crack distribution of continua subject to loads. At the same time several new issues in computer vision and image processing have been studied in depth. The understanding of many of these problems has made significant progress thanks to new methods developed in calculus of variations, geometric measure theory and partial differential equations. In particular, new technical tools have been introduced and successfully applied. For example, in order to describe the geometrical complexity of unknown patterns, a new class of problems in calculus of variations has been introduced together with a suitable functional setting: the free–discontinuity problems and the special BV and BH functions. The conference held at Villa Olmo on Lake Como in September 1994 spawned successful discussion of these topics among mathe-

maticians, experts in computer science and material scientists. Some of the lectures were devoted to differential or variational modeling of image segmentation, domain partitions, surface flow by mean curvature, homogenization of composite materials, phase transformations, crystals, metastability of martensite, interaction between fracture and damage, magnetostrictive materials. In most cases theoretical and numerical analyses of these models were provided. In other lectures specific problems in calculus of variations were isolated and discussed with view to understanding and solving the main technical issues stemming from the above models: noncoercive functionals, integral functionals with exponential growth or with different upper and lower growth estimates, new capacity notions connected with homogenization of nonlinear elliptic operators in perforated domains, Gconvergence, energy functionals depending on both sets and functions, functions with special bounded variation, free gradient discontinuities. This volume contains a set of contributions based on 17 of the 21 invited talks delivered during the conference. Most of them present original and unpublished results, while the others give surveys of new fields which are rapidly evolving at present.

KEYES, D.E., SAMEH, A., VENKATAKRISHNAN, V. (editors), *Parallel Numerical Algorithms.* Kluwer Academic Publishers, Dordrecht Boston London, 1997, 395p. ISBN 0–7923–4282-8.

The book is prefaced by the three editors, goes on with an acknowledgement to Amiran Harten (a professor of Mathematics at Tel–Aviv University on a consultant to ICASE, who died of a massive heart attack on August 5, 1994) and an introduction to parallel numerical algorithms by KEYES, D.E. The thirteen articles are included into four chapters as follows:

Chapter I. *Linear Systems,* includes: 1. On Parallel Numerical Software Libraires by DEMMEL, J.W., surveys the design principles for writing numerical linear algebra software for high performance computers, especially parallel computers. The sub–chapters are: High performance architectures; Software tools; Matrix multiplication; Gaussian elimination; Future work. Software from the LAPACK and ScaLAPACK projects is used, with 34 references to the literature in the field. 2. Parallel Direct Methods for Sparse Linear Systems by HEAT, M.T. presents an overview of parallel direct methods for solving sparse systems of linear equations, focusing on symmetric positive definite systems. Is investigated the performance implication of important difference between dense and sparse systems. The sub–chapters are: Cholesky Factorization Algorithm, Three Forms of Cholesky Factorization, Data Dependence, Sparse Matrices, Matrix Ordering, Solving Sparse

SPD Systems, Sparse Cholesky Factorization Algorithms, Elimination Tree, Multifrontal Method, Levels of Parallelism Available, Distributed Sparse Factorization, Parallel Ordering, Parallel Triangular Solution, A Fully Parallel Sparse Solver. The article concludes with a 57 title bibliography. 3. Linear Systems Solvers: Sparse Iterative Methods by VAN der VORST, H.A. and CHAN, T.F. presents an overview of a number of related itertive methods for the solution of linear systems; these methods are of Krylov projection type. The sub–chapters are: Iterative Methods Versus Direct Methods, A Basic Iterative Method, The Conjugate Gradient Method (Sketch of the background, computational notes, parallelism and data locality in CG), Nonsymmetric Problems (normal equations, GMRES, parallel aspects of GMRES, GMRES in combination with other conjugate schemes), Biconjugate gradients (gradient Squared, Bi–CGSTAB1, Bi-CGSTAB2 and variants). The article also includes several computational schemes and ends with a 33–title bibliography. 4. Implicitly Restarted Arnoldi/Lanczos Methods for Large Scale Eigenvalue Calculation by SORENSEN, D.C. provides an overview of the numerical solution of large–scale algebraic eigenvalue problems; especially methods called Krylov subspace projection methods (Lanczos method; Arnoldi method and the recently developed variant of the Arnoldi/Lanczos scheme). The sub–chapters are: Eigenvalues, Power Iterations and Spectral Transformations (single vector power iterations, spectral transformations, inverse iteration); Krylov Subspaces and Projection Methods; The Arnoldi Factorization; Restarting the Arnoldi Method (explicit and implicit restarting); The Generalized Eigenvalue Problem; Software, Performance and Parallel Computation. Examples are provided, and the article ends with a 50-titles bibliography.

Chapter II, *Preconditioning,* includes the articles: 1. Approximate and Incomplete Factorization by CHAN, T.F. and VAN der VORST, H.A. makes an overview of a particular class of preconditioners known as incomplete factorization, approximating the exact LU factorization of a given matrix A by disallowing certain fill–ins. The sub–chapters are: Introduction and General References, Point ILU and MILU preconditioners, Variants (more fill, relaxed ILU, shifted ILU, ILQ, Multilevel and re–ordered ILU, block ILU and MILU, PDE–based approximate factored schemes), Parallel Aspects (re–ordering, series expansions, domain decomposition). A few examples are mentioned, and a rich literature (110 titles) is relied on. 2. Parallel Multigrid Methods by JONES, J.E. and McCORMICK, S.F. Multigrid methods have proved to by among the fastest numerical methods for solving a broad class of problems, from many types of PDEs to problems with no continuous origin. The aim here is to provide a brief overview of this active and abundant field of research. Three large sub–chapters (Multigrid

Basics: V cycle algorithm, full multigrid algorithm (FMG), computational complexity; Parallel Multigrid: Parallel computations within levels, concurrent multigrid algorithms, adaptive refinement; Multigrid Methods for Time–Dependent Problems: FMG methods for time–stepping algorithms, parabolic multigrid methods) and a 45-titles bibliography make up this article. 3. Domain Decomposition Methods for Partial Differential Equations by SMITH, B.F., summarizes the basic ideas of domain decomposition methods. Next to an extensive Introduction there are the following: Background and Model Problems, Overlapping Methods (Local corrections, classical block methods, one–level and multilevel overlapping Schwarz methods), Nonoverlapping Methods (Substructuring and iterative substructuring, Neumann–Dirichlet preconditioner, Neumann–Neumann method, balancing, coupling PDEs of different character), and 54 titles for references.

Chapter III, *Fast Application of Operators,* includes the articles: 1. Parallel FFTs by PELZ, R.B. reviews parallel fast Fourier transform (FFT) algorithms. The sub–chapters are: FFTs for a Complex Sequence (Serial FFTs, communication, parallel FFTs), Generalizations (A1-D DFT as a 2-D DFT, generalized i-cycles, Bluestein's FFT), Multidimensional Transforms, FFTs of Symmetric Sequences. There is a 23 titles bibliography. 2. Some Empirical Results on Using Multipole–Accelerated iterative Methods to Solve 3-D Potential Integral Equations by KORSMEYER, F.T., PHILLIPS, J., NABORS, K. and WHITE, J. briefly describe the now well–known multipole–accelerated algorithm for solving potential integral equations, and present computational results from solving several three–dimensional problems. The sub–chapters are: Boundary Integral Formulation (discretization, linear system solution), Multipole Acceleration of Boundary Integral Equations, Computational Examples. For reference there are 27 titles. 3. Multiresolution Representation and Numerical Algorithms: A Brief Review by HARTEN, A. review recent developments in techniques for representing data in terms of its local scale components. The sub–chapters are: Nested Discretization (with 2 examples), General Multiresolution Representation Schemes (with one example), Multiresolution Representation of a 2-Dimensional Array, Multiresolution Algorithm for Matrix–Vector–Multiplication, Multiresolution From of Numerical Schemes, Multiresolution Application of Opertors (integral transforms and equations, initial value problems), an Appendix: The Dual MR Scheme, and a 22-titles bibliography.

Chapter IV, *Parallel Tools, Environments, and Benchmarking* includes: 1. Graph partitioning Algorithms with Application to Scientific Computing by POTHEN, A. describes the terminology associated with graph partitioning, the complexity of computing good separators, and graphs that have

good separators; then it deals with early algorithms for graph partitioning, followed by three new algorithms based on geometric, algebraic and multilevel ideas. Finally, the author focuses on the application of graph partitioning to obtain nested disection ordering for solving sparse linear systems in parallel. The sub–chapters are: Background and Graphs with Good Separators, Early Partitioning Algorithms(the Kernighan–Lin algorithm, level–structure partitioning, the greedy algorithm, inertial algorithm), the Spectral Partitioning Algorithm, The Geometric Algorithm, A Multilevel Algorithm, Bipartition and the Quadratic Assignment Problem, Application of Graph Partitioning. 110 titles are quoted. 2. An Introduction to Performance Debugging for Parallel Computers by GROPP, W.D. dwels on some sources of poor parallel performance, ways to identify them in an application and a few ways to address these issues. The sub–chapters are: Identifying Performance Problems, Locating Problems (communication performance, timing variations), Deficiency Analysis, Event Log Tools. In references 10 titles are mentioned. 3. NAS Parallel Benchmark Results by BAILEY, D.H. The NAS Parallel Benchmarks have been developed at NASA Ames Research Center to study the performance of parallel supercomputers. The eight benchmark problems are specified in a "pencil and paper" fashion. The complete details of the problem to be solved are given in a technical document, and, except for a few restrictions, benchmarkers are free to select the language constructs and the implementation techniques best suited for a particular system. Only two titles are mentioned in references, but there are twelve tables with results.

C. Chiruţă

BARLOW, M.T. and BINGHAM, N.H., *Stochastic Analysis,* Proceedings of the Durham Symposium on Stochastic Analysis, 1990, Cambridge University Press, Cambridge, New York, Port Chester, Melbourne, Sydney, 1991, 375p, ISBN 0–521–42533–6.

În cartea de faţă sunt prezentate câteva dintre lucrările dezbătute la Simpozionul de analiză stochastică ţinut la Universitatea Durham în perioada 11–21 iulie 1990 sub auspiciile Societăţii de matematică din Londra. Tematica de bază a fost conţinută în conferinţele ţinute de Aldous, Dawson, Kesten, Meyer, Sznitman şi Varadhan. Trei dintre ele sunt reţinute în carte: The Continuum random tree; an overview (D. Aldous), Some caricatures of multiple contact diffusion–limited aggregation and the η–model (H. Kesten), On long excursions of Brownian motion among Poissonian obsta-

cles (A.-S. Sznitman). Pe lângă aceste lucrări au mai fost selectate 11 dintre cele 26 lucrări prezentate în cadrul simpozionului şi anume: An evolution equation for th intersection local times of superprocesses (R.J. Adler and M. Lewin), Harmonic morphisms and the resurrection of Markov processes (P.J. Fitzsimmons), Statistics of local time and excursions for the Ornstein–Uhlenbeck process (J. Hawkes and A. Truman), L^p–Chen forms on loop spaces (J.D.S. Jones and R. Leandre), Convex geometry and nonconfluent Γ–martingales I; tightness and strict convexity (W.S. Kendal), Limits on random measures and stochastic difference equations related to mixing array of random variables (H. Kunita), Characterizing the weak convergence of stochastic integrals (T.G. Kurtz and P.Protter), Stochastic differential equations involving positive noise (T.Lindstrøm, B. Øksendal and J. Ubøe), Feeling the shape of a manifold with Brownian motion – the last word in 1990 (M.A. Pinsky), Decomposition of Dirichlet processes on Hilbert space (M. Röckner and T.-S. Zhang), A supersymmetric Feynman–Kac formula (A. Rogers).

C. Amihăesei

SHEVRIN, L.N., OVSYANNIKOV, A.J., *Semigroups and Their Subsemigroup Lattices,* Kluwer Academic Publishers, Dordrecht Boston London 1996, ISBN 0–7923–4221–6.

This book is a completely revised and updated translation of the Russian work "Semigroups and Their Subsemigroup Lattices I,II, Ural State University Press, Sverdlovsk, 1990, 1991.

The contents gives a systematized and comprehensive presentation of the results accumulated hitherto concerning subsemigroup lattices. Specifically the following main aspects are treated in the book: – the description of the structure of semigroups S for which the lattice of subsemigroups Sub S satisfies given lattice – theoretic conditions; – the examination of the lattices Sub S with S belonging to a given class of semigroups; – the investigation of the properties of the Semigroups which are retained by the isomorphisms between the subsemigroup lattices and the discovery of semigroups determined by the subsemigroup lattices. These aspects cover the most important investigations carried until now in the area of subsemigroup lattices. Necessary preliminary facts on semigroups in general, inverse semigroups and lattices are given. Each chapter ends with a list of exercices and some open questions are marked. This volume will appeal to mathematicians with interest ranging in ordered algebraic structures, lattices, group theory, general

mathematical systems or mathematical logic.

I. Tofan

CHARLES HOLLAND, W., MARTINEZ, J. (eds.): *Ordered Algebraic Structures;* Kluwer Academic Publishers, 1997; ISBN 0–7923–4377–8.

This volume is the Proceedings of the Curaçao Conference sponsored by the Caribbean Mathematics Foundation, which held from 26^{th} of June through 30^{th} of June, 1995. The book contains a blend of expository material and research papers which provides a sample of recent advances on the themes addressed by the conference. The discourse in ordered algebra has been significantly affected (in the last period) by other disciplines, and this volume is representative of that trend. It contains: 1. A Survey of f–Rings and some of their Generalizations, by M. Henriksen; 2. Reflections on the Global Dimensions of Rings of Continuous Functions, by R.T. Finn; 3. Continuous Ultraproducts, by N.L. Alling; 4. Actions on Archimedian Lattice–Ordered Groups with Strong Unit, by R.N. Ball & J.N. Hagler; 5. Pointfree Topology and the Spectra of f–Rings, by B. Banaschewski; 6. Commutative Singular f–Rings, by R.T. Finn, J. Martinez & W.W. McGovern; 7. Torsion Classes of Lattice–Ordered Groups with Invariant Radicals, by P.F. Conrad & M.R. Darnel; 8. The Automorphism Group of Hahn Groups, by M. Droste & R. Göbel; 9. The Laterally σ–Complete Reflection of an Archimedian Lattice–Ordered Group, by A.W. Hager & J. Martinez; 10. A κ–Kochen–Ershov Principles for Valued and Ordered Vector Spaces, by F.-V. & S. Kuhlmann; 11. Quasi–normal f–Rings, by S. Larson; 12. Rings of Continuous Functions as Real Closed Rings, by N. Schwartz; 13. on Universal Words for Automorphism Group of Linear Orders, by L.V. Welch & S.A. Adeleke. This book will be of great interest to mathematicians whose work includes ordered algebraic structures, topology of rings of functions or general mathematical systems.

I. Tofan

VAN GLABBEEK, R.J.H., *Comparative concurrency semantics and refinement of actions,* CWI Tract no. 109, Amsterdam, 1996, 285p., ISBN 90–6196–454–7.

This book is about comparative concurrency semantics, and it consists

of seven chapters which are all based on separate papers. All chapters can be read independently, although for motivation it may be helpful to read the introduction. Chapter I gives an overview of the literature on semantic equivalences for concrete sequential processes, motivated in terms of the observable behaviour of systems, and according to some testing scenario. In the second chapter it is shown how semantic notions can be used in protocol verification and other applications. This chapter is entirely algebraic in style and uses various axiom systems. The third chapter is devoted to the generalization of bisimulation equivalence to a setting with internal moves. In the fourth chapter an operator for the refinement of actions is defined on four causality based models for concurrent systems. The last three chapters consider various bisimulation equivalences based on partial orders and interleaving, and show that some of them are preserved by refinement whereas the others are not. The book is of interest for all those working in concurrency – an area of research usually located in computer science. However, much theory in the field of concurrency applies equally well to many other systems.

Gabriel Ciobanu

BAYLIS, W.E. (Editor), *Clifford (Geometric) Algebras. With Applications in Physics, Mathematics, and Engineering,* Basel, Birkhäuser Verlag 1996, 536p., ISBN 3–7643–3868–7.

Cartea de faţă reprezintă o iniţiativă de mare valoare şi este foarte bine venită deoarece interesul pentru aceste algebre şi mai ales pentru numeroasele lor aplicaţii este foarte mare. Acest interes a făcut ca cercetători din variate domenii să solicite cu insistenţă un cadru organizat de studiu şi informare în acest domeniu. Acesta a fost motivul pentru care Canadian Association of Physicists a organizat în anul 1995 o Şcoală de vară de fizică teoretică. Expunerile făcute cu această ocazie de specialişti dintre cei mai renumiţi au fost incluse în acest volum. În carte vom găsi prezentate, într-o formă foarte accesibilă, atât elementele teoretice matematice în legătură cu algebrele Clifford cât şi principalele direcţii actuale de aplicare a acestora în matematică, fizică teoretică şi inginerie. Cei interesaţi au la îndemână un manual excelent prin conţinut şi formă care oferă posibilitatea familiarizării cu acet important aparat matematic şi mai ales cu perspectivele utilizării lui în cercetările viitoare. Cititorul este condus cu multă măiestrie pe întinsul celor 33 capitole de la elemente de istorie a apariţiei şi evoluţiei cercetărilor acestor algebre până la problemele actuale de fizică teoretică sau probleme de inginerie în care intervin acestea. Pentru a ne face o imagine asupra

diversităţii problematicii abordate citătm câteva titluri de capitole: Algebre clifford şi operatori spinoriali, Electromagnetism, Fizica electronului, Gravitate, Relativitate generală, Hipergravitate, Proprietăţi ale algebrelor Clifford pentru particule fundamentale ş.a. Apreciez cartea ca o reuşită deosebită atât din punct de vedere al conţinutului cât şi a realizării grafice şi o recomand cu căldură şi începătorilor şi celor avansaţi interesaţi de acest domeniu de frontieră între matematică, fizică şi inginerie.

Vasile Tamaş

DE KERF, E.A., PIJLS, H.G.J. (eds.), *Proceedings seminar 1989–1990, Mathematical structures in field theory.* CWI SYLLABUS 39/1996 Amsterdam Netherlands, 163p, ISBN 90–6196–448–2.

În ultima perioadă de timp au căpătat o amploare fără precedent preocupările pentru aplicaţiile unor structuri algebrice în cercetări de fizică teoretică. Astfel au apărut numeroase modele matematice bazate pe unele algebre liniare care oferă un cadru foarte potrivit cercetărilor teoretice. Cartea de faţă este o ilustrare foarte potrivită a acestor idei. Ea constituie o selecţie a unor expuneri făcute în cadrul seminarului "Mathematical structures in field theory" ţinut la Universitatea din Amsterdam în ultimii ani. Editorii au reuşit o excelentă alegere şi aranjare a materialului care a asigurat o surprinzătoare unitate a cărţii. Cele 6 capitole realizate de autori diferiţi constituie 6 etape de împletire ingenioasă a unor probleme dificile din fizică cu unele algebre obţinându-se modele matematice de mare utilitate. La început se face o foarte utilă iniţiere în precuantizare iar apoi este prezentată algebra Clifford asociată unui spaţiu Minkowski cunoscută sub numele de algebră space–time (STA). Autorul urmăreşte cu perseverenţă să pună la punct un model matematic cu care să se descrie ecuaţia lui Dirac din teoria particolelor. În continuare utilizându-se un model clasic se face o descriere a fenomenului quark. Compactificarea toroidală în spaţii Minkowski este abordată folosindu-se două algebre importante. Dată fiind importanţa algebrelor Hopf în situaţia de faţă s-a considerat necesară introducerea unui capitol special destinat lor şi unor importante cazuri particulare utile. În ultima parte sunt prezentate aplicaţii ale algebrelor Poisson, Jordan–Lie ş.a. Considerăm că lucrarea reprezintă o realizare de mare valoare şi un imbold pentru succese viitoare şi credem că este utilă atât începătorilor cât şi celor avansaţi în aplicarea metodelor algebrelor liniare în fizică.

Vasile Tamaş

NEITTAANMÄKI, P., TIBA, D., *Optimal Control of Nonlinear Parabolic Systems, Theory, Algorithms and Applications,* Marcel Dekker, Inc., New York, Basel, Hong Kong, 1994, XV+399 p, ISBN 0–8247–9081–2.

Nonlinear differential systems and, in particular, nonlinear evolution equations are adequate models for problems arising in many modern applications. The book introduces the reader to this important field of research and to some of its applications. The authors focus their attention on optimal control problems governed by nonlinear parabolic systems including semilinear equations, variational inequalities and systems with phase transitions. The aim is twofold: first the authors discuss a theoretial approach to the subject and then they present numerical methods (with convergence proofs or error estimates) necessary in computerizing optimal control processes. Several examples are worked out in detail in order to demonstrate the usefulness of the proposed methods. The study of this subject is at the interface of the following fields: nonlinear partial differential equations, nonlinear functional analysis, calculus of variations and optimal control, finite element method, nonlinear programming, nonsmooth optimization.

Chapter I beings with several motivating examples of nonlinear parabolic systems and control problems. Among others, applications in biochemistry, chemistry, biology, multiphase Stefan–type problems, thermo–chemical flow phenomena, continuous casting of steel, crystal growth and sterilization of canned foods are given. In Chapter II some fundamental theoretical results for convenience in later references are given. Properties of monotone operators, generalized gradients, nonlinear semigroups and parabolic variational inequalities are briefly summarized. Chapter III is devoted to numerical methods for parabolic systems. The approach is based on the finite element method, the idea of which is first briefly described. The emphasis is on providing algorithms for solving nonlinear parabolic systems and, in particular, multiphase Stefan–like systems. In Chapter IV the authors deal with optimal control problems of systems governed by evolution differential inclusions. The existence of optimal pairs as well as optimality conditions are proved. A regularization technique is presented and the relation between the solutions to the original and the regularized problems is studied. The authors concretize the abstract results by applying them to certain Stefan–type optimal control problems, proving controllability results for the free boundary, and deriving special properties of the optimal control. Chapter V presents numerical methods for optimal control problems. The study begins with the distributed control of linear parabolic systems with the convex cost functional. Estimates of the distance between the optimal controls of the continuous and discrete optimization problem are inferred as

in the approximation of the state equation. In Chapter VI state constrained optimal control problems are dealt with. First the optimality conditions are presented. Section 2 evidences the fact that there is a strong relationship between state constrained control problems and problems governed by variational inequalities. This gives a new approximation method for state constrained control problems. The admissibility question is also discussed. In many applications it is an important and difficult task to find an admissible pair (or at least a pair which is approximately admissible in a sense to be made precise), when state and control constraints are given. The last section presents a different and very recent approach in state constrained control problems. It gives a new insight both into the proof of optimality conditions and into numerical algorithms.

The reader interested only in nonlinear parabolic equations and their numerical solution may confine himself to the first three chapters. For the related control problems, Chapter IV and V are to be recommended, while Chapter VI contains more advanced topics. The book is intended for postgraduate students and researchers interested in nonlinear evolution systems, optimal control theory, numerical analysis, optimization, and applications in science and engineering.

G. Dimitriu

DOLD, A., ECKMANN, B., TAKENS, F. (editors) *Optimal Control of Nonsmooth Distributed Parameter Systems,* Lecture Notes in Mathematics 1459, Springer Verlag, Berlin, Heidelberg, New York, London, Paris, Tokyo, Hong Kong, Barcelona, 1990, V+159p. ISBN 3–540–53524–1.

The present work may be inscribed as a contribution to the general effort of research in nonsmooth optimization problems associated with nonlinear partial differential equations. More precisely, the main aim of these notes is to examine distributed control problems governed by nonlinear evolution equations (parabolic or hyperbolic), in the absence of differentiability properties. In this setting, a special emphasis is given to nonlinear hyperbolic problems which are less discussed in the literature.

The material of the book is divided into three parts, after the type of the nonlinear term which occurs in the state system: semilinear and quasilinear problems, variational inequalities, free boundary problems. Among the different topics underlined throughout the work, the author refers to: existence of optimal pairs, first order necessary conditions, general and efficient approximation procedures. Each time when this is possible applications to regularity or bang–bang results for the optimal control are indicated. Exam-

ples clarifying and motivating the theory are included in every chapter. The introductory chapter contains some prerequisites in the theory of monotone operators, convex analysis, generalized gradients, Sobolev spaces and nonlinear differential equations. Chapter II deals with optimal control problems governed by partial differential equations containing nonlinear terms which are locally Lipschitzian. These problems are neither convex, nor smooth. The general procedure to obtain the optimality conditions is the adapted penalization method. It consists in the approximation of the given problem by a family of smooth optimization problems and it allows the characterization of all the optimal pairs. The results presented here play an important role in the next chapters, in the study of optimal control problems with state equation involving unbounded nonlinear operators: variational inequalities, free boundary problems. Section 3 of the chapter III presents the difficult problem of the optimal control for the vibrating string with obstacle, in the setting of singular control systems. Section 4 is devoted to the analysis of the relationship between the control problems governed by variational inequalities and the problems with state constraints, which leads to efficient approximation methods and to bang–bang results for the optimal control. The large area of aplication of this approach is justified in the next section by the investigation of an optimal design problem with unilateral conditions and state constraints. In the last section of the chapter III is examined an optimal control problem governed by a semilinear elliptic variational inequality. One of the main motivations of the interest for the theory of variational inequalities is given by their intimate relationship with the free boundary problems. These are partial differential equations with boundary and initial conditions, on a domain with an unknown moving part of the boundary and having to be found together with the solution. Chapter IV is devoted exclusively to the study of the optimal control problems governed by two–phase Stefan problems. The choice is also motivated by the special interest for applications of this type of problem.

The book is based mainly on the results obtained by the author during several years, but it also gives a survey of the existing literature in this area of research, via numerous comments, references and comparisons.

G. Dimitriu

DODSON, C.T.J., PARKER, P.E., *A User's Guide to Algebraic Topology,* Kluwer Academic Publishers, 1997, Mathematics and Its Applications, 387, xii, 405 pp.

Aşa cum arată şi titlul, cartea este un ghid la îndemîna celor interesaţi

în a afla elemente despre diverse noţiuni şi rezultate din topologia algebrică. Unele teoreme au şi demonstraţii, astfel că anumite porţiuni pot fi folosite şi în lecţiile de topologie algebrică la facultăţile de matematică. Principalele subiecte abordate sunt din: teoria homotopiei, omologiei şi coomologiei, teoria fasciculelor, teorii spectrale, spaţii fibrate, teoria obstrucţiilor. Prezentarea este destul de amănunţită pe alocuri şi se bazează atât pe aspecte intuitive cât şi pe construcţii axiomatice. După o succintă trecere în revista a problemelor extensiei şi ridicării, se introduce homotopia, homotopia relativă şi se dau aplicaţii în teoria CW-complexelor. In continuare, sunt prezentate problemele generale ale teoriilor de omologie şi de coomologie şi se tratează aspecte concrete în cazurile omologiei singulare cubice şi simpliciale, coomologiei de Rham, coomologiei Cech. După o scurtă prezentare a fasciculelor, sunt date câteva rezultate legate de şiruri spectrale. In ce priveşte teoria fibratelor, se prezintă generalităţi legate de definiţia unui fibrat local trivial, fibrate induse şi rezultate legate de problema clasificării fibrărilor. Sunt date descrieri pentru teoria obstrucţiilor, spaţiile Eilenberg–MacLane, descompunerea Moore–Postnikov, cofunctorii de homotopie, invarianţii Postnikov. Sunt prezentate aplicaţii în legătură cu posibilitatea rezolvării unor probleme de existenţa şi nonexistenţa din domeniile algebrei, analizei matematice, topologiei generale şi diferenţiale, geometriei diferenţiale, topologiei liniare pe porţiuni. Cele patru appendixuri se ocupă de probleme din algebra (categorii şi functori, grupuri), topologie (separare, spaţii compacte, paracompacte, conexe etc.), varietăţi şi spaţii fibrate (calcul diferenţial pe varietăţi, fibrate, metrici riemanniene, conexiuni), tabele de grupuri de homotopie pentru sfere, grupuri clasice reale şi complexe, grupuri Lie simple exceptionale, varietăţi Stiefel reale. Este o carte utilă pentru cercetătorii din domeniu.

V. Oproiu

DUGGAL, K.L., BEJANCU, A., *Lightlike Submanifolds of Semi–Riemannian Manifolds and Applications,* Kluwer Academic Publishers Group, 1996, viii, 300 pp. Mathematics and Its Applications, 364, USD 158.

Teoria subvarietăţilor (în particular, cea a hipersuprafeţelor) în spaţii Riemann constituie o generalizare naturală a teoriei suprafeţelor din spaţiul euclidian 3-dimensional. Multe din proprietăţile suprafeţelor, studiate chiar cu circa două secole în urmă, se regăsesc în cazul subvarietăţilor în spaţii Riemann. In cazul când spaţiul ambiant este numai semiriemannian (pseudoriemannian) sau riemannian degenerat, lucrurile se complică, având în

vedere că ceea ce se induce pe subvarietate poate să nu mai fie o metrică riemannianã sau semiriemannianã, ci se poate ajunge la o degenerare şi că ordinul de degenerare poate să varieze odată cu punctul curent. Există cazul subvarietăţilor de tip spaţial (în special cazul hipersuprafeţelor de tip spaţial în varietăţi Lorentz) când metrica indusă este pozitiv definită şi al celor de tip temporal (în special în cazul curbelor în varietăţi Lorentz) când metrica indusă este negativ definită. Acestea sunt cazuri generice, în sensul că deformări mici ale subvarietăţilor nu le deteriorează calităţile în discuţie. Cazul când metrica indusă este degenerată ne furnizează subvarietăţile de tip luminos (termenul este luat din teoria relativităţii unde, pentru semnale luminoase, pătratul intervalului spaţio–temporal este nul). Evident, subvarietăţile de tip luminos sunt nişte cazuri singulare de subvarietăţi, prin urmare nu au o natură generică. Exemple de asemenea subvarietăţi sunt greu de obţinut, iar, în teoria relativităţii generale, când gravitaţia variază în timp, anumite subvarietăţi pot fi de tip luminos doar într-un moment, în momentul imediat următor putând să-şi schimbe calitatea. In cartea de faţă autorii au încercat să adapteze anumite metode din studiul subvarietăţilor (nedegenerate) în spaţii Riemann (în spaţii semiriemanniene) la cazul celor de tip luminos. După două capitole conţinând noţiuni şi rezultate preliminare din teoria spaţiilor semieuclidiene, a varietăţilor diferenţiabile, în general, şi a varietăţilor semiriemanniene, în special, se trece la studiul geometriei curbelor nule în spaţii Lorentz, obţinându-se formule de tip Frenet. In continuare, sunt studiate hipersuprafeţele şi subvarietăţile de tip luminos în spaţii semiriemanniene, depăşindu-se unele dificultăţi cu ajutorul unor distribuţii ecran. În capitolele 6 şi 7 sunt studiate subvarietăţile de tip luminos în varietăţi Kaehler indefinite şi varietăţi Lorentz reperate. În acest context sunt cercetate proprietăţile eventualelor CR-structuri induse. În ultimele două capitole sunt stabilite unele conexiuni între hipersuprafeţele de tip luminos şi electromagnetism (în contextul relativităţii speciale) şi gravitaţie (în contextul relativităţii generale).

V. Oproiu

ROSENFELD, B., *Geometry of Lie Groups,* Kluwer Academic Publishers, 1997, Mathematics and its Applications, vol. 393, xviii, 293 pp.

In carte sunt prezentate numeroase aspecte legate de geometriile subordonate grupurilor Lie simple din clasele A_n, B_n, C_n, D_n ca şi grupurilor Lie excepţionale G_2, F_4, E_6, E_7, E_8. Intr-o primă fază sunt expuse noţiunile şi rezultatele fundamentale din cadrul fiecărei geometrii, apoi, sunt date indicaţii relative la dezvoltarea geometriei diferenţiale corespunzătoare. Car-

tea este bazată pe conţinutul altor două cărţi ale autorului, apărute în limba rusă: Spaţii multidimensionale, Nauka, Moscova, 1966 şi Spaţii neeuclidiene, Nauka, Moscova, 1969, un manuscris în rusă, nepublicat pâna în prezent, unele rezultate ale investigaţiilor ştiinţifice ale autorului şi pe experienţa didactică a autorului la numeroase universităţi din fosta Uniune Sovietică şi Pennsylvania State University, SUA.

După un capitol introductiv despre structuri algebrice şi topologice, varietăţi diferenţiabile, geometrie riemanniană, grupuri topologice şi Lie şi spaţii omogene şi simetrice, se trece la prezentarea aspectelor fundamentale din teoria algebrelor şi a grupurilor Lie. Sunt studiate algebrele asociative comutative, algebrele asociative necomutative, algebrele alternative, algebrele Lie şi grupurile Lie, algebrele Jordan şi elastice şi reprezentările grupurilor Lie simple. In capitolul al II-lea sunt prezentate mai multe aspecte din geometriile afină şi proiectivă, cum ar fi: spaţii afine şi proiective peste algebre şi interpretările reale şi complexe ale acestora, transformări afine şi proiective, drepte, m-plane şi hiperplane, hipercuadrice, complexe liniare, configuraţii proiective, geometrii finite. În continuare, în capitolele III, IV şi V sunt discutate diverse aspecte din geometriile euclidiană, pseudoeuclidiană, conformă, pseudoconformă, eliptică, hiperbolică, pseudoeliptică, pseudohiperbolică, quasieliptică, quasihiperbolică şi quasieuclidiană: grupurile de mişcări şi similitudini, m-plane, poliedre, hipercuadrice, hipersfere, trigonometriile aferente, formele spaţiale, aplicaţii în fizică. Probleme asemănătoare sunt discutate pentru geometriile simplectică şi quasisimplectică precum şi pentru geometriile subordonate grupurilor Lie excepţionale G_2, F_4, E_6, E_7 şi E_8. Cercetând grupurile necompacte din clasele F_4, E_6, E_7, E_8 sunt obţinute proprietăţi ale geometriilor metasimplectice. Avem de a face cu o carte de referinţă unde cititorul interesat poate găsi numeroase aspecte şi rezultate din geometriile cu grupul fundamental de transformări, unul din grupurile enumerate mai sus şi poate constitui o sursă de informaţii pentru alte investigaţii.

V. Oproiu

CHRISS, N., GINZBURG, V., *Representation Theory and Complex Geometry,* Birkhäuser, 1997, x, 495 pp.

Cartea de faţă a fost scrisă pe baza lecţiilor ţinute de al doilea autor la Universitatea din Chicago din 1993. Dorinţa autorilor a fost să ofere unor cititori interesaţi o cale cât mai directă către teoria reprezentărilor, evitînd expunerea detaliată a unor fundamentări teoretice care, prin lungime şi dificultate pot să descurajeze. După ce cititorul se convinge de frumuseţea şi

utilitatea teoriei, poate să abordeze cu mai multă hotărâre problemele fundamentale expuse în alte materiale, pentru completarea cunoştinţelor necesare în abordarea riguroasă a problematicii reprezentărilor. Primele două capitole sunt introductive şi tratează argumente legate de geometria simplectică (varietăţi simplectice, algebre Poisson, structuri Poisson rezultate din algebre necomutative, aplicaţia moment, varietăţi coizotrope, familii lagrangiene) şi un mozaic din geometria şi topologia algebrică relativ la: Nullstellensatz a lui Hilbert, varietăţi afine, C^*-acţiuni pe varietăţi proiective, omologie Borel–Moore şi convoluţia în aceasta. In continuare sunt prezentate aspecte şi rezultate din teoria grupurilor complexe semisimple şi teoria Springer pentru $U(sl_n)$. Din domeniul K-teoriei echivariante sunt discutate probleme legate de: rezoluţii echivariante, specializarea în K-teoria echivariantă, complexul Koszul şi izomorfismul Thom, formula Küneth, caracterul Chern, varietăţi de steaguri, varietăţi Steinberg, polinoame armonice, varietăţi orbitale, teorema lui Kostant asupra inelelor polinomiale. In legătură cu algebrele Hecke şi K-teoria sunt discutate grupurile Weyl afine şi algebrele Hecke, polinoamele Hilbert şi varietăţile orbitale, algebra Hecke pentru SL_2. In ultimul capitol sunt prezentate probleme din teoria reprezentărilor algebrelor de convoluţie. Este o carte utilă pentru iniţierea în domeniul teoriei reprezentărilor.

V. Oproiu

JARDINE, J.F., *Generalized Etale Cohomology Theories,* Progress in Mathematics vol.146, Birkhäuser Verlag, Basel. Boston. Berlin, 1997, X+318p, Index. ISBN 3–7643–5494–1, 0–8176–5494–1.

Topologia etală este cel mai important exemplu de topologie Grothendieck în raport cu care se pot defini invarianţi omologici şi omotopici pentru varietăţi algebrice şi scheme. O coomologie etală este o coomologie a fasciculelor în topologia etală. O asemenea coomologie se defineşte cu ajutorul functorilor derivaţi. O teorie de coomologie etală generalizată este o teorie care este reprezentată printr-un prefascicul de spectre pe un sit etal pentru o varietate algebrică. Exemplele mai importante sunt coomologia etală şi K–teoria.

Cartea de faţă este o monografie cu caracter tehnic, axată pe o demonstraţie completă şi originală a teoremei de descendenţă a coomologiei etale pentru K–teoria Bott, datorată lui Thomason. Este vorba de un rezultat major din K-teorie, stabilit de către R.W. Thomason [*Algebraic K-theory and étale cohomology*, Ann. Scient. Ec. Norm. Sup., 4^e série 18(1985), 437–552]. Demonstraţia are un caracter combinatoric şi deci algebric şi

ocazionează o serie de idei majore asupra teoriei omotopiei prefasciculelor de spectre pe situri etale. Demonstraţia dată de către autorul cărţii teoremei lui Thomason permite stabilirea legăturii acesteia cu conjectura Lichtenbaum–Quillen asupra grupurilor de omotopie stabilă. Cele şapte capitole ale cărţii se referă, pe lângă obiectivul său principal, la: smash produse de spectre şi prefascicule, fibrări şi cofibrări, K-teoria prefasciculelor, coomologie ordinară Galois şi coomologie Čech generalizată şi periodicitatea Bott în K-teorie.

Cartea prezintă interes pentru specialişti în K-teoria algebrică.

I.Pop