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NON–WEAKLY SUSLINIAN EXAMPLE*

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Abstract. Yonping Luo posed a problem if a locally connected continuum which can be written as an uncountable disjoint union of proper subcontinua only finitely many of which are degenerate, is weakly Suslinian. We give the negative answer in Example.

Weakly Suslinian continua. A *continuum* means a nonempty compact connected metric space. A continuum X is said to be *weakly Suslinian* provided that X is not the union of more than one pairwise disjoint nondegenerate subcontinua of X. YONPING LUO in [1] posed a problem:

Problem. *If a locally connected continuum X can be written as an uncountable disjoint union of proper subcontinua only finitely many of which are degenerate, is X weakly Suslinian*?

We give the negative answer:

Example. *Let a cardinal λ, $1 \leq \lambda \leq \aleph_0$ be given. Then there is a non–weakly Suslinian continuum X which can be written as an uncountable disjoint union of proper subcontinua, exactly λ of which are degenerate.*

Proof. For $x \in [0,1]$ denote $d(x) = \operatorname{dist}(x, D)$, where $D = \{0\} \cup \cup\{2^{-n}\}_{n=0}^{\infty}$. Clearly d is nonnegative and continuous on $[0,1]$ vanishing just on D. We denote M the unit square in $\mathbb{R}^2$ and $Y = \{(x,y) \in M : d(x) \leq \leq y \leq 1\}$. If $\lambda = \aleph_0$ we set $X = Y$, and for finite λ we set

$$X = Y \cap \{(x,y) \in M : x \geq 3 \cdot 2^{-\lambda-1}\}.$$

Let X inherit the Euclidean topology from the plane. Clearly X is a locally connected continuum. For $t \in [0,1]$ we denote $V_t = \{(x,y) \in M : x = t\}$ the

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vertical segments in M. We see that $X = \bigcup_{0\le x\le 1} (X\cap V_x)$ is the decomposition of X into nondegenerate subcontinua which shows that X is not weakly Suslinian. For $t \in [0,1]$ we denote $H_t = \{(x,y) \in M : y = t\}$ the horizontal segments in M. Now $X = \bigcup_{0\le x\le 1} (X \cap H_y)$ is the decomposition of X into subcontinua where the only degenerate subcontinua we obtain for $y = 0$. The cardinality of the set of such degenerate subcontinua equals λ. See Figure. ∎

Figure (Example)

We can naturally ask the following:

Question. *Does there exists a weakly Suslinian, locally connected continuum, which can be written as an uncountable disjoint union of proper subcontinua only finitely many of which are degenerate*?

REFERENCES

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