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SOME THEOREMS IN THE BENDING THEORY OF POROUS THERMOELASTIC PLATES

BY

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Abstract. In the context of the linear theory of thermoelastic materials with voids, we study the dynamic bending of Mindlin type plates. We formulate the corresponding boundary–initial–value problem and establish a reciprocal theorem. Then we present two variational theorems of Gurtin type.

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1. Introduction. The Nunziato–Cowin theory for the treatment of porous elastic solids in which the interstices (or pores) are void of material was established in [1,2]. In this theory, the bulk mass density is written as the product of two fields, the matrix material density field and the volume fraction field. Capriz and Podio–Guidugli [3] remarked that the theory proposed by Nunziato and Cowin can be regarded as a particular case of the theory of materials with microstructure. The linear theory of thermoelastic materials with voids was investigated by Ieşan [4]. Due to its intended applications in the mechanics of granular materials and manufactured porous bodies, the theory of elastic solids with voids has been intensively studied in the last two decades.

The present paper is concerned with the bending of thermoelastic thin plates made from an isotropic and homogeneous material with voids. We employ a plate theory of Mindlin type, taking into account the transverse shear deformation in the flexural motions of plates. In [5] Mindlin introduced a plate model with transverse shear deformation. Since then, a large number of papers have been devoted to the study of the bending theory for Mindlin type plates. Several articles have investigated the behaviour of plates made from materials with microstructure (see, e.g., [6]). The theory of plates is presented in detail in the work of NAGHDI [7].

The bending theory of thermoelastic plates made from a material with voids was derived by BÎRSAN [8]. Some results concerning the study of porous plates and shells were presented in [9,10].

In the first part of the present article we remind the fundamental equations for the bending of thermoelastic porous plates established in [8]. Then, the corresponding boundary–initial–value problem is expressed in a form in which the initial conditions are incorporated into the field equations. Using this formulation of the problem, a reciprocal theorem is derived. In the final section we prove two theorems which provide a variational characterization of the solution of the boundary–initial–value problem.

2. Basic equations. In this section we summarize the fundamental equations that govern the bending of porous thermoelastic plates of Mindlin type. For a detailed derivation of these equations we refer to [8].

We denote by B the interior of a right cylinder of length h_0 with open cross-section Σ and smooth lateral boundary Π. Let $\mathcal{C}$ be the boundary of Σ. We assume that the region B is occupied by a linearly thermoelastic material with voids. The length h_0 is supposed to be small enough so that the body represents a thin plate of uniform thickness h_0. The rectangular Cartesian coordinate frame is chosen such that x_1Ox_2 is the middle plane of the plate.

Let $\boldsymbol{u}$ be the displacement vector field, φ the change in volume fraction field and θ the temperature field measured from the constant absolute temperature T_0 of the reference state. Following [5,6,7], we study the states of bending characterized by

$$
\begin{aligned}
&u_\alpha = x_3 v_\alpha(x_1,x_2,t), \quad u_3 = w(x_1,x_2,t),\\
&\varphi = x_3\psi(x_1,x_2,t), \qquad \theta = x_3 T(x_1,x_2,t) \quad \text{on } B\times\mathcal{T},
\end{aligned}
\tag{1}
$$

where $\mathcal{T} = [0,t_1)$ is a given interval of time (which may also be $[0,\infty)$). Throughout this paper, Latin subscripts range over the integers $1,2,3$, whereas Greek subscripts are confined to the range $1,2$. The usual summation and differentiation conventions are also employed.

We say that (v_α, w, ψ, T) is an *admissible process* on $\Sigma\times\mathcal{T}$ provided: (i) v_α, w and ψ are of class C^2 on $\Sigma\times\mathcal{T}$; (ii) T is of class $C^{2,1}$ on $\Sigma\times\mathcal{T}$;

(iii) v_α, w, ψ and T are of class C^1 on $\overline{\Sigma}\times\mathcal{T}$.
Let us denote by $\mathcal{D}$ the set of all admissible processes.

The linear strain measures are

$$\varepsilon_{\alpha\beta} = \frac{1}{2}(v_{\alpha,\beta} + v_{\beta,\alpha}), \quad \gamma_\alpha = v_\alpha + w_{,\alpha}. \tag{2}$$

The quantities γ_α represent the angles of rotation of the cross–sections x_α = constant about the middle surface (see, e.g., [11]).

For thermoelastic porous plates made from an isotropic and homogeneous material, we have the following constitutive equations

$$\begin{aligned}
&M_{\alpha\nu} = I[\lambda\varepsilon_{\rho\rho}\delta_{\alpha\nu} + 2\mu\varepsilon_{\alpha\nu} + b\psi\delta_{\alpha\nu} - \beta T\delta_{\alpha\nu}],\\
&N_\alpha = \mu\gamma_\alpha, \quad H_\nu = \alpha I\psi_{,\nu},\\
&G = I(-b\varepsilon_{\rho\rho} - \xi\psi + mT), \quad \sigma = I(\beta\varepsilon_{\rho\rho} + m\psi + aT),\\
&Q_\alpha = kIT_{,\alpha}, \quad \Gamma = \alpha\psi, \quad R = kT,
\end{aligned} \tag{3}$$

where $\lambda, \mu, b, \beta, \alpha, \xi, m, a$ and k are constitutive constants and $I = h_0^3/12$. For the significance of the functions $M_{\alpha\nu}, N_\alpha, H_\nu, G, \sigma, Q_\alpha, \Gamma$ and R which appear in relations (3), we refer to [8]. By virtue of the second law of thermodynamics, the coefficient k is nonnegative. We denote by ρ the reference mass density.

The balance equations for the bending of porous plates are

$$\begin{aligned}
&M_{\beta\alpha,\beta} - h_0 N_\alpha + f_\alpha = \rho I\ddot{v}_\alpha,\\
&N_{\alpha,\alpha} + f = \rho\ddot{w},\\
&H_{\alpha,\alpha} + G - h_0\Gamma + \ell = \rho\kappa I\ddot{\psi},\\
&Q_{\alpha,\alpha} - h_0 R + S = T_0\dot{\sigma},
\end{aligned} \tag{4}$$

where f_α, f, ℓ and S are prescribed functions representing resultant external body loads and heat supply (see [8]).

We consider the following mixed boundary conditions

$$\begin{aligned}
&v_\alpha = \widetilde{v}_\alpha \text{ on } \overline{\mathcal{C}}_1\times\mathcal{T}, \quad M_{\beta\alpha}n_\beta = \widetilde{M}_\alpha \text{ on } \mathcal{C}_2\times\mathcal{T},\\
&w = \widetilde{w} \text{ on } \overline{\mathcal{C}}_3\times\mathcal{T}, \quad N_\alpha n_\alpha = \widetilde{N} \text{ on } \mathcal{C}_4\times\mathcal{T},\\
&\psi = \widetilde{\psi} \text{ on } \overline{\mathcal{C}}_5\times\mathcal{T}, \quad H_\nu n_\nu = \widetilde{H} \text{ on } \mathcal{C}_6\times\mathcal{T},\\
&T = \widetilde{T} \text{ on } \overline{\mathcal{C}}_7\times\mathcal{T}, \quad Q_\alpha n_\alpha = \widetilde{Q} \text{ on } \mathcal{C}_8\times\mathcal{T},
\end{aligned} \tag{5}$$

where $\boldsymbol{n}$ designate the outward unit normal of $\partial\Sigma$ and $\mathcal{C}_i$ $(i = 1, 2, .., 8)$ are subsets of $\partial\Sigma$ such that $\overline{\mathcal{C}}_1 \cup \mathcal{C}_2 = \overline{\mathcal{C}}_3 \cup \mathcal{C}_4 = \overline{\mathcal{C}}_5 \cup \mathcal{C}_6 = \overline{\mathcal{C}}_7 \cup \mathcal{C}_8 = \mathcal{C}$ and $\mathcal{C}_1 \cap \mathcal{C}_2 = \mathcal{C}_3 \cap \mathcal{C}_4 = \mathcal{C}_5 \cap \mathcal{C}_6 = \mathcal{C}_7 \cap \mathcal{C}_8 = \emptyset$.
We assume that the prescribed functions $\widetilde{v}_\alpha, \widetilde{w}, \widetilde{\psi}$ and $\widetilde{T}$ are continuous on their domains of definition, whereas the given functions $\widetilde{M_\alpha}, \widetilde{N}, \widetilde{H}$ and $\widetilde{Q}$ are piecewise regular and continuous in time.

The initial conditions are

$$
\begin{aligned}
&v_\alpha(x_\gamma, 0) = v^0_\alpha(x_\gamma), \quad \dot{v}_\alpha(x_\gamma, 0) = v^1_\alpha(x_\gamma),\\
&w(x_\gamma, 0) = w^0(x_\gamma), \quad \dot{w}(x_\gamma, 0) = w^1(x_\gamma),\\
&\psi(x_\gamma, 0) = \psi^0(x_\gamma), \quad \dot{\psi}(x_\gamma, 0) = \psi^1(x_\gamma),\\
&\sigma(x_\gamma, 0) = \sigma^0(x_\gamma), \quad (x_\gamma) \in \Sigma,
\end{aligned} \tag{6}
$$

where the functions appearing in the right–hand sides are given continuous functions.

The boundary–initial–value problem associated with the bending of thermoelastic porous plates consists of equations (2)–(6). We mention that the field equations (2)–(4) can be expressed in terms of the functions v_α, w, ψ and T in the following form

$$
\begin{aligned}
&I[\mu\Delta v_\alpha + (\lambda + \mu)v_{\nu,\nu\alpha} + b\psi_{,\alpha} - \beta T_{,\alpha}] - \mu h_0(v_\alpha + w_{,\alpha}) + f_\alpha = \rho I \ddot{v}_\alpha,\\
&\mu\Delta w + \mu v_{\alpha,\alpha} + f = \rho\ddot{w},\\
&I[\alpha\Delta\psi - bv_{\nu,\nu} + mT] - (\xi I + \alpha h_0)\psi + \ell = \rho\kappa I\ddot{\psi},\\
&I[k\Delta T - \beta T_0\dot{v}_{\alpha,\alpha} - mT_0\dot{\psi}] - kh_0T + S = aIT_0\dot{T},
\end{aligned} \tag{7}
$$

where Δ is the Laplace operator.

In [8] we have studied the solutions of the above boundary–initial–value problem and we have proved some uniqueness and continuous dependence results.

3. Reciprocity. In this section we derive a reciprocal theorem using the method presented in [12]. For two arbitrary scalar fields u and v on $\Sigma \times \mathcal{T}$ that are continuous in time, we denote by $u * v$ the convolution of u and v

$$[u * v](x, t) = \int_0^t u(x, t - \tau)v(x, \tau)d\tau$$

and by $\bar{u}$ the function defined on $\Sigma \times \mathcal{T}$ by

$$\bar{u}(x,t) = \int_0^t u(x,\tau) d\tau.$$

We introduce the functions

$$e(t) = 1,\ j(t) = t,\ t \in \mathcal{T}.$$

Taking the convolution of equations $(4)_{1,2,3}$ with j and of equation $(4)_4$ with e, we find

$$\begin{aligned} & j * (M_{\beta\alpha,\beta} - h_0 N_\alpha) + F_\alpha = \rho I v_\alpha, \\ & j * N_{\alpha,\alpha} + F = \rho w, \\ & j * (H_{\beta,\beta} + G - h_0 \Gamma) + L = \rho \kappa I \psi, \\ & e * (Q_{\alpha,\alpha} - h_0 R) + W = T_0 \sigma, \end{aligned} \tag{8}$$

where the functions F_α, F, L and W are defined by

$$\begin{aligned} & F_\alpha = j * f_\alpha + \rho I (t v_\alpha^1 + v_\alpha^0), \quad F = j * f + \rho (t w^1 + w^0), \\ & L = j * \ell + \rho \kappa I (t \psi^1 + \psi^0), \quad W = e * S + T_0 \sigma^0. \end{aligned} \tag{9}$$

We remark that equations (8) are equivalent to the balance equations (4) together with the initial conditions (6).

Let us consider two external data systems $\mathcal{L}^{(\alpha)} = \{ f_\gamma^{(\alpha)}, f^{(\alpha)}, \ell^{(\alpha)}, S^{(\alpha)}, \widetilde{v}_\gamma^{(\alpha)}, \widetilde{M}_\gamma^{(\alpha)}, \widetilde{w}^{(\alpha)}, \widetilde{N}^{(\alpha)}, \widetilde{\psi}^{(\alpha)}, \widetilde{H}^{(\alpha)}, \widetilde{T}^{(\alpha)}, \widetilde{Q}^{(\alpha)}, v_\gamma^{0(\alpha)}, v_\gamma^{1(\alpha)}, w^{0(\alpha)}, w^{1(\alpha)}, \psi^{0(\alpha)}, \psi^{1(\alpha)}, \sigma^{0(\alpha)} \}$ and let $p^{(\alpha)} = \{ v_\gamma^{(\alpha)}, w^{(\alpha)}, \psi^{(\alpha)}, T^{(\alpha)}, M_{\beta\gamma}^{(\alpha)}, N_\beta^{(\alpha)}, H_\beta^{(\alpha)}, G^{(\alpha)}, \sigma^{(\alpha)}, Q_\beta^{(\alpha)}, \Gamma^{(\alpha)}, R^{(\alpha)} \}$ be solutions of the boundary–initial–value problem corresponding to the data systems $\mathcal{L}^{(\alpha)}$ $(\alpha = 1,2)$, respectively. We denote by

$$M_\gamma^{(\alpha)} = M_{\beta\gamma}^{(\alpha)} n_\beta,\ N^{(\alpha)} = N_\beta^{(\alpha)} n_\beta,\ H^{(\alpha)} = H_\beta^{(\alpha)} n_\beta,\ Q^{(\alpha)} = Q_\beta^{(\alpha)} n_\beta$$

and let $F_\gamma^{(\alpha)}, F^{(\alpha)}, L^{(\alpha)}, W^{(\alpha)}$ be the quantities defined in (9) corresponding to the two data systems $\mathcal{L}^{(\alpha)}$ $(\alpha = 1,2)$.

We introduce the notation

$$\begin{aligned} E_{\alpha\beta}(r,s) = & M_{\gamma\sigma}^{(\alpha)}(r) \varepsilon_{\gamma\sigma}^{(\beta)}(s) + h_0 N_\sigma^{(\alpha)}(r) \gamma_\sigma^{(\beta)}(s) - G^{(\alpha)}(r) \psi^{(\beta)}(s) + \\ & + H_\gamma^{(\alpha)}(r) \psi_{,\gamma}^{(\beta)}(s) + h_0 \Gamma^{(\alpha)}(r) \psi^{(\beta)}(s) - \sigma^{(\alpha)}(r) T^{(\beta)}(s), \end{aligned} \tag{10}$$

for all $r, s \in \mathcal{T}$, and $\alpha, \beta \in \{1, 2\}$, where

$$\varepsilon_{\gamma\sigma}^{(\alpha)} = \frac{1}{2}(v_{\gamma,\sigma}^{(\alpha)} + v_{\sigma,\gamma}^{(\alpha)}),\ \gamma_{\sigma}^{(\alpha)} = v_{\sigma}^{(\alpha)} + w_{,\sigma}^{(\alpha)}. \tag{11}$$

From the constitutive equations (3) one can easily prove that

$$E_{\alpha\beta}(r, s) = E_{\beta\alpha}(s, r). \tag{12}$$

On the other hand, from the relations (10), (11) and the balance equations $(4)_{1,2,3}$ and $(8)_4$ we obtain

$$\begin{aligned}
&\int_{\Sigma} E_{\alpha\beta}(r, s)da = \int_{\mathcal{C}} [M_{\gamma}^{(\alpha)}(r)v_{\gamma}^{(\beta)}(s) + h_0 N^{(\alpha)}(r)w^{(\beta)}(s) + \\
&+ H^{(\alpha)}(r)\psi^{(\beta)}(s) - \frac{1}{T_0}\overline{Q}^{(\alpha)}(r)T^{(\beta)}(s)]dl + \int_{\Sigma}[f_{\gamma}^{(\alpha)}(r)v_{\gamma}^{(\beta)}(s) + \\
&+ h_0 f^{(\alpha)}(r)w^{(\beta)}(s) + \ell^{(\alpha)}(r)\psi^{(\beta)}(s) - \frac{1}{T_0}W^{(\alpha)}(r)T^{(\beta)}(s)]da - \\
&- \int_{\Sigma}\rho[I\ddot{v}_{\gamma}^{(\alpha)}(r)v_{\gamma}^{(\beta)}(s) + h_0\ddot{w}^{(\alpha)}(r)w^{(\beta)}(s) + \kappa I\ddot{\psi}^{(\alpha)}(r)\psi^{(\beta)}(s)]da + \\
&+ \frac{k}{T_0}\int_{\Sigma}[h_0\overline{T}^{(\alpha)}(r)T^{(\beta)}(s) + I\overline{T}_{,\gamma}^{(\alpha)}(r)T_{,\gamma}^{(\beta)}(s)]da.
\end{aligned} \tag{13}$$

If we put $r = \tau$ and $s = t - \tau$, relations (12) and (13) yield

$$\begin{aligned}
&\int_{\mathcal{C}}(M_{\gamma}^{(1)} * v_{\gamma}^{(2)} + h_0 N^{(1)} * w^{(2)} + H^{(1)} * \psi^{(2)} - e * \frac{1}{T_0}Q^{(1)} * T^{(2)})dl + \\
&+ \int_{\Sigma}[f_{\gamma}^{(1)} * v_{\gamma}^{(2)} + h_0 f^{(1)} * w^{(2)} + \ell^{(1)} * \psi^{(2)} - \frac{1}{T_0}W^{(1)} * T^{(2)}]da - \\
&- \int_{\Sigma}\rho[I\ddot{v}_{\gamma}^{(1)} * v_{\gamma}^{(2)} + h_0\ddot{w}^{(1)} * w^{(2)} + \kappa I\ddot{\psi}^{(1)} * \psi^{(2)}]da = \\
&= \int_{\mathcal{C}}(M_{\gamma}^{(2)} * v_{\gamma}^{(1)} + h_0 N^{(2)} * w^{(1)} + H^{(2)} * \psi^{(1)} - e * \frac{1}{T_0}Q^{(2)} * T^{(1)})dl + \\
&+ \int_{\Sigma}[f_{\gamma}^{(2)} * v_{\gamma}^{(1)} + h_0 f^{(2)} * w^{(1)} + \ell^{(2)} * \psi^{(1)} - \frac{1}{T_0}W^{(2)} * T^{(1)}]da - \\
&- \int_{\Sigma}\rho[I\ddot{v}_{\gamma}^{(2)} * v_{\gamma}^{(1)} + h_0\ddot{w}^{(2)} * w^{(1)} + \kappa I\ddot{\psi}^{(2)} * \psi^{(1)}]da.
\end{aligned} \tag{14}$$

Taking the convolution of equation (14) with j and making use of relations of the type

$$j * \ddot{w} = w(t) - (tw^1 + w^0),$$

we are led to the following reciprocal theorem.

Theorem 1. *Let* $p^{(\alpha)}$ *be a solution of the boundary–initial–value problem corresponding to the external data system* $\mathcal{L}^{(\alpha)}$ $(\alpha = 1, 2)$*. Then the following relation holds*

$$
\begin{aligned}
&\int_{\mathcal{C}} j*[M_\gamma^{(1)}*v_\gamma^{(2)} + h_0 N^{(1)}*w^{(2)} + H^{(1)}*\psi^{(2)} - e*\frac{1}{T_0}Q^{(1)}*T^{(2)}]dl+\\
&+\int_\Sigma [F_\gamma^{(1)}*v_\gamma^{(2)} + h_0 F^{(1)}*w^{(2)} + L^{(1)}*\psi^{(2)} - j*\frac{1}{T_0}W^{(1)}*T^{(2)}]da =\\
&=\int_{\mathcal{C}} j*[M_\gamma^{(2)}*v_\gamma^{(1)} + h_0 N^{(2)}*w^{(1)} + H^{(2)}*\psi^{(1)} - e*\frac{1}{T_0}Q^{(2)}*T^{(1)}]dl+\\
&+\int_\Sigma [F_\gamma^{(2)}*v_\gamma^{(1)} + h_0 F^{(2)}*w^{(1)} + L^{(2)}*\psi^{(1)} - j*\frac{1}{T_0}W^{(2)}*T^{(1)}]da.
\end{aligned}
\tag{15}
$$

4. Variational theorems. With a view toward establishing variational theorems of Gurtin type, we follow the method described in [13].

The equation (8) can be put in the form of a vector equation

$$AU = \phi, \tag{16}$$

where we have introduced the vectors

$$U = (v_1, v_2, w, \psi, T), \quad \phi = (F_1, F_2, h_0 F, L, -j*\frac{1}{T_0}W) \tag{17}$$

and the operator A defined on $\mathcal{D}$ by

(18)

$$
\begin{aligned}
&A = (A_1, A_2, A_3, A_4, A_5),\\
&A_\alpha U = \rho I v_\alpha - j*[\mu I \Delta v_\alpha + (\lambda+\mu) I v_{\gamma,\gamma\alpha} - \mu h_0 (v_\alpha + w_{,\alpha}) + bI\psi_{,\alpha} - \beta I T_{,\alpha}],\\
&A_3 U = \rho h_0 w - j*\mu h_0(\Delta w + v_{\alpha,\alpha}),\\
&A_4 U = \rho\kappa I\psi - j*[\alpha I\Delta\psi - bIv_{\gamma,\gamma} - (\xi I + \alpha h_0)\psi + mIT],\\
&A_5 U = j*[-I(\beta v_{\gamma,\gamma} + m\psi + aT) + e*\frac{k}{T_0}(I\Delta T - h_0 T)].
\end{aligned}
$$

In view of the equations (7), we see that U represents a solution of the boundary–initial–value problem if and only if the relation (16) holds and U satisfy the boundary conditions (5).

Let us consider first the case of homogeneous boundary conditions. For $U = (v_\alpha^{(1)}, w^{(1)}, \psi^{(1)}, T^{(1)})$ and $V = (v_\alpha^{(2)}, w^{(2)}, \psi^{(2)}, T^{(2)})$ satisfying the homogeneous boundary conditions corresponding to (5), the reciprocal theorem (15) yields

$$(AU \circledast V) = (U \circledast AV), \tag{19}$$

where we have denoted by $\circledast$ the convolution scalar product defined by

$$(u \circledast v) = \int_\Sigma u_i * v_i \; da,$$

for any vectors u, v continuous on $\Sigma \times \mathcal{T}$. Relation (19) means that the linear operator A is symmetric in convolution. Then we may apply a theorem presented in [13] which asserts that U is a solution of equation (16) if and only if

$$\delta \mathcal{F}(U) = 0,$$

where

$$\mathcal{F}(U) = \frac{1}{2}(AU \circledast U) - (U \circledast \phi). \tag{20}$$

Substituting (17) and (18) into (20) we find the expression of the functional $\mathcal{F}$ in our case

(21)
$$\begin{aligned}
\mathcal{F}(U)=&\frac{1}{2}\int_\Sigma j *[\lambda I v_{\alpha,\alpha} * v_{\beta,\beta} + 2\mu I \varepsilon_{\alpha\beta} * \varepsilon_{\alpha\beta} + \mu h_0 \gamma_\alpha * \gamma_\alpha + 2bI v_{\gamma,\gamma} * \psi -\\
&-2\beta I v_{\gamma,\gamma} * T + \alpha I \psi_{,\gamma} * \psi_{,\gamma} + (\xi I + \alpha h_0)\psi * \psi - 2mI\psi * T - aIT * T]da +\\
&+\frac{1}{2}\int_\Sigma [\rho(I v_\alpha * v_\alpha + h_0 w * w + \kappa I \psi * \psi) - j * e * \frac{k}{T_0}(h_0 T * T + IT_{,\alpha} * T_{,\alpha})]da -\\
&-\int_\Sigma (F_\alpha * v_\alpha + h_0 F * w + L * \psi - j * \frac{1}{T_0} W * T)da.
\end{aligned}$$

Thus, we have obtained the following result.

Theorem 2. *Let $\mathcal{M}$ be the set of all admissible processes $U=(v_\alpha, w, \psi, T)$ which satisfy homogeneous boundary conditions. Then*

$$\delta \mathcal{F}(U) = 0 \quad \text{on} \;\; \mathcal{M},$$

if and only if $U \in \mathcal{M}$ *satisfy the field equations* (7) *and the initial conditions* (6).

In the case of inhomogeneous boundary conditions (5),we define

$$\mathcal{G}(U) = \mathcal{F}(U) -$$
$$-j * \left\{ \int_{\mathcal{C}_2} \widetilde{M_\alpha} * v_\alpha \, dl + \int_{\mathcal{C}_4} h_0 \widetilde{N} * w \, dl + \int_{\mathcal{C}_6} \widetilde{H} * \psi \, dl - \int_{\mathcal{C}_8} e * \frac{1}{T_0} \widetilde{Q} * T \, dl \right\}.$$

The following variational theorem holds.

Theorem 3. *Let* $\mathcal{K}$ *be the set of all admissible processes* $U=(v_\alpha, w, \psi, T)$ *that satisfy the boundary conditions*

$$v_\alpha = \widetilde{v}_\alpha \quad on\ \overline{\mathcal{C}}_1 \times \mathcal{T}, \qquad w = \widetilde{w} \quad on\ \overline{\mathcal{C}}_3 \times \mathcal{T},$$
$$\psi = \widetilde{\psi} \quad on\ \overline{\mathcal{C}}_5 \times \mathcal{T}, \qquad T = \widetilde{T} \quad on\ \overline{\mathcal{C}}_7 \times \mathcal{T}.$$

Then

$$\delta \mathcal{G}(U) = 0 \quad on\ \ \mathcal{K},$$

if and only if U *is a solution of the boundary–initial–value problem.*

Proof. In order to prove the theorem we can use arguments similar to those presented in [13].

Alternatively, we can verify the assertion of the theorem directly, with the help of the relation

(22)
$$\left[\frac{d}{dy}\mathcal{G}(U + yU^*)\right]_{y=0} = -\int_\Sigma \{[j * (M_{\beta\alpha,\beta} - h_0 N_\alpha) + F_\alpha - \rho I v_\alpha] * v_\alpha^* +$$
$$+h_0(j * N_{\alpha,\alpha} + F - \rho w) * w^* + [j * (H_{\beta,\beta} + G - h_0 \Gamma) + L - \rho\kappa I \psi] * \psi^* +$$
$$+j * \frac{1}{T_0}[e * (Q_{\alpha,\alpha} - h_0 R) + W - T_0 \sigma] * T^*\} da +$$
$$+j * \left\{ \int_{\mathcal{C}_2} (M_{\beta\alpha} n_\beta - \widetilde{M_\alpha}) * v_\alpha^* dl + \int_{\mathcal{C}_4} h_0 (N_\alpha n_\alpha - \widetilde{N}) * w^* dl + \right.$$
$$\left. + \int_{\mathcal{C}_6} (H_\beta n_\beta - \widetilde{H}) * \psi^* dl - \int_{\mathcal{C}_8} e * \frac{1}{T_0} (Q_\alpha n_\alpha - \widetilde{Q}) * T^* dl \right\},$$

for every $U^* = (v_\alpha^*, w^*, \psi^*, T^*)$ such that $U + yU^* \in \mathcal{K}$, for all real numbers y. Using (22) and taking into account equations (8), we complete the proof in the classical manner (see [14]). □

Theorems 2 and 3 provide a variational characterization of the solution of the boundary–initial–value problem for the bending of thermoelastic porous plates.

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