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BOOK REVIEWS

Leonhard Euler, A Man to be Reckoned with Text: Andreas K. Heyne, Alice K. Heyne; Illustrations: Elena Pini, 2007, Birkhäuser Verlag, 45pp., Hardcover, CHF 32.90/Euro 19.90/USD 19.95/GB 14.50, ISBN-13:978-3-7643-8332-9.

His ideas turned the mathematical world on its head. He calculated the currents of liquids, the moment of inertia, developed the calculus of variations and the modern number theory. As scientist he achieved extraordinary things in many different fields of mathematics, as well as excelling in numerous other subjects. The importance of his work cannot be overestimated, and his name should be placed alongside those of Newton and Einstein. Even today, his theories and formulas are still used by engineers, architects and statisticians around the world. Engineers all over the world use his formulas every day-whether it be for constructing the hull of the "Alinghi" or for calculating the vibrations of the "Viaduc de Millau", the world's highest motorway bridge. He has, however, a man who loved the peace and quiet of his home life-not easy at the time of the foundation of St. Petersburg, when murdering czars was daily business; or in Berlin at the time of the Silesian wars; and particularly not in the midst of a crowd of children.

The comic by Elena Pini (illustrations) and Alice and Andreas K. Heyne (text) follows the life of the genius from Basel, who, born 300 years ago, would set out to change the scientific world.

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STOUT, EDGAR LEE – *Polynomial Convexity*, 2007, Birkhäuser Verlag, Boston-Basel-Berlin, x+439p., Hardcover, CHF 119.00/Euro 69.90/USD 79.95, ISBN-13:978-0-8176-4537-3.

This book is devoted to an exposition of the theory of polynomially convex sets. A compact subset of $\mathbb{C}^N$ is polynomially convex if it is defined

by a family, finite or infinite, of polynomial inequalities. These sets play an important role in the theory of functions of several complex variables, especially in questions concerning approximation. On the one hand, the present volume is a study of polynomial convexity per se, on the other, it studies the application of polynomial convexity to other parts of complex analysis, especially to approximation theory and the theory of varieties.

Not every compact subsets of $\mathbb{C}^N$ is polynomially convex, but associated with an arbitrary compact sets, say X, is its polynomially convex hull, $\widehat{X}$, which is the intersection of all polynomially convex sets that contain X. Of paramount importance in the study of polynomial convexity is the study of the complementary set $\widehat{X}\backslash X$. The only obvious reason for this set to be nonempty is for it to have some kind of analytic structure, and initially one wonders whether this set always has complex structure is some sense. It is not long before one is disabused of this naive hope; a natural problem then is that of giving conditions under which the complementary set does have complex structure. In a natural class of one-dimensional examples, such analytic structure is found. The study of this class of examples is one of the major directions of the work at hand.

This book is not self-contained. Certainly it is assumed that the reader has some previous exposure to the theory of functions of several complex variables. Here and there we draw on some major results from the theory of Stein manifolds. This seems reasonable in the context: Stein manifolds are the natural habitat of the complex analyst. We draw freely on the elements of real variables, functional analysis, and classical function theory. At certain points in the text, parts of algebraic topology and Morse theory are invoked. For results in algebraic topology that go beyond what one could reasonably expect to meet in an introductory course in the subject, precise references to the textbook literature are given as are references for Morse theory.

Chapter by chapter, the contents of the book can be summarized as follows. Chapter 1 is introductory and contains the initial definitions of the subject, develops some of the tools that will be used in subsequent chapters, and gives illustrative examples. Chapter 2 is concerned mainly with general properties of polynomially convex sets, for the most part properties that are independent that are independent of particular structural requirements. Chapter 3 is a systematic study of the polynomial hull of a one-dimensional set that is connected and has finite length or, more generally, this is contained in a connected set of finite length. For example, in this chapter, it is found that a rectifiable arc is polynomially convex, a result that, despite the simplicity of its formulation, is not at all simple to prove. Also in this chapter the theory of polynomially convex sets is applied to the study of one-dimensional varieties, especially to question of analytic continuation. Chapter 4 continues the study of the polynomially convex hull of one-dimensional sets, this time admitting sets more general than those

considered in Chapter 3, sets that are sometime termed geometrically 1-rectifiable. Chapter 5 studies three distinct subjects that do, though, have some connections with one another. The first concerns certain isoperimetric properties of hulls.

Finally, the hulls of surfaces in strictly pseudoconvex boundaries are considered. Chapter 6 is devoted to approximation questions, mostly on compact sets, but with some consideration of approximation on unbounded sets. Chapter 7 applies ideas of polynomial convexity to the study of one-dimensional subvarieties of strictly pseudoconvex domains, for example the ball. In part, the motivation for this work comes from the well-developed theory of the boundary behaviour of holomorphic functions. Chapter 8 is devoted to some additional topics that either further the subject of polynomial convexity itself or are applications of this theory. This beautiful exposition of a rich and complex theory, which contains much material not available in other texts, is destined to be the standard reference for many years, and will appeal to all those with an interest in multivariate complex analysis.

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ITENBERG, ILIA; MIKHALKIN, GRIGORY; SHUSTIN, EUGENII – *Tropical Algebraic Geometry*, Birkhäuser Verlag, Basel-Boston-Berlin, Volume 35, 2007, Oberwolfach Seminars, viii+103p., 30 illus., CHF 32.90/Euro 19.90, ISBN 978-37643-8309-1.

Tropical Geometry can be viewed as a sort of algebraic geometry with the underling algebra based on the so-called tropical numbers. The tropical numbers (the term "tropical" comes from computer science and commemorates Brazil, in particular a contribution of the Brazilian school to the language recognition problem) are the real numbers enhanced with negative infinity and equipped with two arithmetic operations called tropical addition and tropical multiplication. The tropical addition is the operation of taking the maximum. The tropical multiplication is the conventional addition. These operations are commutative, associative and satisfy the distribution law. It turns out that such tropical algebra describes some meaningful geometric objects, namely, the Tropical Varieties. From the topological point of view the tropical varieties are piecewise-linear polyhedral complexes equipped with a particular geometric structure coming from tropical algebra. From the point of view of complex geometry this geometric structure is the worst possible degeneration of complex structure on a manifold. From the point of view of symplectic geometry the tropical variety is

the result of the Lgrangian collapse of a symplectic manifold(along a singular fibration by Lagrangian tori). Besides a general introduction to tropical geometry, the authors discuss the concepts of complex and non-Arhimedean amoebas, as was as the patchworking construction and enumerative tropical geometry.

The book consists of three chapters. The first chapter, "Introduction to tropical geometry" (by G. Mikhalkin), is a basic treatment of tropical varieties and their relation to classical geometry, in particular the theory of amoebas. Special emphasis is put on tropical curves. The second chapter, "Patchworking of algebraic varieties" (by E. Shustin), deals with patchworking construction in algebraic geometry, the link between real algebraic geometry and tropical geometry. The third chapter, "Applications of tropical geometry to enumerative geometry" (by I. Itenberg), presents various applications, based on Mikhalkin's correspondence theorem of tropical geometry in real and complex enumerative geometry. These applications mostly concern calculation of Gromov-Witten invariants and Welschchinger invariants. Each chapter is supplemented by exercises.

The book is based on the lectures given by the authors at the Oberwolfach Seminar on Tropical Algebraic Geometry in October 2004.

I. Pop

BENZ, WALTER – *Classical Geometries in Modern Contexts. Geometry of Real Inner Product Spaces*, Birkhäuser Verlag, Boston-Basel-Berlin, 2005, xii+242pp., ISBN 3-7643-7371-7.

Se consideră un spaţiu vectorial X de dimensiune (nu neapărat finită) cel puţin egală cu 2, dotat cu un produs interior δ. Sunt preluate aici conceptele de hiperplan, sferă, disc; transformările ortogonale alcătuiesc un grup $O(X)$. Apare un concept suficient de general de translaţie; o translaţie T adăugată lui $O(X)$ determină un grup G de transformări ale lui X. Spaţiul Klein (X, G) este izomorf cu un spaţiu euclidian sau cu unul hiperbolic. (Teorema 7, p. 21). În acest context extrem de general sunt prezentate extensii şi rezultate ale geometriei euclidiene şi hiperbolice din dimensiunile 2 şi 3. Geometrii pe sferă Möbius, Poincaré, Lie, Laguerre sunt prezentate. Un consistent capitol 4 este consacrat transformărilor Lorentz în acest context general.

D. Brânzei

DARVAS, GYÖRGY – *Symmetry,* Birkhäuser Verlag, Basel-Boston-Berlin, 2007, xi+508 pp., 420 illus., 66 in color, Softcover, CHF 105.00/Euro 59.90, ISBN 13:978-3-7643-7554-6.

In this book the author explores recent discoveries related to symmetry and symmetry breaking, discussing the questions and answers they raise in a broad range of disciplines: particles and high energy physics, structural chemistry and biochemistry of proteins, genetic code study, brain research, as well as architectural structures and business decision-making. After some indroductory chapters dealing with symmetry, invariance, harmony, some hystorical survey, symmetry in geometrical decorative art and the golden section, the author presents some interdisciplinary applications: Fibonacci numbers in nature, perfection and beauty, the mystery of fivefold symmetry, viruses and fllerene molecules. Next he talks about cosmological symmetries, sight and hearing, and symmetries and symmetry breaking in inanimate nature. As a road from nature to man, the author presents chirality and cerebral asymmetries. In last chapter he talks about human creativity considering the interdependence of beauty and truth and rationality and impression.

V. Oproiu

BACHMAN, DAVID – *A Geometric Approach to Differential Forms,* Birkhäuser Verlag, Boston-Basel-Berlin, 2006, xvi+133 pp., 39 illus., Softcover, CHF 59.90/Euro 34.90, ISBN 13:978-0-8176-4499-4.

The author intends to make the topic of differential forms accessible to the sophomore level undergraduate, while stille providing material that will be of interest for more advanced students. It is based on the courses on Multivariable Calculus and Vector Calculus. However, the earlier chapters of the book do not require many concepts from linear algebra. Some notions and results are covered in the first section.

The book begins with basic concepts from multivariable calculus such as partial derivatives, gradients and multiple integrals. The second chapter focuses on parametrization of curves, surfaces and three-dimensional regions. The author insists on developing tools which will help students to find parametrizations on their own. After presenting a motivational reason for introducing differential forms that arise naturally when integrating over parametrized domains, the author begins to present the entire machinery of differential forms from a geometric standpoint. The author presents the 1-froms, 2-forms, 3-forms and n-forms and introduces the algebraic computations of products. Next he considers the differential forms of degrees 1,

2, 3 and studies the integration of n-forms on R^n. After presenting exterior differential for differential forms he studies the generalized Stokes theorem. As applications, he presents the obtaining of the Maxwell equations in the differential forms context and the foliations and contact structures. In the last chapter of the book (intended for advanced undergraduate and beginning graduate students) the author focuses on generalizing the theory of differential forms to the setting of abstract manifolds. He presents too a brief introduction to de Rham cohomology.

The book is useful to the undergraduate students interested in the differential geometry as well as to the graduate students interested in finding some genuine insights and applications of the theory of exterior differential forms.

V. Oproiu

RAMIREZ GALARZA, ANA I.; SEADE, JOSÉ – *Introduction to Classical Geometries,* Birkhäuser Verlag, Basel-Boston-Berlin, 2007, x+232 pp., Softcover, CHF 49.90/Euro 29.90, ISBN 13:978-3-7643-7517-1.

The authors intend to propose a specific way to introduce concepts that have arisen from the heyday of the Greek school of geometry to the present day. They work with coordinate models since this facilitates the use of algebraic and analytic results. The adopted viewpoint is that proposed by Felix Klein, i.e. studying geometry via groups of symmetries of the space in question. Mainly, the authors restrict themselves to the case of the plane geometry.

The Euclidean geometry is obtained by studying the properties invariant to symmetries. The authors obtain the rigid transformations and their invariants and presents the cylinders and tori. They study the finite subgroups of E(2) and E(3) and the frieze patterns and tesselations. Next they study, in the same vein, the affine geometry as the set of figures an properties invariant under the affine group. By considering the line at infinity, they obtain the real projective plane for which they state the duality principle. Next they present the shape of $P^2(R)$ and the coordinate charts for $P^2(R)$ and $P^1(C)$. The projective group is related to the cross ratio. Next one studies the projective properties of the conics, the poles and polars, and one introduces the elliptic geometry. The hyperbolic geometry is introduced by considering models of the hyperbolic plane and the transformations of the hyperbolic plane. They consider the hyperbolic metric and study the first results in hyperbolic geometry. At the end they consider surfaces with hyperbolic geometry and tessellations. In the appendices, the author present some results related to the differentiable functions, equivalence relations, the

symmetric group in four symbols, Euclidian postulates, elements of topology and some results on the circle.

V. Oproiu

DE RISI, VINCENZO – *Geometry and Monadology; Leibniz's Analysis Situs and Philosophy of Space,* Birkhäuser Verlag, Basel-Boston-Berlin, 2007, xx+658 pp., CHF 229.00/Euro 139.00, ISBN 13:978-3-7643-7985-8.

In the present book the author reconstructs Leibniz's geometrical sudies, focusing in particular on the research Leibniz carried on in the last years of his life. The main purpose is to is to offer a better understanding of the philosophy of space and in general of the mature Leibnizean metaphysics, through a pressing confrontation with the problems of geometric foundation. The book is divided in four chapters, two of which are devoted to the analysis situs from the point of view of geometry, and two to a general interpretation of Leibiniz's spatial metaphysics. Chapter 1 offers a brief survey of Leibniz's geometrical research meant to elucidate the development and thus the interest, of Leibniz's geometrical results from the period 1712-16 which most of the following investigation will deal with. Chapter 2 attempts to reconstruct, in modern mathematical terms, the main results of Leibniz's analysis of situation and chiefly those most relevant from the philosophical point of view. Chapter 3 attempts to show how situational analysis is a requisite for Leibniz's theory of expression, which constitutes the heart of his monadological metaphysics. Chapter 4 attempts more concrete to found Leibniz's theory of extension. The above four chapters alternate with three interchapters, or additions, that deal with somehow marginal or technical issues having nonetheless some relevance to the general account. In the Appendix, the author transcribes some unpublished Leibinizean manuscrtipts on the analysis of situation.

V. Oproiu

FAUSER, BERTFRIED; TOLKSDORF, JÜRGEN; ZEIDLER, EBERHARD (Eds.) – *Quantum Gravity; Mathematical Models and Experimental Bounds,* Birkhäuser Verlag, Basel-Boston-Berlin, 2007, xvi+336 pp., Hardcover, CHF 149.00/E 89.90, ISBN 13:978-3-7643-7977-3.

The present volume emerged from the 2nd Blaubeuren Workshop Mathematical and Physical Aspects of Quantum Gravity, intended to bring together experts in mathematics and physics to discuss in an open atmosphere

the fundamental questions and the frontier of theoretical physics. The book provides the reader with an overview of the different mathematical attempts to quantize gravity written by leading experts in the field. There are presented too the possible experimental bounds on quantum gravity effects. The volume starts with an overview, presented by Claus Kiefer, on the main road toward a quantum theory of relativity. After a brief motivation why there is a need to consider a quantum theory of gravity, the author discusses several aspects of the different approaches considered in the book. Next he presents a brief summary of some of the main results obtained so far to achieve a quantum version of gravity. In the second article Claus Laemmerzahl reports on the experimental status of quantum gravity effects. Alfredo Macias and Hernando Quevedo review in their contribution the role of time in the process of (canonical) quantization. The approach proposed by Louis Kauffman assumes non-commutative variables. In contrast, the contribution by Shahn Majid uses the framework of Hopf algebras and (bi)covariant calculus on the former to address the problem of quantum gravity within an elaborated algebraic framework. Group field is an elaborated extension of Penrose's spin networks and spin foam. Daniele Oriti critically describes the challenges and achievements of group field theory. In his contribution Mario Paschke gives an overview on the present status of the approaching the quantum theory of gravity by using non-commutative geometry. Romeo Brunetti and Klaus Fredenagen propose a certain background independent axiomatic formulation of perturbative quantum gravity. In his contribution Domenico Giulini provides a well written introduction to the study of representations of the diffeomorphism group for any diffeomorphism invariant theory. Christian Fleischhack studies uniqueness theorems in loop quantum gravity analogous to the famous Stone-von Neumann theorem of ordinary quantum mechanics and Kishore Marathe discusses several aspects of the interplay between topological quantum field theory and quantum gravity. The "principle of fermionic projector" proposed by Felix Finster summarizes the idea to start the formulation of a quantum theory of gravity from a set of points, a certain set of projectors related to these points and a discrete variational principle. In his article Thomas Mohaupt deals with a string theoretical approach to black hole physics. Tekin Dereli and Robin W. Tucker start out from the question of whether there is an effect similar to the photon electric effect related to the quantum gravitational energy. Using the renormalization group Oliver Lauscher and Martin Reuter discuss the existence of non-Gaussian fixed points of the renormalization group flow such that the number of counter terms can be restricted to a finite number. Harald Grosse and Raimar Wulkenhaar close this volume by a summary of their pioneering work on the construction of a specific model of renormalizable quantum field theory on such a so-called "theta-deformed" space time.

V. Oproiu

PESKIR, GORAN; SHIRYAEV, ALBERT – *Optimal Stopping and Free-Boundary Problems,* Lectures in Mathematics, ETH Zürich, Birkhäuser Verlag, Basel-Boston-Berlin, xxii+500 p., ISBN 3-7643-2419-8.

The book is treating the probabilistic problems of optimal stopping and the connection with free-boundary problems. A special attention is given to the examples and applications in various areas.

The book consists of 8 chapters and an Introduction. Chapter 1 contains a basic exposition of the general theory of optimal stopping in both discrete and continuous cases and covers the martingale and Markovian approaches.

Chapter 2 is an introduction to the theory of stochastic processes, mainly martingales and Markov processes, as well as basic transformations.

Chapter 3 develops the connection with the free boundary problems and Chapter 4 presents various methods of solution such as:the reduction to free boundary problems, the superharmonic characterization, the methods of time change, space change, measure change, integral equations. The following chapters are devoted to the various applications as follows: Chapter 5 treats the optimal stopping in stochastic analysis: Wald, Bessel, Doob, Hardy-Littlewood, Burkholder-Davis-Gundy inequalities.

Chapter 6 studies the optimal stopping problems in mathematical statistics: sequential testing problem for Wiener process with infinite and finite horizon and for Poisson process with infinite horizon. Chapters 7 and 8 deal with the optimal stopping in mathematical finance and financial engineering.

The book is very well and clearly written and may be very useful for a broad range of readers, from beginners in the field to advanced people and researchers.

C. Lefter

DRÁBEK, PAVEL; MILOTA, JAROSLAV – *Methods of Nonlinear Analysis. Applications to Differential Equations*, Birkhäuser Verlag, Basel-Boston-Berlin, 2007, xii+568 p., 78 illus., Hardcover, CHF 119.00/Euro 69.90, ISBN 13: 978-3-7643-8146-2.

Volumul de faţă este o expunere la două nivele a metodelor Analizei Neliniare şi a aplicaţiilor acestora în teoria Ecuaţiilor Diferenţiale. Materialul de bază, auto-conţinut, este prezentat în partea principală a celor 7 capitole ale lucrării; iar materialul mai avansat, în diferite apendice ale unor secţiuni din aceleaşi capitole.

Organizarea materialului de bază este după cum urmează. Capitolele 1-3 sunt dedicate introducerii unor noţiuni şi rezultate de bază din Analiza Funcţională liniară şi neliniară (operatori liniari, principiul contracţiilor,

calcul diferenţial şi integral pe spaţii normate). În Capitolul 4 sunt tratate proprietăţi locale ale aplicaţiilor diferenţiabile (teorema funcţiei inverse, teorema funcţiilor implicite, teorema rangului şi unele rezultate din teoria varietăţilor diferenţiabile); tot acolo este prezentată şi teorema bifurcaţiei locale a lui Crandall şi Rabinowitz. Capitolul 5 are ca obiect prezentarea metodelor topologice şi de monotonie ale analizei neliniare (teorema de punct fix a lui Brouwer şi Schauder, teorema lui Sard, o abordare analitică a gradului unei aplicaţii, operatori monotoni şi metoda iteraţiilor monotone bazată pe super- şi sub-soluţii). În Capitolul 6 sunt prezentate metodele variaţionale: extreme locale şi globale, metoda multiplicatorilor Lagrange şi aplicaţii la probleme de valori proprii, teorema "mountain pass" şi teorema "saddle point". Toate aceste rezultate sunt acompaniate de exemple din teoria problemelor la limită pentru ecuaţii diferenţiale ordinare. In fine, Capitolul 7 se ocupă cu unele applicaţii ale metodelor precedente în studiul problemelor la limită pentru ecuaţii cu derivate parţiale neliniare.

În ce priveşte materialul mai avansat, notăm doar temele mai importante tratate: metoda Newton abstractă (Appendix 3.2A), analiza pe varietăţi (Appendices 4.3A-4.3D), teoreme de punct fix pentru operatori necompacţi (Appendix 5.1A), teoreme de bifurcaţie globală (Appendix 5.2A), generalizarea gradului topologic Leray-Schauder (Appendix 5.2B), extinderea teoriei operatorilor monotoni (Appendix 5.3A), teorema Krein-Rutman (Appendix 5.4A), metoda Ritz (Appendix 6.2A), super- şi sub-soluţii (Appendix 6.2B), teorema bifurcaţiei "potenţiale" (Appendices 6.3A-6.3B), generalizarea teoremei "mountain pass" (Appendix 6.4A), extinderea multiplicatorilor Lagrange (Appendix 6.4B), generalizarea teoremei "saddle point" (Appendix 6.5A), applicaţii ale teoriilor precedente la probleme la limită pentru ecuaţii cu derivate parţiale (Appendices 7.5A-7.7A).

Cartea se încheie cu un sumar al metodelor descrise de-a lungul capitolelor, o listă de aplicaţii tipice ale acestora, o comparaţie a rezultatelor de bifurcaţie apărute în cadrul expunerii, o listă de simboluri, un index de nume şi noţiuni, precum şi o bibliografie minimală asupra temelor tratate care cuprinde 137 titluri.

Prin modul atractiv de prezentare, variatatea temelor si mulţimea aplicaţiilor, cartea este utilă tuturor specialiştilor în Analiza Funcţională neliniară şi aplicaţiile acesteia.

M. Turinici

BEZIAU, JEAN-YVES (Ed.) – *Logica Universalis. Towards a General Theory of Logic* (Second edition), Birkhäuser Verlag, Basel-Boston-Berlin,

2007, 246p., Softcover, CHF 78.00/Euro 49.90, ISBN-13: 978-3-7643-8353-4.

Cartea cu titlul mai sus menţionat adună o serie de contribuţii importante şi recente din domeniul logicii universale. Materialul acesteia a fost divizat în trei părţi. Partea I se deschide cu un articol de sinteză asupra logicii abstracte generale (J.-Y. Beziau) şi continuă cu prezentarea diferitelor contexte pentru o teorie generală a logicii: teoria modelelului abstract (M. Garcia-Matos şi J. Vaananen), abordarea topologică (S. Lewitzka), contextul algebric (R. Jansana) si cel categorial (P. Ageron). Lucrările din Partea II se ocupă cu o problemă centrală a logicii universale: identitatea dintre structurile logice. Astfel, după o prezentare a sistemelor logice echipolente (C. Caleiro şi R. Goncalves) se arată că fiecare echivalenţă logică poate fi formalizată prin intermediul conceptului de instituţie (T. Mossakowski, J. Goguen, R. Diaconescu şi A. Tarlecki) iar apoi, că identitatea dintre logici poate fi introdusă cu ajutorul pre-ordinilor (L. Strassburger). În fine, Partea III este destinată prezentării diferitelor instrumente şi concepte utile în studiul logicilor: matrici non-deterministe (A. Avron), matrici multi-valuate (C. Caleiro, W. Carnielli, M. Coniglio şi J. Marcos) operatorul Tarski de consecinţă (D. Makinson) şi discriminarea logică (L. Humberstone).

Cartea este utilă tuturor celor interesaţi de înţelegerea teoriilor logice deja existente şi construirea de noi asemenea teorii.

M. Turinici