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THE FIXED POINT METHOD TO STABILITY PROPERTIES OF A FUNCTIONAL EQUATION OF JENSEN TYPE*

BY

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Abstract. We use the fixed point method to obtain stability theorems of Ulam-Hyers type for a functional equation of Jensen type. The method uses the fixed point alternative as a meaningful device on the road to a better understanding of the stability property, plainly related to some fixed point of a concrete operator.

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1. Introduction and preliminaries. The study of stability problems for functional equations originated from a question of S. M. Ulam concerning the stability of group homomorphisms: *Let G be a metric group with a metric d. Given $\varepsilon > 0$, does there exist a $k > 0$ such that for every function $f : G \to G$ satisfying the inequality*

$$d(f(x \cdot y), f(x) \cdot f(y)) < \varepsilon, \forall x, y \in G,$$

there exists an automorphism a of *G with*

$$d(f(x), a(x)) < k\varepsilon \ ?$$

D. H. Hyers in 1941 gave an affirmative answer to the question of Ulam for Cauchy equations in Banach spaces. Then Aoki [2], Bourgin [4] as well

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as other authors considered the stability problem with unbounded Cauchy differences (see e.g. [11], [23], [13] and [14]). Their results include the following

Theorem 1.1. (Hyers-Aoki-Gajda). *Suppose that E is a real normed space, F is a real Banach space and $f : E \to F$ is a given function, such that the following condition holds*

$$(1_p) \qquad \|f(x+y) - f(x) - f(y)\|_F \leq \theta(\|x\|_E^p + \|y\|_E^p), \forall x, y \in E,$$

for some $p \in [0, \infty)\backslash\{1\}$. Then there exists a unique additive function $a : E \to F$ such that

$$(2_p) \qquad \|f(x) - a(x)\|_F \leq \frac{2\theta}{|2 - 2^p|} \|x\|_E^p, \forall x \in E.$$

This phenomenon is called *generalized Ulam-Hyers stability*. It is worth mentioning that the proofs used the idea conceived by D. H. Hyers: the additive function $a : E \to F$ is constructed, starting from the given function f, by the following formulae:

$$(2_{p<1}) \qquad a(x) = \lim_{n\to\infty} \frac{1}{2^n} f(2^n x), \text{ if } p < 1;$$

$$(2_{p>1}) \qquad a(x) = \lim_{n\to\infty} 2^n f\left(\frac{x}{2^n}\right), \quad \text{if } p > 1.$$

This method is called *the direct method* or *Hyers method*. It was often used to construct solutions to and is seen to be a powerful tool for studying the stability of many functional equations (cf. [12], [15] and [16] for details).

On the other hand, in [21], [7] and [5] a *fixed point method* was identified, by showing that many theorems concerning the stability of Cauchy and Jensen equations are consequences of the fixed point alternative. Subsequently, the method has been successfully used, e.g., in [6], [8], [9], [22], [20], [17] or [18], to other types of functional equations.

In the present paper we give, a *generalized Ulam-Hyers stability* property (or Ulam-Hyers-Bourgin stability) of the single variable equation

$$(1.1) \qquad w \circ f \circ \eta = f.$$

Namely, if f is mapping an Abelian group G into a complete normed space and verifies a *slightly perturbed* single variable equation, i.e. controlled by a

mapping $\varphi : G \times G \times G \to [0, \infty)$ with suitable properties, then there exists a unique *solution* of (1.1) fittingly approximating f.

As an application of the above result, the generalized stability of the following functional equation of Jensen type

$$\begin{aligned} &3f\left(\frac{x+y+z}{3}\right) + f(x) + f(y) + f(z) \\ &= 2\left(f\left(\frac{x+y}{2}\right) + f\left(\frac{y+z}{2}\right) + f\left(\frac{z+x}{2}\right)\right) \end{aligned} \tag{1.2}$$

is obtained.

Our stability theorem for the equation (1.2) extends the results of (Trif, [25]), where Aoki-Rassias stability theorems for the control functions φ given by $\varphi(x, y) = \delta(1 - i) + \varepsilon \cdot (||x||^p + ||y||^p)$, $(\varepsilon, \delta, p \geq 0$ are fixed, $p \neq 1$, $i = 0$ if $0 < p < 1$ and $i = 1$ if $p > 1)$, are obtained. For $\varphi(x, y) = \delta$ an Ulam-Hyers stability result for (1.2) is deduced.

It is worth noticing that our method introduces a metrical context and better clarifies the ideas of stability, which is seen to be plainly related to some fixed point of a suitable operator. In fact, our control conditions (below denoted by (H_i) and (H_i^*)), are responsible for the following fundamental facts: On the one hand, they ensure

1) the *contraction property* of a Schröder type operator J and

2) the first two successive approximations, f and Jf, to be at a *finite distance*.

And, on the other hand,

3) they force the fixed point of J to be a *solution of the initial equation*.

For the sake of completeness, we recall the following

Theorem 1.2. (The alternative of fixed point [19], see also [24], ch. 5]). *Suppose we are given a complete generalized metric space $(\mathcal{E}, d)$ -i.e. one for which d may assume infinite values- and a strictly contractive mapping $A : \mathcal{E} \to \mathcal{E}$, with the Lipschitz constant L. Then, for each given element $f \in \mathcal{E}$,*

either

(A_1) $d(A^n f, A^{n+1} f) = +\infty, \ \forall n \geq 0,$

or

(A_2) *There exists a natural number n_0 such that*

(A_{20}) $d(A^n f, A^{n+1} f) < +\infty, \forall n \geq n_0;$

(A_{21}) *The sequence* $(A^n f)$ *is convergent to a fixed point* f^* *of* A;
(A_{22}) f^* *is the unique fixed point of* A *in the set*
$\mathcal{E}^\star = \{g \in \mathcal{E},\ d(A^{n_0} f, g) < +\infty\}$;
(A_{23}) $d(g, f^*) \leq \frac{1}{1-L} d(g, Ag), \forall g \in \mathcal{E}^\star$.

Remark 1.1. The fixed points f^*, if any, need not be uniquely determined *in the whole space* $\mathcal{E}$ and do depend on the initial guess f.

For convenience, we recall an auxiliary stability result in [10] (see also [1] and [3]) for the single variable equation $w \circ g \circ \eta = g$, where

1. the unknown is a mapping $g : G \to Y$.
2. η is a self-mapping of the Abelian group G;
3. w is a Lipschitzian self-mapping (with Lipschitz constant ℓ_w) of the $\beta-$normed space Y, where, as usual, a mapping $\|\cdot\|_\beta : Y \to \mathbb{R}_+$, with $\beta \in (0, 1]$, is called a $\beta-norm$ iff it has the properties (n_β^I) : $||y||_\beta = 0 \Leftrightarrow y = 0$, (n_β^{II}) : $||\lambda \cdot y||_\beta = |\lambda|^\beta \cdot ||y||_\beta$, and (n_β^{III}) : $||y + z||_\beta \leq ||y||_\beta + ||z||_\beta$, for all $y, z \in Y$, and $\lambda \in \mathbb{K}$.

Theorem 1.3. *Suppose that* $f : G \to Y$ *satisfies*

$$(C_\psi) \qquad \|(w \circ f \circ \eta)(x) - f(x)\|_\beta \leq \psi(x),\ \forall x \in G,$$

with some fixed mapping $\psi : G \to [0, \infty)$. *If there exists* $L < 1$ *such that*

$$(H_\psi) \qquad \ell_w \cdot (\psi \circ \eta)(x) \leq L\psi(x), \forall x \in G,$$

then there exists a unique mapping $c : G \to Y$ *which satisfies both the equation*

$$(E_{w,\eta}) \qquad (w \circ c \circ \eta)(x) = c(x), \forall x \in G$$

and the estimation

$$(Est_\psi) \qquad \|f(x) - c(x)\|_\beta \leq \frac{\psi(x)}{1 - L}, \forall x \in G.$$

Namely, the solution mapping c *can be acquired through the Hyers method:*

$$c(x) = \lim_{n \to \infty} \left(w^n \circ f \circ \eta^n\right)(x), \forall x \in G.$$

2. Applications to the Jensen type functional equation. We will consider the following equation, where the "unknowns" are functions $f : X \to Y$, between two vector spaces:

$$\begin{aligned} &3f\left(\frac{x+y+z}{3}\right) + f(x) + f(y) + f(z) \\ &= 2\left(f\left(\frac{x+y}{2}\right) + f\left(\frac{y+z}{2}\right) + f\left(\frac{z+x}{2}\right)\right), \end{aligned} \tag{1.2}$$

for all $x, y, z \in X$.

Trif [25] showed that the equation (1.2) is equivalent to the Jensen functional equation

$$2f\left(\frac{x+y}{2}\right) = f(x) + f(y).$$

He also found the general solution of (1.2) and he proved the stability results of Aoki-Rassias type for the above equation, with unbounded control conditions invoking sums of powers of norms.

Let us define

$$\begin{aligned} D_f(x,y,z) : &= 3f\left(\frac{x+y+z}{3}\right) + f(x) + f(y) + f(z) \\ &- 2\left(f\left(\frac{x+y}{2}\right) + f\left(\frac{y+z}{2}\right) + f\left(\frac{z+x}{2}\right)\right), \end{aligned}$$

for all $x, y, z \in X$.

The following generalized Ulam-Hyers stability result extends Theorems 3.1, 3.2 & 3.3 in [25]. To prove our result we will use Theorem 1.3.

Theorem 2.1. *Let X be a normed space, Y a Banach space, and $r_i = \begin{cases} 4, i = 0 \\ \frac{1}{4}, i = 1 \end{cases}$. Suppose that the mapping $f : X \to Y$ satisfies the condition $f(0) = 0$ and an inequality of the form*

$$\|D_f(x,y,z)\|_Y \leq \varphi(x,y,z), \forall\, x, y, z \in X, \tag{J_φ}$$

where $\varphi : X \times X \times X \to [0, \infty)$ is a given function.

If there exists $L = L(i) < 1$ such that the mapping

$$x \to \Omega(x) = \varphi\left(-\frac{x}{2}, -\frac{x}{2}, x\right)$$

has the property

$$(H_i) \qquad \Omega(x) \leq L \cdot r_i \cdot \Omega\left(\frac{x}{r_i}\right), \forall\, x \in X,$$

and if the mapping φ satisfies

$$(H_i^*) \qquad \lim_{n\to\infty} \frac{\varphi\left(r_i^n x, r_i^n y, r_i^n z\right)}{r_i^n} = 0, \forall\, x, y \in X,$$

then there exists a unique additive mapping $a : X \to Y$ such that

$$(Est_i) \qquad \|f(x) - a(x)\|_Y \leq \frac{L^{1-i}}{1-L}\Omega(x), \forall\, x \in X.$$

Proof. Note that $r_0 = 4$ if (H_0) holds, and $r_1 = 4^{-1}$ if (H_1) holds. We set $y = x$ and $z = -2x$ in the relation (J_φ) and we see that

$$(2.1) \qquad ||f(-2x) - 4f\left(-\frac{x}{2}\right)||_Y \leq \varphi(x, x, -2x), \forall x \in X.$$

If the hypothesis (H_0) holds, and if we replace x by $-2x$ in the above relation we obtain

$$(J_{\Omega,0}) \qquad \left\|\frac{f(4x)}{4} - f(x)\right\|_Y \leq \frac{\varphi(-2x, -2x, 4x)}{4} = \frac{\Omega(4x)}{4}, \forall x \in X.$$

If the hypothesis (H_1) holds, and we replace x by $-\frac{x}{2}$ in the relation (2.1) then we obtain

$$(J_{\Omega,1}) \qquad \left\|f(x) - 4f\left(\frac{x}{4}\right)\right\|_Y \leq \varphi\left(-\frac{x}{2}, -\frac{x}{2}, x\right) = \Omega(x), \forall\, x \in X.$$

Now we can apply Theorem 1.3 (for $\beta = 1$), with $w : X \to Y$, $\eta : X \to X$, $\psi : X \to [0, \infty)$,

$$w(x) := \frac{x}{r_i},\ \eta(x) := r_i x,\ \psi(x) := \frac{\Omega(4^{1-i}x)}{4^{1-i}}$$

and $r_i = \begin{cases} 4, i = 0 \\ \frac{1}{4}, i = 1 \end{cases}$. Clearly, $l_w = \frac{1}{r_i}$ and, by using $(J_{\Omega,i})$ and the hypothesis (H_i), we obtain that (C_ψ) and (H_ψ) hold.

Then there exists a unique mapping $a : X \to Y$,

$$a(x) := \lim_{n\to\infty} \left(w^n \circ f \circ \eta^n\right)(x) = \lim_{n\to\infty} \frac{f\left({r_i}^n x\right)}{{r_i}^n}, \forall\, x \in X, \tag{2.2}$$

which satisfies the following equation

$$(w \circ a \circ \eta)(x) = a(x) \Leftrightarrow a\left(4x\right) = 4a\left(x\right),\ \forall\, x \in X$$

and the inequality

$$\|f(x) - a(x)\|_Y \leq \frac{\psi(x)}{1-L} = \frac{1}{1-L} \cdot \frac{\Omega(4^{1-i}x)}{4^{1-i}} \leq \frac{L^{1-i}}{1-L} \cdot \Omega(x), \forall\, x \in X.$$

The statement that a is an additive mapping follows immediately from (J_φ) and (2.2): If in (J_φ) we replace x by ${r_i}^n x$, y by ${r_i}^n y$ and z by ${r_i}^n z$, then we obtain

$$\left\| \frac{D_f({r_i}^n x, {r_i}^n y, {r_i}^n z)}{{r_i}^n} \right\|_Y \leq \frac{\varphi\left({r_i}^n x, {r_i}^n y, {r_i}^n z\right)}{{r_i}^n},$$

for all x, y, z in X.
Taking into account the hypothesis (H_i^*) and letting $n \to \infty$, we get

$$D_a(x, y, z) = 0, \forall\ x, y \in X,$$

hence a satisfies *the equation* (1.2). But Trif proved in [25] that a mapping f is a solution of (1.2) iff is an additive function, so our proof is finished.□

The above Theorem holds in the case $r_0 = 4$ even if $f(0) \neq 0$:

Theorem 2.2. *Let X be a normed space and Y a Banach space. Suppose that the mapping $f : X \to Y$ satisfies an inequality of the form*

$$\|D_f(x, y, z)\|_Y \leq \varphi(x, y, z), \forall\, x, y, z \in X, \tag{J_φ}$$

where $\varphi : X \times X \times X \to [0, \infty)$ is a given function.

If there exists $L < 1$ such that the mapping

$$x \to \Omega(x) = \varphi\left(-\frac{x}{2}, -\frac{x}{2}, x\right)$$

has the property

$$\Omega(x) \leq L \cdot 4 \cdot \Omega\left(\frac{x}{4}\right), \forall\, x \in X, \tag{H_0}$$

and if the mapping φ has the property

$$(H_0^*) \qquad \lim_{n\to\infty} \frac{\varphi\left(4^n x, 4^n y, 4^n z\right)}{4^n} = 0, \forall\, x, y \in X,$$

then there exists a unique additive mapping $a : X \to Y$ which satisfies the inequality

$$(Est_0) \qquad \|f(x) - f(0) - a(x)\|_Y \leq \frac{L}{1-L}\Omega(x), \forall\, x \in X.$$

Proof. Let $g : X \to Y$ be the function defined by $g(x) = f(x) - f(0)$. Then $g(0) = 0$ and, since $D_g(x, y, z) = D_f(x, y, z)$ for all $x, y, z \in X$, we have

$$(2.3) \qquad \|D_g(x, y, z)\|_Y \leq \varphi(x, y, z), \forall\, x, y, z \in X.$$

Now, we can apply Theorem 2.1 for the mapping g (instead of f) and we obtain the existence and the uniqueness of an additive mapping a which satisfies the estimation relation (Est_0). □

Remark 2.1. *The case $r_1 = \frac{1}{4}$ does not generate a result similar to Theorem* 2.2. *In fact, if we apply Theorem* 2.1 *for the mapping $g(x) := f(x) - f(0), \forall\, x \in X$, we obtain*

$$a(x) := \lim_{n\to\infty} \frac{g\left(r_1^n x\right)}{r_1^n} = \lim_{n\to\infty} 4^n f\left(\frac{x}{4^n}\right) - \lim_{n\to\infty} 4^n f\left(0\right), \forall\, x \in X,$$

which is infinite for $f(0) \neq 0$.

As consequences of the above theorems, we obtain the Aoki-Rassias stability for equation (1.2) (see also Trif, [25], Theorems 3.1 and 3.3):

Let X be a normed linear space, Y a Banach space, δ, ε two non-negative real numbers,

$$(p, q) \in \begin{cases} [0, 1) \times [0, 1), \text{ if } i = 0 \\ (1, \infty) \times (1, \infty), \text{ if } i = 1 \end{cases}$$

and

$$r_i := \begin{cases} 4, \text{ if } i = 0 \\ \frac{1}{4}, \text{ if } i = 1 \end{cases}.$$

Corollary 2.3. *If the function $f : X \to Y$ satisfies*

$$\|D_f(x,y,z)\| \leq \delta(1-i) + \varepsilon\left(||x||^p + ||y||^p + ||z||^p\right),\ \forall x, y \in X,$$

there exists an additive mapping $a : G \to Y$,

$$a(x) := \lim_{n\to\infty} \frac{f({r_i}^n x)}{{r_i}^n},$$

such that

$$\|f(x) - a(x) - (1-i)f(0)\| \leq \frac{(1-i)\delta}{4^{1-p}-1} + \frac{\varepsilon \cdot ||x||^p}{|2^{1-p}-1|},\ \forall x \in X. \tag{2.4}$$

Proof. Having in mind Theorem 2.2 for $i = 0$ and Theorem 2.1 for the case $i = 1$, we consider a complete normed space Y and we take the following control function

$$\varphi(x,y,z) = \begin{cases} \delta + \varepsilon\left(||x||^p + ||y||^p + ||z||^p\right), & \text{if } p,q \in [0,1), \\ \varepsilon\left(||x||^p + ||y||^q + ||z||^p\right) & \text{if } p,q \in (1,\infty), \end{cases}$$

for all $x, y, z \in X$. Then, obviously, we can write

$$\frac{\varphi\left({r_i}^n x, {r_i}^n y, {r_i}^n z\right)}{{r_i}^n}$$

$$= \delta(1-i){r_i}^{-n} + {r_i}^{n(p-1)} \cdot \varepsilon \|x\|^p + {r_i}^{n(p-1)} \cdot \varepsilon \|y\|^p + {r_i}^{n(p-1)} \varepsilon \|z\|^p \xrightarrow[n\to\infty]{} 0,$$

that is the hypothesis (H_0^*) / (H_1^*) from Theorems 2.2/2.1 hold.

On the other hand

$$\Omega(x) := \varphi\left(-\frac{x}{2}, -\frac{x}{2}, x\right) = (1-i)\delta + \varepsilon \|x\|^p \cdot \frac{1+2^{p-1}}{2^{p-1}}, \forall x \in X.$$

Case 0. If $i = 0 (r_0 = 4)$, then $0 \leq p < 1$, and the hypothesis (H_0) from Theorem 2.2 is satisfied, with $L = 4^{p-1} < 1$.

Case 1. If $i = 1 (r_1 = \frac{1}{4})$, then $p > 1$ and the relation (H_1) from Theorem 2.1 holds, with $L = 4^{1-p} < 1$.

Therefore the hypotheses of Theorem 2.2 are verified in the Case 0, and those of Theorem 2.1 are satisfied in the Case 1. Then there exists an *unique solution* $a : X \to Y$ of equation (1.2)

$$a(x) := \lim_{n\to\infty} \frac{f({r_i}^n x)}{{r_i}^n},\ \forall x \in X,$$

such that (2.4) is satisfied. □

As a particular case of Theorem 2.2, for $\varphi(x,y) = \delta$, we obtain the stability result of Ulam-Hyers type in [25]:

Corollary 2.4. *Let X a normed space and Y a Banach space. Suppose that the mapping $f : X \to Y$ satisfies an inequality of the form*

$$\|D_f(x,y,z)\|_Y \leq \delta, \ \forall\, x,y,z \in X.$$

Then there exists an unique additive mapping $a : X \to Y$ which satisfies the inequality

$$\|f(x) - f(0) - a(x)\|_Y \leq \frac{\delta}{3}, \forall\, x \in X.$$

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