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CERTAIN FINITE INTEGRALS INVOLVING A PRODUCT OF SOME TRANSCEDENTAL FUNCTIONS

BY

V.B.L. CHAURASIA and PINKY LATA

Abstract. The aim of the present paper is to establish a finite integral involving the product of generalized Lauricella function and two H-functions of several complex variables (Srivastava, Panda and Olkha, Chaurasia). By assigning suitable values to the parameters, the main result can be reduced to G and F functions. Specially, we are using the series representation of H-function given by Olkha and Chaurasia.

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1. Introduction. The series representation for the H-function of several complex variables studied by Olkha and Chaurasia [3] is given as follows:

$$H[z_1,\dots,z_r]=H^{0,\lambda':(u',v');\dots;(u^{(r)}),v^{(r)})}_{A',C':[B',D'];\dots;[B^{(r)},D^{(r)}]}\left[\begin{array}{l}\left[(a);\theta',\dots,\theta^{(r)}\right];\left[b':\phi'\right];\dots;\left[b^{(r)}:\phi^{(r)}\right];\\ \\ \left[(c):\psi',\dots,\psi^{(r)}\right];\left[d':\delta'\right];\dots;\left[d^{(r)}:\delta^{(r)}\right];\end{array}\begin{array}{l}\\ \\ z_1,\dots,z_r\end{array}\right]$$

$$=\sum_{m_i=1}^{u^{(i)}}\sum_{n_i=0}^{\infty}\Phi_1\Phi_2\frac{\prod_{i=1}^{r}(z_i)^{U_i}(-1)^{\sum_{i=1}^{r}(n_i)}}{\prod_{i=1}^{r}(\delta^{(i)}_{m_i}n_i!)},\tag{1.1}$$

where

$$\Phi_1=\frac{\prod_{j=1}^{\lambda'}\Gamma(1-a_j+\sum_{i=1}^{r}\theta_j^{(i)}U_i)}{\prod_{j=\lambda'+1}^{A'}\Gamma(a_j-\sum_{i=1}^{r}\theta_j^{(i)}U_i)\prod_{j=\lambda'+1}^{C'}\Gamma(1-c_j+\sum_{i=1}^{r}\psi_j^{(i)}U_i)}\tag{1.2}$$

$$\Phi_2 = \frac{\prod_{j=1,j\neq m_i}^{u^{(i)}} \Gamma(d_j^{(i)} - \delta_j^{(i)} U_i) \prod_{j=1}^{v^{(i)}} \Gamma(1 - b_j^{(i)} + \phi_j^{(i)} U_i)}{\prod_{j=u^{(i)}+1}^{D^{(i)}} \Gamma(1 - d_j^{(i)} + \delta_j^{(i)} U_i) \prod_{j=v^{(i)}+1}^{B^{(i)}} \Gamma(b_j^{(i)} - \phi_j^{(i)} U_i)} \tag{1.3}$$

and

$$U_i = \frac{d_{m_i}^{(i)} + n_i}{\delta_{m_i}^{(i)}}, i = 1, \ldots, r, \tag{1.4}$$

which is valid under the following conditions

$$\delta_{m_i}^{(i)}[d_j^{(i)} + p_i] \neq \delta_j^{(i)}[d_{m_i}^{(i)} + n_i] \tag{1.5}$$

for $j \neq m_i, m_i = 1, \ldots, u^{(i)}; p_i, n_i = 0, 1, 2 \ldots; z_i \neq 0,$

$$\nabla_i = \sum_{j=1}^{A'} \theta_j^{(i)} - \sum_{j=1}^{C'} \psi_j^{(i)} + \sum_{j=1}^{B^{(i)}} \phi_j^{(i)} - \sum_{j=1}^{D^{(i)}} \delta_j^{(i)} < 0, \ \forall i = 1, \ldots, r. \tag{1.6}$$

Srivastava and Panda [5] have introduced the multivariable H-function

$$H_{[}y_1, \ldots, y_R] = H^{0,\lambda:(\alpha',\beta');\ldots;(\alpha^{(R)},\beta^{(R)})}_{A,C:[M',N'];\ldots;[M^{(R)},N^{(R)}]}$$

$$\left[\begin{array}{ll} [(g) : \gamma', \ldots, \gamma^{(R)}]; [q' : \eta']; \ldots; [q^{(R)} : \eta^{(R)}]; & \\ & y_1, \ldots, y_R \\ \left[(f); \xi', \ldots, \xi^{(R)}\right]; [p' : \varepsilon']; \ldots; [p^{(R)}; \varepsilon^{(R)}]; & \end{array} \right]. \tag{1.7}$$

For the sake of brevity

$$T_i = \sum_{j=1}^{A} \gamma_j^{(i)} - \sum_{j=1}^{C} \xi_j^{(i)} + \sum_{j=1}^{M^{(i)}} \eta_j^{(i)} - \sum_{j=1}^{N^{(i)}} \varepsilon_j^{(i)} \leq 0, \tag{1.8}$$

$$\begin{aligned} \Omega_i \ = & - \sum_{j=\lambda+1}^{A} \gamma_j^{(i)} - \sum_{j=1}^{C} \xi_j^{(i)} + \sum_{j=1}^{\beta^{(i)}} \eta_j^{(i)} - \sum_{j=\beta^{(i)}+1}^{M^{(i)}} \eta_j^{(i)} \\ & + \sum_{j=1}^{\alpha^{(i)}} \varepsilon_j^{(i)} - \sum_{j=\alpha^{(i)}+1}^{N^{(i)}} \varepsilon_j^{(i)} > 0, \end{aligned} \tag{1.9}$$

$$|arg(y_i)| < \frac{T_i \pi}{2}, \ \ \forall i = 1, \ldots, R. \tag{1.10}$$

2. The main integral transformation. We derive the following integral transformation for the multivariable H-function defined by SRIVASTAVA and PANDA [4] ([see also [3]).

(2.1)

$$\int_0^1 x^{\varepsilon-1}(1-x)^{\beta} H[y_1x^{k_1},\ldots,y_Rx^{k_R}]H[z_1x^{h_1},\ldots,z_rx^{h^{h_r}}]F^{v:P';\ldots;P^{(s)};0;0}_{\sigma:Q';\ldots;Q^{(s)};1;1}$$

$$\left[\begin{array}{l}[(\alpha_v):a';\ldots;a^{(s)},\gamma,\gamma]:[(l'):\rho'];\ldots;[(l^{(s)}):\rho^{(s)}];[\ldots];[\ldots];\\ \\ \left[(\beta_\sigma):b';\ldots;b^{(s)},\mu,\mu\right]:[(m'):\tau'];\ldots;\left[(m^{(s)}):\tau^{(s)}\right];\end{array}\right]dx$$

$$\left[\begin{array}{c}z_1',\ldots,z_s',-xt,(1-x)t\\ [\alpha+1:1];[\beta+1;1];\end{array}\right]$$

$$=\sum_{n,n_j=0}^{\infty}\sum_{m_i=1}^{u^{(i)}}\frac{\prod_{j=1}^{v}(\sigma_j)_{n\gamma_j}\prod_{i=1}^{r}(z_i)^{U_i}(-1)^{\sum_{i=1}^{r}(n_j)}(-t)^n\Gamma(\beta+n+1}{\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j}\prod_{i=1}^{r}(\delta_{m_i}^{(i)}n_i!)(\alpha+1)_n(\beta+1)_nn!}\Phi_1\Phi_2$$

$$F^{v:P';\ldots;P^{(s)}}_{\sigma:Q';\ldots;Q^{(s)}}\left[\begin{array}{l}[(\alpha_v+n\gamma_v):a';\ldots;a^{(s)}]:[(l'):\rho'];\ldots;[(l^{(s)});\rho^{(s)}];\\ \\ \left[(\beta_\sigma+n\mu_\sigma):b';\ldots;b^{(s)}\right]:[(m'):\tau'];\ldots;\left[m^{(s)};\tau^{(s)}\right];\end{array}\right.$$

$$\left.\begin{array}{c}z_1',\ldots,z_s'\\ \\ \end{array}\right]$$

$$H^{0,\lambda+2:(\alpha',\beta');\ldots;(\alpha^{(R)},\beta^{(R)})}_{A+2,C+2:[M',N'];\ldots;[M^{(R)},N^{(R)}]}$$

$$\left[\begin{array}{l}[1-\varepsilon-\sum_{i=1}^{r}h_iU_i;k_1,\ldots,k_R];[1-\varepsilon+\alpha-\sum_{i=1}^{r}h_iU_i;k_1,\ldots,k_R];\\ \left[(f):\xi',\ldots,\xi^{(R)}\right];[-\beta-n-\varepsilon-\sum_{i=1}^{r}h_iU_i:k_1,\ldots,k_R];\\ [(g):\gamma',\ldots,\gamma^{(R)}];[(q'):\eta'];\ldots;[(q^{(R)}):\eta^{(R)}];\\ \\ [1-\varepsilon+\alpha+n-\sum_{i=1}^{r}h_iU_i:k_1,\ldots,k_R];[(p'):\varepsilon'];\ldots;[(p^{(R)}):\varepsilon^{(R)}];\end{array}\right.$$

$$\left.\begin{array}{r}\\ \\ y_1,\ldots,y_R\\ \\ \end{array}\right],$$

where $Re(\beta)>-1; Re[\varepsilon+\sum_{i=1}^{R}k_i(\frac{p_j^{(i)}}{\varepsilon_j^{(i)}})]>0;$

$j=1,\ldots,\alpha^{(i)}; k_i>0; h_l>0; T_i>0; |arg(y_i)|<\frac{T_i\pi}{2};$

$i=1,\ldots,R; l=1,\ldots,r; |t|<1$, where the series on the right hand side is convergent and is given by (1.8).

3. Proof. In order to prove (2.1), we start with the following result [6]:

(3.1)

$$F^{v:P';\ldots;P^{(S)};0;0}_{\sigma:Q';\ldots;Q^{(S)};1;1}\left[\begin{array}{l}[(\alpha_v):a';\ldots;a^{(s)},\gamma,\gamma]:[(l'):\rho'];\ldots;[(l^{(s)};\rho^{(s)}];\\ \left[(\beta_\sigma):b';\ldots;b^{(s)},\mu,\mu\right]:[(m'):\tau'];\ldots;[(m^{(s)}:\tau^{(s)}];\end{array}\right.$$

$$\left.\begin{array}{l}[\ldots];[\ldots];\\ z_1',\ldots,z_s',-xt,(1-x)t\\ [\alpha+1:1]\,;[\beta+1;1];\end{array}\right]$$

$$=\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{v}(\alpha_j)_{n\gamma_j}t^nP_n^{(\alpha,\beta)}(1-2x)}{\prod_{j=1}^{\sigma}(b_j)_{n\mu_j}(\alpha+1)_n(\beta+1)_n}$$

$$F_{\sigma:Q';\ldots;Q^{(s)}}^{v:P';\ldots;P^{(s)}}\left[\begin{array}{l}[(\alpha_v+n\gamma_v):a';\ldots;a^{(s)}]:[(l'):\rho'];\ldots;[(l^{(s)};\rho^{(s)}];\\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad z_1',\ldots,z_s'\\ \left[(\beta_\sigma+n\mu_\sigma):b';\ldots;b^{(s)}\right]:[(m'):\tau']\,;\ldots;[(m^{(s)}):\tau^{(s)}];\end{array}\right]$$

where $\gamma_i a_j^{(i)};i=1,\ldots,v$ and $j=1,\ldots,s$;

$\mu_i b_j^{(i)};i=1,\ldots,\sigma$ and $j=1,\ldots,s$;

$\rho_i\tau_k^{(j)};i=1,\ldots,P^{(j)};j=1,\ldots,s$; and $k=1,\ldots,Q^{(j)}$ are all real and positive, (α_v) stands for the sequence of parameters $\alpha_1,\ldots,\alpha_v;(l^{(j)})$ stands for the sequence of $P^{(j)}$ parameters $l_1^{(j)},\ldots,l_{p^{(j)}}^{(j)},j=1,\ldots,s$, with similar interpretation for (β_σ) and $(m^{(j)})$, $j=1,\ldots,s$ and F represents the generalized Lauricella function of s-complex variables $z_1',\ldots,z_s'$ given by Srivastava and Daoust [8]. Now to prove (2.1), we multiply both sides of (3.1) by $x^{\varepsilon-1}(1-x)^\beta H[y_1x^{k_1},\ldots,y_Rx^{k_R}]H[z_1x^{h_1},\ldots,z_rx^{h_r}]$ and integrate with respect to x from 0 to 1.

Now express the $H[z_1x^{h_1},\ldots,z_rx^{h_r}]$ in series form [3], given by (1.1) and interchange the order of integrations and summations (which is permissible under the stated conditions). The announced formula is obtained by evaluating the innermost integral with the help of following result [5]:

$$\text{(3.2)}\qquad \int_0^1 x^\varepsilon(1-x)^\beta H[y_1x^{k_1},\ldots,y_Rx^{k_R}]P_n^{\alpha,\beta}(1-2x)dx$$

$$=\frac{(-1)^n\Gamma(\beta+n+1)}{n!}H_{A+2,C+2:[M',N'];\ldots;[M^{(R)},N^{(R)}]}^{0,\lambda+2:(\alpha',\beta');\ldots;(\alpha^{(R)},\beta^{(R)})}$$

$$\left[\begin{array}{l}[-\varepsilon;k_1,\ldots,k_R],[-\varepsilon+\alpha:k_1,\ldots,k_R],[(g):\gamma',\ldots,\gamma^{(R)}];\\ \left[(f):\xi',\ldots,\xi^{(R)}\right];[-\beta-n-\varepsilon-1:k_1,\ldots,k_R],[\alpha+n-\varepsilon:k_1,\ldots,k_R]\end{array}\right.$$

$$\left.\begin{array}{l}[(q'):\eta'];\ldots;[(q^{(R)}):\eta^{(R)}];\\ \qquad\qquad y_1,\ldots,y_R\\ [(p'):\varepsilon']\,;\ldots;[(p^{(R)}):\varepsilon^{(R)}];\end{array}\right],$$

where

$$Re(\beta) > -1; Re[\varepsilon + \sum_{i=1}^{R} k_i(\frac{p_j^{(i)}}{\varepsilon_j^{(i)}})] > -1; j = 1, \ldots, \alpha^{(i)}; k_i > 0; T_i > 0;$$

$|arg(y_i)| < \frac{T_i\pi}{2}, i = 1, \ldots, R$ and T_i is given by (1.8).

4. Particular cases.

(1) Reducing the H-function of several complex variables to the G-function by putting $\gamma', \ldots, \gamma^{(R)} = \eta', \ldots, \eta^{(R)} = \xi', \ldots, \xi^{(R)} = \varepsilon', \ldots, \varepsilon^{(R)} = k_1, \ldots, k_R = \omega_1, \ldots, \omega_R$ in (2.1), we get the following result:

$$(4.1) \qquad \int_0^1 x^{\varepsilon-1}(1-x)^{\beta} H[z_1 x^{h_1}, \ldots, z_r x^{h_r}] F_{\sigma:Q';\ldots;Q^{(s)};1;1}^{v:P';\ldots;P^{(s)};0;0}$$

$$\left[\begin{array}{l} [(\alpha_v) : a'; \ldots; a^{(s)}, \gamma, \gamma] : [(l') : \rho']; \ldots; [(l^{(s)}) : \rho^{(s)}]; [\ldots]; [\ldots]; \\ \\ (\beta_\sigma) : b'; \ldots; b^{(s)}, \mu, \mu] : [(m') : \tau']; \ldots; \end{array} \right.$$

$$\left. \begin{array}{r} z_1', \ldots, z_s', -xt, (1-x)t] \\ \left[(m^{(s)}); \tau^{(s)}\right]; [\alpha+1:1]; [\beta+1:1]; \end{array} \right]$$

$$G_{A,C:[M',N'];\ldots;[M^R,N^{(R)}]}^{0,\lambda:(\alpha',\beta');\ldots;(\alpha^{(R)},\beta^{(R)})} \left[\begin{array}{c} (g) : (q'); \ldots; (q^{(R)}); \\ y_1^{1/\omega_1} x, \ldots, y_R^{1/\omega_R} x \\ (f) : (p'); \ldots; (p^{(R)}); \end{array} \right] dx$$

$$= \sum_{n,n_i=0}^{\infty} \sum_{m_i=1}^{u^{(j)}} \frac{\prod_{j=1}^{v}(\alpha_j)_{n\gamma_j} \prod_{i=1}^{r}(z_i)^{U_i}(-1)^{\sum_{i=1}^{r}(n_i)}(-t)^n \Gamma(\beta+n+1)}{\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j} \prod_{i=1}^{r}(\delta_{m_i}^{(i)} n_i!)(\alpha+1)_n(\beta+1)_n n!} \Phi_1 \Phi_2$$

$$F_{\sigma:Q';\ldots;Q^{(s)}}^{v:P';\ldots;P^{(s)}} \left[\begin{array}{l} [(\alpha_v + n\gamma_v) : a'; \ldots; a^{(s)}] : [(l') : \rho']; \ldots; [(l^{(s)}) : \rho^{(s)}]; \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad z_1', \ldots, z_s' \\ \left[(\beta_\sigma + n\mu_\sigma) : b'; \ldots; b^{(s)}\right]; [(m') : \tau']; \ldots; [(m^{(s)}); \tau^{(s)}]; \end{array} \right]$$

$$G_{A+2,C+2:[M',N']:\ldots;[M^{(R)},N^{(R)}]}^{0,\lambda+2:(\alpha',\beta');\ldots;(\alpha^{(R)},\beta^{(R)})}$$

$$\left[\begin{array}{l} [1-\varepsilon-\sum_{i=1}^{r} h_i U_i], [1-\varepsilon+\alpha-\sum_{i=1}^{r} h_i U_i], (g); (q'); \ldots; (q^{(R)}); \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad y_1^{1/\omega_1}, \ldots, y_R^{1/\omega_R} \\ \\ (f); [-\beta-n-\varepsilon-\sum_{i=1}^{r} h_i U_i], [1-\varepsilon+\alpha+n-\sum_{i=1}^{r} h_i U_i]; (p'); \ldots; (p^{(R)}); \end{array} \right]$$

which holds under the same conditions obtainable from (2.1) and $\omega_1, \ldots, \omega_R > 0$.

(2) Reducing the H-function of several complex variables to the generalized Lauricella function [8] by putting $\lambda = A, \alpha^{(i)} = M^{(i)}, N^{(i)} = N^{(i)} + 1, \forall 1, \ldots, R$ in (2.1), we get the following result:

$$(4.2) \qquad \int_0^1 x^{\varepsilon-1}(1-x)^{\beta} H[z_1 x^{h_1}, \ldots, z_r x^{h_r}] F_{\sigma:Q';\ldots;Q^{(s)};1;1}^{v:P';\ldots;P^{(s)};0;0}$$

$$\left[\begin{array}{l} [(\alpha_v) : a'; \ldots; a^{(s)}, \gamma, \gamma] : [(l') : \rho']; \ldots; [(l^{(s)}) : \rho^{(s)}]; [\ldots]; [\ldots] \\ \\ \left[(\beta_\sigma) : b'; \ldots; b^{(s)}, \mu, \mu\right] : [(m') : \tau']; \ldots; [(m^{(s)} : \tau^{(s)}]; \\ \\ \qquad\qquad z_1', \ldots, z_s', -xt, (1-x)t \\ \qquad\qquad [\alpha+1:1]; [\beta+1:1]; \end{array} \right]$$

$$F_{C:N';\ldots;N^{(R)}}^{A:M';\ldots;M^{(R)}} \left[\begin{array}{l} [1-(g) : \gamma', \ldots, \gamma^{(R)}] : [1-(q') : \eta']; \ldots; [1-(q^{(R)}) : \eta^{(R)}] \\ \\ \left[1-(f) : \xi', \ldots, \xi^{(R)}\right] : [1-(p') : \varepsilon']; \ldots; \\ \\ \qquad\qquad -y_1 x^{k_1}, \ldots, -y_R x^{k_R} \\ \qquad\qquad \left[1-(p^{(R)}) : \varepsilon^{(R)}\right]; \end{array} \right]$$

$$= \sum_{n,n_i=0}^{\infty} \sum_{m_i=1}^{u^{(i)}} \frac{\prod_{j=1}^{v}(\alpha_j)_{n\gamma_j} \prod_{i=1}^{r}(z_i)^{U_i}(-1)^{\sum_{i=1}^{r}(n_i)}(-t)^n \Gamma(\beta+n+1)}{\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j} \prod_{i=1}^{r}(\delta_{m_i}^{(i)} n_i!)(\alpha+1)_n(\beta+1)_n n!} \Phi_1 \Phi_2$$

$$F_{\sigma:Q';\ldots;Q^{(s)}}^{v:P';\ldots;P^{(s)}} \left[\begin{array}{l} [(\alpha_v + n\gamma_v) : a'; \ldots; a^{(s)}] : [(l') : \rho'] : \ldots; [(l^{(s)}) : \rho^{(s)}]; \\ \qquad\qquad z_1', \ldots, z_s' \\ \left[(\beta_\sigma + n\mu_\sigma) : b'; \ldots; b^{(s)}\right] : [(m') : \tau']; \ldots; [(m^{(s)}); \tau^{(s)}]; \end{array} \right]$$

$$\cdot \frac{\Gamma(\varepsilon - \sum_{i=1}^{r} h_i U_i)}{\Gamma(1+\beta+\varepsilon+n+\sum_{i=1}^{r} h_i U_i)} \frac{\Gamma(\varepsilon-\alpha+\sum_{i=1}^{r} h_i U_i)}{\Gamma(\varepsilon-\alpha+n+\sum_{i=1}^{r} h_i U_i)}$$

$$F_{C+2:N';\ldots;N^{(R)}}^{A+2:M';\ldots;M^{(R)}} \left[\begin{array}{l} [\varepsilon+\sum_{i=1}^{r} h_i U_i; k_1, \ldots, k_R], [\varepsilon-\alpha+\sum_{i=1}^{r} h_i U_i; k_1, \ldots, k_R], \\ \left[1-(f) : \xi', \ldots, \xi^{(R)}\right]; [1+\beta+\varepsilon+n+\sum_{i=1}^{r} h_i U_i; k_1, \ldots, k_R], \\ [1-(g) : \gamma', \ldots, \gamma^{(R)}]; [1-(q') : \eta']; \ldots; [1-(q^{(R)}) : \eta^{(R)}] \\ \qquad\qquad -1 y_1, \ldots, -y_R \\ [\varepsilon-\alpha-n+\sum_{i=1}^{r} h_i U_i; k_1, \ldots, k_R]; [1-(p') : \varepsilon']; \ldots; [1-(p^{(R)}) : \varepsilon^{(R)}] \end{array} \right]$$

which is valid under the same conditions as (2.1).

(3) Taking $i = 1, 2$ in (4.2) then the Lauricella function reduces to the Kampé de Fériet function [7] and we get the following result:

$$(4.3)\qquad \int_0^1 x^{\varepsilon-1}(1-x)^{\beta} H[z_1x^{h_1},\dots,z_rx^{h_r}]F^{v:P';\dots;P^{(S)};0;0}_{\sigma:Q';\dots;Q^{(S)};1;1}$$

$$\left[\begin{array}{l}[(\alpha_v):a';\dots;a^{(s)},\gamma,\gamma]:[(l'):\rho'];\dots;[(l^{(s)}:\rho^{(s)}];\\ \\ \left[(\beta_\sigma):b';\dots;b^{(s)},\mu,\mu\right]:[(m'):\tau'];\dots;[m^{(s)}:\tau^{(s)}];\\ \qquad\qquad [\dots];[\dots];\\ \qquad\qquad z'_1,\dots,z'_s,-xt,(1-x)t\\ \qquad\qquad [\alpha+1:1];[\beta+1:1];\end{array}\right]$$

$$S^{A:M';M''}_{C:N';N''}\left[\begin{array}{l}[(g):\gamma',\gamma'']:[(q'):\eta'];[(q''):\eta''];\\ \qquad\qquad\qquad\qquad y_1x^{k_1},y_2x^{k_2}\\ [(f):\xi',\xi'']:[(p'):\varepsilon'];[(p''):\varepsilon''];\end{array}\right]dx$$

$$= \sum_{n,n_i=0}^{\infty}\sum_{m_i=1}^{u^{(i)}} \frac{\prod_{j=1}^{v}(\alpha_j)_{n\gamma_j}\prod_{i=1}^{r}(z_i)^{u_i}(-1)^{\sum_{i=1}^{r}(n_i)}(-t)^n\Gamma(\beta+n+1)}{\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j}\prod_{i=1}^{r}(\delta^{(i)}_{m_i}n_i!)(\beta+1)_n n!}\Phi_1\Phi_2$$

$$F^{v:P';\dots;P^{(s)}}_{\sigma:Q';\dots;Q^{(s)}}\left[\begin{array}{l}[(\alpha_v+n\gamma_v):a';\dots;a^{(s)}]:[(l'):\rho'];\dots;[(l^{(s)}):\rho^{(s)}];\\ \qquad\qquad\qquad\qquad\qquad\qquad z'_1,\dots,z'_s\\ \left[(\beta_\sigma+n\mu_\sigma):b';\dots;b^{(s)}\right]:[(m'):\tau'];\dots;[(m^{(s)}):\tau^{(s)}];\end{array}\right]$$

$$S^{A+2:M';M''}_{C+2:N';N''}\left[\begin{array}{l}[1-\varepsilon-\sum_{i=1}^{r}h_iU_i;k_1,k_2],[1-\varepsilon+\alpha-\sum_{i=1}^{r}h_iU_i;k_1,k_2],\\ [(f):\xi',\xi'']\,;[-\beta-\varepsilon-n-\sum_{i=1}^{r}h_iU_i;k_1,k_2],\\ \qquad [(g):\gamma',\gamma''];[(q'):\eta'];[(q''):\eta''] \qquad\qquad\qquad\qquad\\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad y_1,y_2\\ \qquad [1-\varepsilon+\sigma+n-\sum_{i=1}^{r}h_iU_i;k_1,k_2]\,;[(p'):\varepsilon'];[(p''):\varepsilon'']\end{array}\right],$$

which is valid under the same conditions as (2.1).

(4) By taking $\lambda = A = C = 0$ in (2.1), we obtain the following result involving the product of R mutually independent H-functions:

$$(4.4)\qquad \int_0^1 x^{\varepsilon-1}(1-x)^{\beta} H[z_1x^{h_1},\dots,z_Rx^{h_r}]F^{v:P';\dots;P^{(s)};0;0}_{\sigma;Q';\dots;Q^{(s)};1;1}$$

$$\left[\begin{array}{l}[(\alpha_v):a';\dots;a^{(s)},\gamma,\gamma]:[(l'):\rho'];\dots;[(l^{(s)}):\rho^{(s)}];[\dots];[\dots]\\ \\ (\beta_\sigma):b';\dots;b^{(s)},\mu,\mu]:[(m'):\tau'];\dots;[(m^{(s)}):\tau^{(s)}];\end{array}\right.$$

$$\left.\begin{matrix} z_1', \ldots, z_s', -xt, (1-x)t \\ [\alpha+1:1]\,;[\beta+1:1] \end{matrix}\right]$$

$$\prod_{i=1}^{R}\left\{ H_{M^{(i)},N^{(i)}}^{\alpha^{(i)},\beta^{(i)}}\left[y_i x^{(i)} \middle| \begin{matrix} [(q^{(i)});\eta^{(i)}] \\ \left[(p^{(i)}):\varepsilon^{(i)}\right] \end{matrix} \right] \right\} dx$$

$$= \sum_{n,n_i=0}^{\infty}\sum_{m_i=1}^{u^{(i)}} \frac{\prod_{j=1}^{v}(\alpha_j)\prod_{i=1}^{r}(z_i)^{U_i}(-1)^{\sum_{i=1}^{r}(n_i)}(-t)^n\Gamma(\beta+n+1)}{\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j}\prod_{i=1}^{r}(\delta_{m_i}^{(i)}n_i!)(\alpha+1)_n(\beta+1)_n n!}\Phi_1\Phi_2$$

$$F_{\sigma:Q';\ldots;Q^{(s)}}^{v:P';\ldots;P^{(s)}}\left[\begin{matrix} [(\alpha_v+n\gamma_v):a';\ldots;a^{(s)}]:[(l'):\rho'];\ldots;[(l^{(s)}):\rho^{(s)}]; \\ z_1', \ldots, z_s' \\ \left[(\beta_\sigma+n\mu_\sigma):b';\ldots;b^{(s)}\right]:[(m'):\tau'];\ldots;[m^{(s)};\tau^{(s)}]; \end{matrix} \right]$$

$$H_{2,2:[M',N'];\ldots;[M^{(R)},N^{(R)}]}^{0,2:(\alpha',\beta');\ldots;(\alpha^{(R)},\beta^{(R)})}$$

$$\left[\begin{matrix} [1-\varepsilon-\sum_{i=1}^{r}h_iU_i:k_1,\ldots,k_R];[1-\varepsilon+\alpha-\sum_{i=1}^{r}h_iU_i:k_1,\ldots,k_R]; \\ [-\beta-n-\varepsilon-\sum_{i=1}^{r}h_iU_i:k_1,\ldots,k_R]\,;[1-\varepsilon+\alpha+n \end{matrix} \right.$$

$$\left. \begin{matrix} [(q'):\eta';\ldots;[(q^{(R)}):\eta^{(R)}]; \\ y_1, \ldots, y_R \\ -\sum_{i=1}^{r}h_iU_i;k_1,\ldots,k_R];[(p'):\varepsilon']\,;\ldots;[(p^{(R)}):\varepsilon^{(R)}]; \end{matrix} \right]$$

which holds under the same conditions as (2.1).

(5) By putting $z_1' = \ldots = z_s' = 0$ in (2.1), we get the following integral involving generalized Kampé de Fériet function [7,8]:

$$(4.5)\qquad \int_0^1 x^{\varepsilon-1}(1-x)^\beta H[z_1x^{h_1},\ldots,z_rx^{h_r}]H[y_1x^{k_1},\ldots,y_Rx^{k_R}]$$

$$S_{\sigma:1;1}^{v:0;0}\left[\begin{matrix} [(\alpha_v):\gamma,\gamma]:[\ldots];[\ldots]; \\ -xt,(1-x)t \\ [(\beta_\sigma):\mu,\mu]:[\alpha+1:1];[\beta+1:1]; \end{matrix} \right] dx$$

$$= \sum_{n,n_i=0}^{\infty}\sum_{m_i=1}^{u^{(i)}} \frac{\prod_{j=1}^{v}(\alpha_j)_{n\gamma_j}\prod_{i=1}^{r}(z_i)^{U_i}(-1)^{\sum_{i=1}^{r}(n_i)}(-t)^n\Gamma(\beta+n+1)}{\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j}\prod_{i=1}^{r}(\delta_{m_i}^{(i)}n_i!)(\alpha+1)_n(\beta+1)_n n!}$$

$$H_{A+2,C+2:,[M',N'];\ldots;[M^{(R)},N^{(R)}]}^{0,\lambda+2:(\alpha',\beta');\ldots;(\alpha^{(R)},\beta^{(R)})}$$

$$\left[\begin{matrix} [1-\varepsilon-\sum_{i=1}^{r}h_iU_i;k_1,k_2];[1-\varepsilon+\alpha-\sum_{i=1}^{r}h_iU_i;k_1,\ldots,k_R]; \\ \left[(f):\xi',\ldots,\xi^{(R)}\right]\,;[-\beta-n-\varepsilon-\sum_{i=1}^{r}h_iU_i;k_1,\ldots,k_R];[1-\varepsilon+\alpha+n \end{matrix} \right.$$

$$\left.\begin{matrix}[(g):\gamma',\ldots,\gamma^{(R)}];[(q'):\eta'];\ldots;[(q^{(R)}):\eta^{(R)}]; \\ -\sum_{i=1}^{r}h_iU_i:k_1,\ldots,k_R];[(p'):\varepsilon'];\ldots;[(p^{(R)}):\varepsilon^{(R)}];\end{matrix}\right.\begin{matrix}\\ y_1,\ldots,y_R\end{matrix}\Bigg],$$

which holds under the same conditions as (2.1).

(6) If we replace v by $v+2$ and take $v=\sigma=2, \alpha_1=\alpha', \alpha_2=\alpha'', \alpha_3=\alpha+1, \alpha_4=\beta+1=\beta_2$ and choose $\gamma_i=1=\mu_j$, where $i=1,\ldots,v+2$ and $j=1,\ldots,\sigma$ also taking $h_1=\ldots=h_r=0$ in result (4.5), then it reduces to a known result obtained by Olkha and Chaurasia ([2, eq.(3.2)], when $n_1=\ldots=n_R=0$).

(7) By taking $h_1=\ldots=h_r=0$ in result (4.5), we get a known result obtained by Chaurasia and Godika ([1, eq.(3.2)], when $n_1=\ldots=n_R=0$).

(8) When we take $h_1=\ldots=h_r=0$ in result (4.4), then it reduces to a known result obtained by Chaurasia and Godika ([1, eq.(3.3)], when $n_1=\ldots=n_R=0$).

(9) If we put $r=1, h_1=\ldots=h_r=0$ in result (4.4), we get a known result obtained by Chaurasia and Godika ([1, eq.(3.3)],when $n_1=\ldots=n_R=0$).

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Department of Mathematics,
University of Rajasthan,
Jaipur-302004, Rajasthan,
INDIA