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BOOK REVIEWS

CLARK, J.; LOMP, C.; VANAJA, N.; WISBAUER, R. – *Lifting Modules, Supplements and Projectivity in Module Theory*, Frontiers in Mathematics, Birkhäuser Verlag, Basel-Boston-Berlin, 2006, xiv+394 p., CHF 78.00/EUR 48.00, ISBN 3-7643-7572-8.

This monograph provides a comprehensive and up-to-date account of the theory of lifting modules and essential related topics.

The extending modules, which generalize injective modules, have been thoroughly studied and are well documented in the literature. The dual concept of lifting modules, a generalization of projective supplemented modules, is less present in book form studies. This volume consists largely of results that appear for the first time in book form and is thus a worthy tool for mathematicians interested in this area.

The first chapter reviews basic notions and techniques in module theory which are fundamental for the rest of the book: injectivity, small submodules, the radical, coclosure, projectivity, hollow dimension. Chapter 2 presents torsion theory aspects pertaining to lifting modules. Decomposability properties that are particularly relevant to lifting modules are presented in chapter 3: exchange rings, internal exchange property, LE-modules, local summands, stable rank 1 and cancellation, decomposition uniqueness.

Chapter 4 is devoted to supplements in modules, an essential topic for the chapters that follow. The semilocal modules and rings are studied and weak supplements are introduced. Finite hollow dimension and its connections with weak supplements is studied, as well as modules with semilocal endomorphism rings. Section 20 of this chapter treats supplements and finitely, cofinitely, respectively amply supplemented modules. The connections with hollow submodules and hollow dimension are studied. The remaining section of this chapter deals with modules for which every submodule have a unique coclosure (UCC modules).

Lifting modules are finally introduced in Chapter 5. The indecomposable lifting modules are shown to be the hollow modules and some sufficient conditions for a lifting module to possess an indecomposable decomposition are given. The interplay between chain conditions and the lifting property

is studied. The next sections investigate the lifting property with respect to direct sums (finite and infinite); in particular, a LE-module M such that the direct sum of any family of copies of M is lifting is proved to be local and self-projective. The indecomposable summands of modules with this property (called $\Sigma-$modules) are looked at in the next sections. Section 26 studies some remarkable classes of lifting modules: weakly discrete, quasi-discrete, discrete, and strongly discrete modules and gives characterizations and properties regarding decomposition, factor modules, and direct sums for each class. The *Harada modules* (modules M such that every injective module in $\sigma[M]$ is lifting) *co-Harada modules* (modules M such that every projective module in $\sigma[M]$ is Σ-extending) are the subject of section 27. Some results involving quasi-Frobenius, Harada and co-Harada rings are generalized here in the framework of lifting modules theory. This duality is further developed in the next section, considering modules M for which extending (or all) modules in $\sigma[M]$ are lifting.

The Appendix contains two graph theory topics, Hall's Marriage Theorem and König's Graph Theorem, used in Chapter 3.

Many sections include a paragraph of Comments and some exercises.

C. Volf

COSTA, DAVID G. – *An Invitation to Variational Methods in Differential Equations*, Birkhäuser, Boston-Basel-Berlin, xii+138p., 2007, ISBN-13:978-0-8176-4535-9.

The book treats the variational methods with applications to boundary value problems for semilinear partial differential equations of elliptic type.

The volume of the book is relatively restraint compared to the big amount of research directions in the field but the author succeeds to point out the main tools and methods and to give suggestive examples. Thus, the main subjects that are treated in the book are the direct minimization, the mountain pass theorem, minimization problems with constraints, a duality principle, as well as variational problems with symmetry and Lusternik-Schnirelman theory. Last two chapters are dedicated to the problems with lack of compactness, either in unbounded domain as well as in bounded domains. The book has a large bibliography, very useful for those wishing to continue their research in the field. Exercise sections are also provided to some of the chapters.

As a conclusion, the book is clearly written and may be very useful for students and appropriate for a one semester course support.

C. Lefter