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UNSTEADY FLOW OF A CONDUCTING DUSTY FLUID BETWEEN TWO PARALLEL PLATES STARTED IMPULSIVELY FROM REST

BY

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Abstract. The unsteady laminar flow of an electrically conducting viscous incompressible fluid between two infinitely extended non-conducting parallel plates under the influence of a uniform magnetic field has been considered. The lower and upper plates are started impulsively from rest and thereafter move with different but uniform velocities. The velocity profiles for conducting fluid and non-conducting dust particles have been determined. The expression for discharge per unit breadth of the plate and skin-friction at lower plate are obtained. Finally the effect of strength of magnetic field on velocity profiles at fixed time has been discussed.

Mathematics Subject Classification 2000: 76T10, 76T15.

Key words: Frenet frame field system, parallel plates, dusty fluid, velocity of dust phase and fluid phase, conducting dusty fluid, magnetic field.

1. Introduction. The influence of dust particles on viscous flows has great importance in petroleum industry and in the purification of crude oil. Other important applications of dust particles in boundary layer, include soil erosion by natural winds and dust entrainment in a cloud during nuclear explosion. Also such flows occur in a wide range of areas of technical importance like fluidization, flow in rocket tubes, combustion, paint spraying and more recently blood flows in capillaries.

The flow of a viscous, incompressible and electrically conducting fluid in presence of an external magnetic field due to the impulsive motion of an infinite flate plate has been discussed by Rossow [13]. Ong and Nicholls [10] have extend the problem to cover the case of flow near an infinite wall which executes simple harmonic motion parallel to itself. Saffman [14] formulated the basic equations for the flow of dusty fluid. Regarding the plate problems, Liu [9] have studied the flow produced by the motion of an infinite plane in a steady fluid occupying the semi-infinite space above it. Later, Baral [5] has discussed the plane parallel flow of conducting dusty gas.

To investigate the kinematical properties of fluid flows in the field of fluid mechanics some researchers like Kanwal [8], Truesdell [15], Indrasena [7], Purushotham [11], Bagewadi, Shantharajappa and Gireesha [1, 2, 3] have applied differential geometry techniques. Further, recently the authors [2, 3] have studied two-dimensional dusty fluid flow in Frenet frame field system. In the present study, the flow of an electrically conducting viscous incompressible fluid which suspended non-conducting small spherical dust particles due to the impulsive motion of two infinitely extended non-conducting parallel plates in the presence of a uniform transverse magnetic field fixed relative to the fluid is discussed. The lower and upper plates start moving with uniform velocities u_0 and v_0 respectively in the same direction. Initially both the conducting fluid and the dust particles are assumed to be at rest. Applying Laplace transform technique, the velocity fields for fluid and dust particles have been obtained. Also the drag on lower the plate and total volume flow in between the plates have been calculated. Finally results have been represented graphically for different values of u_0 and v_0.

2. Equations of motion. The modified Saffman's [14] equations for the conducting dusty gas and non-conducting dust particle are:
For fluid phase

$$\nabla.\overrightarrow{u} = 0 \tag{2.1}$$

$$\frac{\partial \overrightarrow{u}}{\partial t} + (\overrightarrow{u}.\nabla)\overrightarrow{u} = -\rho^{-1}\nabla p + \nu\nabla^2\overrightarrow{u} + \frac{kN}{\rho}(\overrightarrow{v} - \overrightarrow{u}) + \frac{1}{\rho}(\overrightarrow{J} \times \overrightarrow{B}). \tag{2.2}$$

For dust phase

$$\nabla.\overrightarrow{v} = 0 \tag{2.3}$$

$$\frac{\partial \overrightarrow{v}}{\partial t} + (\overrightarrow{v}.\nabla)\overrightarrow{v} = \frac{k}{m}(\overrightarrow{u} - \overrightarrow{v}). \tag{2.4}$$

We have following nomenclature:

$\overrightarrow{u}-$velocity of the fluid phase, $\overrightarrow{v}-$velocity of dust phase, $\rho-$density of the gas, $p-$pressure of the fluid, $N-$number density of dust particles, $\nu-$kinematic viscosity, $k = 6\pi a\mu-$Stoke's resistance (drag coefficient), $a-$spherical radius of dust particle, $m-$mass of the dust particle, $\mu-$the co-efficient of viscosity of fluid particles, $t-$time and $\overrightarrow{J}$ and $\overrightarrow{B}$ given by Maxwell's equations and Ohm's law, namely,

$$\nabla\times\overrightarrow{H} = 4\pi\overrightarrow{J},\ \nabla\times\overrightarrow{B} = 0,\ \nabla\times\overrightarrow{E} = 0,\ \overrightarrow{J} = \sigma[\overrightarrow{E} + \overrightarrow{u}\times\overrightarrow{B}]. \tag{2.5}$$

Here $\overrightarrow{H}-$magnetic field, $\overrightarrow{J}-$current density, $\overrightarrow{B}-$magnetic Flux, $\overrightarrow{E}$-electric field and $\sigma-$ the electrical conductivity of the fluid.

It is assumed that the effect of induced magnetic fields produced by the motion of the electrically conducting gas is negligible and no external electric field is applied. With those assumptions the magnetic field $\overrightarrow{J}\times\overrightarrow{B}$ of the body force in (2.2) reduces simply to $-\sigma B_0^2\overrightarrow{u}$, where B_0-the intensity of the imposed transverse magnetic field.

Let $\overrightarrow{s}, \overrightarrow{n}, \overrightarrow{b}$ be triply orthogonal unit vectors tangent, principal normal, binormal respectively to the spatial curves of congruences formed by fluid phase velocity and dusty phase velocity lines respectively, Geometrical relations are given by Frenet formulae [4]

$$\begin{aligned}
&(2.6)\quad i) && \frac{\partial\overrightarrow{s}}{\partial s} = k_s\overrightarrow{n},\ \frac{\partial\overrightarrow{n}}{\partial s} = \tau_s\overrightarrow{b} - k_s\overrightarrow{s},\ \frac{\partial\overrightarrow{b}}{\partial s} = -\tau_s\overrightarrow{n};\\
&ii) && \frac{\partial\overrightarrow{n}}{\partial n} = k'_n\overrightarrow{s},\ \frac{\partial\overrightarrow{b}}{\partial n} = -\sigma'_n\overrightarrow{s},\ \frac{\partial\overrightarrow{s}}{\partial n} = \sigma'_n\overrightarrow{b} - k'_n\overrightarrow{n};\\
&iii) && \frac{\partial\overrightarrow{b}}{\partial b} = k''_b\overrightarrow{s},\ \frac{\partial\overrightarrow{n}}{\partial b} = -\sigma''_b\overrightarrow{s},\ \frac{\partial\overrightarrow{s}}{\partial b} = \sigma''_b\overrightarrow{n} - k''_b\overrightarrow{b};\\
&iv) && \nabla.\overrightarrow{s} = \theta_{ns} + \theta_{bs};\ \nabla.\overrightarrow{n} = \theta_{bn} - k_s;\ \nabla.\overrightarrow{b} = \theta_{nb},
\end{aligned}$$

where $\partial/\partial s$, $\partial/\partial n$ and $\partial/\partial b$ are the intrinsic differential operators along fluid phase velocity (or dust phase velocity) lines, principal normal and binormal. The functions (k_s, k'_n, k''_b) and $(\tau_s, \sigma'_n, \sigma''_b)$ are the curvatures and torsion of the above curves and θ_{ns} and θ_{bs} are normal deformations of these spatial curves along their principal normal and binormal respectively.

3. Formulation and solution of the problem. The present discussion considers a viscous incompressible, dusty fluid bounded by two infinite flat plates seperated by a distance h. The upper and lower plates are moving with the constant speeds u_0 and v_0 respectively. Both the fluid and the dust particle clouds are supposed to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size. The number density of the dust particles is taken as a constant throughout the flow. It is assumed that the dust particles are electrically nonconducting and neutral. The motion of the dusty fluid is due to magnetic field of uniform strength B_0. Under these assumptions the flow will be a parallel flow in which the streamlines are along the tangential direction and the velocities are varies along binormal direction and with time t, since we extended the fluid to infinity in the principal normal direction.

By virtue of system of equations (2.6) the intrinsic decomposition of equations (2.2) and (2.4) give the following forms

$$\frac{\partial u_s}{\partial t} = \nu \left[\frac{\partial^2 u_s}{\partial b^2} - C_r u_s \right] + \frac{kN}{\rho}(v_s - u_s) - Du_s \tag{3.1}$$

$$2u_s^2 k_s = \nu \left[2\sigma_b'' \frac{\partial u_s}{\partial b} - u_s k_s^2 \right] \tag{3.2}$$

$$0 = \nu \left[u_s k_s \tau_s - 2k_b'' \frac{\partial u_s}{\partial b} \right] \tag{3.3}$$

$$\frac{\partial v_s}{\partial t} = \frac{k}{m}(u_s - v_s) \tag{3.4}$$

$$2v_s^2 k_s = 0 \tag{3.5}$$

where $D = \frac{\sigma B_0^2}{\rho}$ and $C_r = (\sigma_b'^2 + k_n'^2 + k_b'^2 + \sigma_b''^2)$ is called curvature number [3].

From equation (3.5) we see that $v_s^2 k_s = 0$ which implies either $v_s = 0$ or $k_s = 0$. The choice $v_s = 0$ is impossible, since if it happens then $u_s = 0$, which shows that the flow doesn't exist. Hence $k_s = 0$, it suggests that the curvature of the streamline along tangential direction is zero. Thus no radial flow exists.

Equation (3.1) and (3.4) are to be solved subject to the initial and boundary conditions;

$$\left.\begin{cases} \text{Initial condition;} & \text{at } t=0;\ u_s=0, v_s=0 \\ \text{Boundary condition;} & \text{for } t>0;\ u_s=u_0,\ \text{at } b=0 \\ \text{and } u_s=v_0 \text{ at } b=h & \end{cases}\right\}. \tag{3.6}$$

We define Laplace transformations of u_s and v_s as

$$U=\int_0^\infty e^{-xt}u_s dt \quad \text{and} \quad V=\int_0^\infty e^{-xt}v_s dt. \tag{3.7}$$

Applying the Laplace transform to equations (3.1), (3.4) and to boundary conditions, then by using initial conditions one obtains

$$xU=\nu\left[\frac{\partial^2 U}{\partial b^2}-C_r U\right]+\frac{l}{\tau}(V-U)-DU \tag{3.8}$$

$$xV=\frac{1}{\tau}(U-V) \tag{3.9}$$

$$U=\frac{u_0}{x},\ \text{at } b=0 \text{ and } U=\frac{v_0}{x} \text{ at } b=h \tag{3.10}$$

where $l=\frac{mN}{\rho}$ and $\tau=\frac{m}{k}$. Equation (3.9) implies

$$V=\frac{U}{1+x\tau}. \tag{3.11}$$

Eliminating V from (3.8) and (3.11) we obtain the following equation

$$\frac{d^2U}{db^2}-Q^2U=0 \tag{3.12}$$

where $Q^2=(C_r+\frac{x}{\nu}+\frac{D}{\nu}+\frac{xl}{\nu(1+x\tau)})$.

The velocities of fluid and dust particle are obtained by solving the equation (3.12) subjected to the boundary conditions (3.10) as follows

$$U=\frac{1}{x}\left\{\frac{v_0\sin h(Qb)-u_0\sin h(Q(b-h))}{\sin h(Qh)}\right\}.$$

Using U in (3.11) we obtain V as

$$V = \frac{1}{x(1+x\tau)} \left\{ \frac{v_0 \sin h(Qb) - u_0 \sin h(Q(b-h))}{\sin h(Qh)} \right\} .$$

By taking inverse Laplace transform to U and V, one can obtain

$$\begin{aligned} u_s &= \frac{v_0 \sin h(yb) - u_0 \sin h(y(b-h))}{\sin h(yh)} \\ &+ \frac{2\pi\nu}{h} \sum_{n=0}^{\infty} n(-1)^n \left[u_0 \sin\left(\frac{n\pi}{h}(b-h)\right) - v_0 \sin\left(\frac{n\pi}{h} b\right) \right] \\ &\times \left[\frac{e^{x_1 t}(1+x_1\tau)^2}{x_1\left[(1+x_1\tau)^2 + l\right]} + \frac{e^{x_2 t}(1+x_2\tau)^2}{x_2\left[(1+x_2\tau)^2 + l\right]} \right] \end{aligned}$$

$$\begin{aligned} v_s &= \frac{v_0 \sin h(yb) - u_0 \sin h(y(b-h))}{\sin h(yh)} + (u_0 - v_0) e^{-\frac{t}{\tau}} \\ &+ \frac{2\pi\nu}{h} \sum_{n=0}^{\infty} n(-1)^n \left[u_0 \sin\left(\frac{n\pi}{h}(b-h)\right) - v_0 \sin\left(\frac{n\pi}{h} b\right) \right] \\ &\times \left[\frac{e^{x_1 t}(1+x_1\tau)}{x_1\left[(1+x_1\tau)^2 + l\right]} + \frac{e^{x_2 t}(1+x_2\tau)}{x_2\left[(1+x_2\tau)^2 + l\right]} \right] \end{aligned}$$

where

$$\begin{aligned} x_1 &= -\frac{1}{2\tau}\left(1 + l + D\tau + \nu C_r \tau + \nu\tau\frac{n^2\pi^2}{h^2}\right) \\ &+ \frac{1}{2\tau}\sqrt{\left(1+l+D\tau+\nu C_r\tau+\nu\tau\frac{n^2\pi^2}{h^2}\right)^2 - 4\tau\left(C_r\nu + D + \nu\frac{n^2\pi^2}{h^2}\right)} \\ x_2 &= -\frac{1}{2\tau}\left(1 + l + D\tau + \nu C_r \tau + \nu\tau\frac{n^2\pi^2}{h^2}\right) \\ &- \frac{1}{2\tau}\sqrt{\left(1+l+D\tau+\nu C_r\tau+\nu\tau\frac{n^2\pi^2}{h^2}\right)^2 - 4\tau\left(C_r\nu + D + \nu\frac{n^2\pi^2}{h^2}\right)} \\ y &= \frac{C_r\nu + D}{\nu}. \end{aligned}$$

Flux and the shearing stress: The discharge of the conducting dusty fluid per unit breadth of the plate is

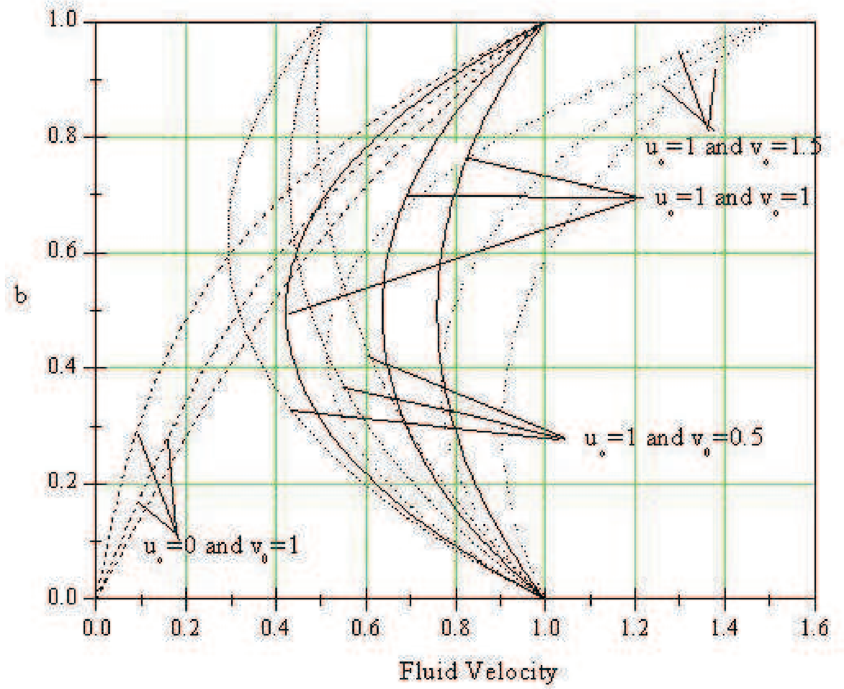
$$\begin{aligned} q &= \int_0^h u_s db \\ q &= \frac{(v_0 - u_0)}{y}[\coth(yh) - cosech(yh)] \\ &+2\nu \sum_{n=0}^{\infty} (-1)^n [u_0((-1)^n - 1) - v_0((-1)^n - 1)] \\ &\times \left[\frac{e^{x_1 t}(1 + x_1 \tau)}{x_1 \left[(1 + x_1 \tau)^2 + l\right]} + \frac{e^{x_2 t}(1 + x_2 \tau)}{x_2 \left[(1 + x_2 \tau)^2 + l\right]} \right]. \end{aligned}$$

The shearing stress at the lower plate is given by

$$\begin{aligned} D_{bs} &= \mu \left(\frac{\partial u}{\partial b} \right)_{b=0} \\ D_{bs} &= \frac{y}{\sin h(yh)} \left(v_0 - u_0 \cos h(yh) \right) \\ &+ \frac{2\pi^2 \nu}{h^2} \left[\frac{e^{x_1 t}(1 + x_1 \tau)}{x_1 \left[(1 + x_1 \tau)^2 + l\right]} + \frac{e^{x_2 t}(1 + x_2 \tau)}{x_2 \left[(1 + x_2 \tau)^2 + l\right]} \right] \sum_{n=0}^{\infty} n^2 \left(u_0 - v_0 \right). \end{aligned}$$

Conclusion. The velocity profiles for the fluid and dust particles are drawn as in figure 1, which are parabolic. According Frenet approximation of a curve in the osculating plane the path of the curve near origin is parabolic. Hence the results obtained here are analogous to the above [4]. It is concluded that velocity of fluid particles is parallel to velocity of dust particles. Also it is evident from the graphs that, as we increase the strength of the magnetic field, it has an appreciable effect on the velocities of fluid and dust particles. Further one can observe that if the magnetic field is zero then results are in agreement with the plane Couette flow. Also one can find that the drag on the lower plate and the total volume flow in between the plates decreases as magnetic field increases.

If $B_0 = 0$, and the plates are vibrating then the results analogous to [12]. Also if we consider the fluid in absence of pressure gradient and in the presence of magnetic field in [6] the results satisfied with this paper.



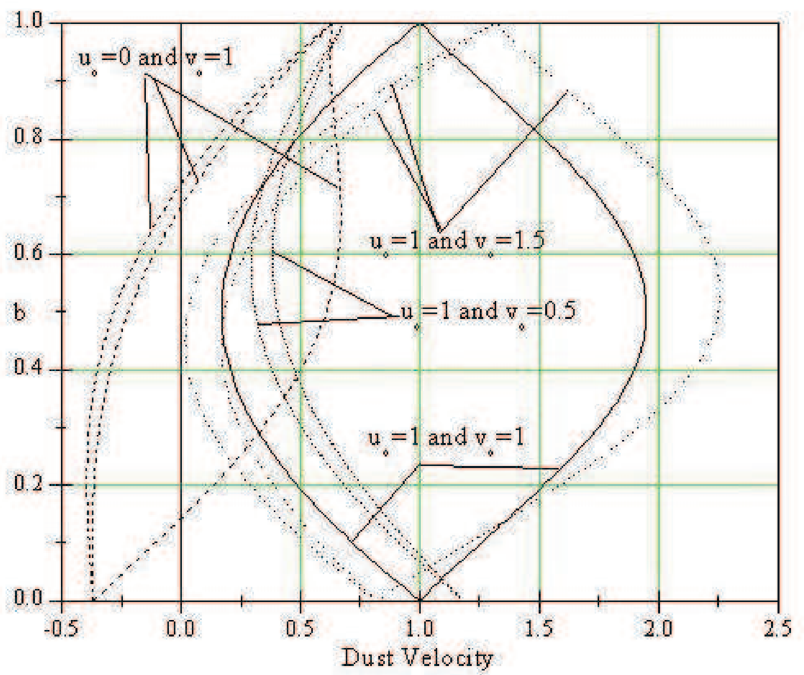


Figure-1: Variation of fluid and dust phase velocity.

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