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FUZZY STRONGLY PRECONTINUOUS FUNCTIONS

BY

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Abstract. In this paper, fuzzy semiopen and fuzzy preopen sets used to define and investigate a new class of functions called fuzzy strongly precontinuous. Relationships between the new class and other classes of functions are established.

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1. Introduction.

Ever since the introduction of fuzzy sets by Zadeh [9], the fuzzy concepts has involved almost all branches of Mathematics. The concept of fuzzy topological space has introduced by Chang [3] in 1968. Since then many fuzzy topologists have extended various notions in classical topology to fuzzy topological spaces. In this paper, fuzzy semiopen sets and fuzzy preopen sets are used to define and investigate a new class of functions called, fuzzy strongly precontinuous functions. Relationships between the new class and other classes of functions are established. Throughout this paper, X and Y are always fuzzy topological spaces. The class of all fuzzy sets on a universe X will be denoted by I^X.

2. Preliminaries.

Now, we introduced some basic notions and results that are used in the sequel.

Definition 2.1. *A fuzzy topology on a nonempty set X is a family δ of fuzzy subsets of X which satisfies the following three conditions:*

(i) $0, 1 \in \delta$;

(*ii*) *If* $g, h \in \delta$, *their* $g \wedge h \in \delta$;

(*iii*) *If* $f_i \in \delta$ *for each* $i \in I$, *then* $\vee_{i \in I} f_i \in \delta$.

The pair (X, δ) is called a fuzzy topological space [3].

Definition 2.2. *Members of* δ *are called fuzzy open sets* [3] *and complement of fuzzy open sets are called fuzzy closed sets* [3], *where the complement of a fuzzy set* A, *denoted by* A^c *is* $1 - A$.

Definition 2.3. [6] *The fuzzy subset* x_a *of a non-empty set* X, *with* $x \in X$ *and* $0 < a \leq 1$ *defined by*

$$x_a(p) = \begin{cases} a, & \text{if } p = x \\ 0, & \text{if } p \neq x \end{cases}$$

is called a fuzzy point in X *with suppost* x *and value* a. *The fuzzy point* x_1 *is called crisp point.*

Definition 2.4. [6] *Let* A *be fuzzy set in* X *and* x_a *a fuzzy point in* X. *We say that* $x_a \in A$ *if and only if* $x_a \leq A$.

Definition 2.5. *A fuzzy set* A *of a fuzzy topological space* X *is said to be fuzzy semiopen* [1] (*resp. fuzzy preopen* [2]) *if and only if* $A \leq \mathrm{Cl}(\mathrm{Int}(A))$ (*resp.* $A \leq \mathrm{Int}(\mathrm{Cl}(A))$), *where* $\mathrm{Cl}(A) = \wedge \{B | B \geq A, B \text{ is fuzzy closed in } X\}$ *and* $\mathrm{Int}(A) = \vee \{B | B \leq A, B \text{ is fuzzy open in } X\}$. *If* A *is fuzzy semiopen* (*resp. fuzzy preopen*), *then* $1 - A$ *is fuzzy semiclosed* (*resp. preclosed*).

Definition 2.6. [8] *Let* X *be a fuzzy topological space and* A *be any fuzzy set in* X. *The fuzzy semi-closure* (*pre-closure*) *of* A *in* X *is denoted by* $s\,\mathrm{Cl}(A)$ ($p\,\mathrm{Cl}(A)$) *as follows:* $s\,\mathrm{Cl}(A)$ ($p\,\mathrm{Cl}(A)$)$= \wedge\{B | B \geq A$, B *is fuzzy semiclosed* (*preclosed*) *in* $X\}$. *Similarly we can define* $s\,\mathrm{Int}(A)$.

Remark 2.7. [8] For a fuzzy set A of X, $1 - s\,\mathrm{Int}(A) = s\,\mathrm{Cl}(1 - A)$.

Remark 2.8. [8] A fuzzy set A is fuzzy semiclosed if and only if $s\,\mathrm{Cl}(A) = A$.

Definition 2.9. *A fuzzy set* A *in* X *is said to be q-coincident* [6] *with a fuzzy set* B, *denoted by* AqB, *if there exists* $x \in X$ *such that* $A(x) + B(x) > 1$. *It is known that* $A \leq B$ *if and only if* A *and* $1 - B$ *are not q-coincident, denoted by* $A\bar{q}(1 - B)$.

Definition 2.10. *A fuzzy set B is a quasi neighbourhood* [6] *(q- neighbourhood, for short) of x_α if and only if there exists a fuzzy open set U such that $x_\alpha q U \leq B$.*

Definition 2.11. *A fuzzy set A in X is said to be a semi-q-neighbourhood* [4] *(semi-q-nbd, for short) of x_α if and only if there exists a fuzzy semiopen set V in X such that $x_\alpha \; q \; V \leq A$.*

Definition 2.12. [3] *Let X and Y be two fuzzy topological spaces. Let $A \in I^X$, $B \in I^Y$. Then $f(A)$ is a fuzzy subset of Y, defined by*
$f(A) : Y \to [0,1]$

$$f(A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(\{y\})} A(x), & \text{if } f^{-1}(\{y\}) \neq \emptyset \\ 0, & \text{if } f^{-1}(\{y\}) = \emptyset \end{cases}$$

and $f^{-1}(B)$ is a fuzzy subset of X, defined by $f^{-1}(B)(x) = B(f(x))$.

Lemma 2.13. [1] *Let $f : X \to Y$ be a function and $\{A_\alpha\}$ be a family of fuzzy sets of Y, then*

(*i*) $f^{-1}(\vee A_\alpha) = \vee \; f^{-1}(A_\alpha)$ *and*

(*ii*) $f^{-1}(\Lambda A_\alpha) = \wedge \; f^{-1}(A_\alpha)$.

Lemma 2.14. [1] *For functions f_i: $X_i \to Y_i$ and fuzzy sets A_i of Y_i, $i = 1, 2$; we have $(f_1 \times f_2)^{-1}(A_1 \times A_2) = f_1^{-1}(A_1) \times f_2^{-1}(A_2)$.*

Lemma 2.15. [1] *Let $g : X \to X \times Y$ be the graph of a function $f : X \to Y$. Then, if A is a fuzzy set of X and B is a fuzzy set of Y, $g^{-1}(A \times B) = A \wedge f^{-1}(B)$.*

Definition 2.16. *A function $f : X \to Y$ is said to be:*

(*i*) *fuzzy irresolute* [8] *if $f^{-1}(V)$ is fuzzy in X for each fuzzy semiopen set V in Y;*

(*ii*) *fuzzy pre-continuous* [2] *if $f^{-1}(V)$ is fuzzy preopen set in X for each fuzzy open set V in Y.*

3. Fuzzy strongly precontinuous functions. We have introduced the following definition

Definition 3.1. *A function $f : X \to Y$ is said to be fuzzy strongly precontinuous if $f^{-1}(V)$ is fuzzy preopen in X for every fuzzy semiopen set V of Y.*

It is clear that every fuzzy strongly precontinuous function is fuzzy precontinuous. But the converse is not true as shown in the following example.

Example 3.2. Let $X = \{a, b, c\}$ and $\tau = \{0, A, 1\}$, where $A = (\frac{a}{0.2}, \frac{b}{0.3}, \frac{c}{0.4})$. Then the identity function $f : (X, \tau) \to (X, \tau)$ is fuzzy precontinuous but not fuzzy strongly precontinuous, since for the fuzzy semiopen set $B = (\frac{a}{0.8}, \frac{b}{0.7}, \frac{c}{0.7})$ of (X, τ), $f^{-1}(B)$ is not fuzzy preopen in (X, τ).

Theorem 3.3. *For a function $f : X \to Y$, the following are equivalent:*

(*i*) *f is fuzzy strongly precontinuous;*

(*ii*) *for each fuzzy point $x_\alpha \in X$ and each fuzzy semiopen set U of Y such that $f(x_\alpha) \leq U$, there exists a fuzzy preopen set V of X such that $x_\alpha \in B$ and $f(B) \leq U$;*

(*iii*) $f^{-1}(U) \leq \mathrm{Int}(\mathrm{Cl}(f^{-1}(U)))$ *for every fuzzy semiopen set U of Y;*

(*iv*) *$f^{-1}(F)$ is fuzzy preclosed in X for every fuzzy semiclosed set F of Y;*

(*v*) $\mathrm{Cl}(\mathrm{Int}(f^{-1}(A))) \leq f^{-1}(s\,\mathrm{Cl}(A))$ *for every fuzzy subset A of Y;*

(*vi*) $f(\mathrm{Cl}(\mathrm{Int}(B))) \leq s\,\mathrm{Cl}(f(B))$ *for every fuzzy subset B of X.*

Proof. (i)⇒(ii) Let x_α be a fuzzy point in X and U be any fuzzy semiopen set of Y such that $f(x_\alpha) \leq U$. Then $x_\alpha \in f^{-1}(f(x_\alpha)) \leq f^{-1}(U)$. Set $V = f^{-1}(U)$, then by (i), V is a fuzzy preopen subset of X such that $x_\alpha \in V$. Now, $f(V) = f(f^{-1}(U)) \leq U$.

(ii)⇒(iii) Let U be any fuzzy semiopen set of Y. Let x_α be any fuzzy point in X such that $f(x_\alpha) \in U$. Then $x_\alpha \in f^{-1}(U)$. By (ii), there exists a fuzzy preopen set V of X such that $x_\alpha \in V$ and $f(V) \leq U$. We obtain $x_\alpha \in V \leq f^{-1}(f(V)) \leq f^{-1}(U)$. This implies that $x_\alpha \in V \leq f^{-1}(U)$. Thus, we have $x_\alpha \in V \leq \mathrm{Int}(\mathrm{Cl}(V)) \leq \mathrm{Int}(\mathrm{Cl}(f^{-1}(U)))$ and hence $f^{-1}(U) \leq \mathrm{Int}(\mathrm{Cl}(f^{-1}(U)))$.

(iii)$\Rightarrow$(iv) Let F be any fuzzy semiclosed subset of Y. Then $1 - F$ is fuzzy semiopen in Y. By (iii), we obtain $f^{-1}(1-F) \leq \mathrm{Int}(\mathrm{Cl}(f^{-1}(1-F)))$. Then $1 - f^{-1}(F) \leq \mathrm{Int}(\mathrm{Cl}(1 - f^{-1}(F))) = 1 - \mathrm{Cl}(\mathrm{Int}(f^{-1}(F)))$ and hence $f^{-1}(F)$ is fuzzy preclosed in X.

(iv)$\Rightarrow$(v) Let A be any fuzzy subset of Y. Since $s\,\mathrm{Cl}(A)$ is a fuzzy semiclosed subset of Y, then $f^{-1}(s\,\mathrm{Cl}(A))$ is fuzzy preclosed in X and hence $\mathrm{Cl}(\mathrm{Int}(f^{-1}(s\,\mathrm{Cl}(A)))) \leq f^{-1}(s\,\mathrm{Cl}(A))$. Therefore, we obtain $\mathrm{Cl}(\mathrm{Int}(f^{-1}(A)))$ $\leq f^{-1}(s\,\mathrm{Cl}(A))$.

(v)$\Rightarrow$(vi) Let B be any fuzzy subset of X. By (v), we have $\mathrm{Cl}(\mathrm{Int}(B)) \leq \mathrm{Cl}(\mathrm{Int}(f^{-1}(f(B)))) \leq f^{-1}(s\,\mathrm{Cl}(f(B)))$ and hence $f(\mathrm{Cl}(\mathrm{Int}(B))) \leq s\,\mathrm{Cl}(f(B))$.

(vi)$\Rightarrow$(i) Let U be any fuzzy semiopen subset of Y. Since $f^{-1}(1-U) = 1 - f^{-1}(U)$ is a fuzzy subset of X and by (vi), we obtain $f(\mathrm{Cl}(\mathrm{Int}(f^{-1}(1 - V)))) \leq s\,\mathrm{Cl}(f(f^{-1}(1-U))) \leq s\,\mathrm{Cl}(1-U) = 1 - s\,\mathrm{Int}(U) = 1 - U$ and hence $1 - \mathrm{Int}(\mathrm{Cl}(f^{-1}(U))) = \mathrm{Cl}(\mathrm{Int}(1 - f^{-1}(U))) = \mathrm{Cl}(\mathrm{Int}(f^{-1}(1-U))) \leq f^{-1}(f(\mathrm{Cl}(\mathrm{Int}(f^{-1}(U))))) \leq f^{-1}(1-U) = 1 - f^{-1}(U)$. Therefore, we have $f^{-1}(U) \leq \mathrm{Int}(\mathrm{Cl}(f^{-1}(U)))$ and hence $f^{-1}(U)$ is fuzzy preopen in X. Thus, f is fuzzy strongly precontinuous. □

Lemma 3.4. *Let A be a fuzzy subset of a fuzzy space X. Then A is fuzzy semiopen if and only if* $\mathrm{Cl}(A) = \mathrm{Cl}(\mathrm{Int}(A))$.

Proof. Necessity. Suppose A is fuzzy semiopen. Then by definition $A \leq \mathrm{Cl}(\mathrm{Int}(A))$ and so $\mathrm{Cl}(A) \leq \mathrm{Cl}(\mathrm{Int}(A))$. On the other hand, we have $\mathrm{Int}(A) \leq A$ and hence $\mathrm{Cl}(\mathrm{Int}(A)) \leq \mathrm{Cl}(A)$. Consequently, we obtain $\mathrm{Cl}(A)$ $= \mathrm{Cl}(\mathrm{Int}(A))$.

Sufficiency: By hypothesis, $\mathrm{Int}(A) \leq A \leq \mathrm{Cl}(A) = \mathrm{Cl}(\mathrm{Int}(A))$. Hence A is fuzzy semiopen. □

Lemma 3.5. *If A is a nonzero fuzzy semiopen set, then* $\mathrm{Int}\,A \neq 0$.

Proof. Since A is fuzzy semiopen, by Lemma 3.4, we have $\mathrm{Cl}(A) = \mathrm{Cl}(\mathrm{Int}(A))$. Suppose $\mathrm{Int}(A) = 0$. Then we have $\mathrm{Cl}(A) = 0$ and hence $A = 0$. This contrary to the hypothesis. Therefore, $\mathrm{Int}(A)$ is a non zero fuzzy subset of X. □

Lemma 3.6. [1] *Let $(X_i, \tau_i)_{i\in\wedge}$ be any family of fuzzy topological spaces and λ_i be fuzzy subset of X_i for all $i \in \wedge$. Then (i)* $\mathrm{Int}(\Pi\lambda_i) = \Pi\,\mathrm{Int}(\lambda_i)$ *if $\lambda_i = X_i$ except for a finite $i \in \wedge$ and* $\Pi\,\mathrm{Int}(\lambda_i) \neq 0$.

(ii) $\mathrm{Cl}(\Pi\lambda_i) = \Pi\,\mathrm{Cl}(\lambda_i)$.

Theorem 3.7. [2] *Let X and Y be a fuzzy topological spaces such that X is product related to Y. Then the product $\lambda \times \mu$ of a fuzzy preopen set λ in X and a fuzzy preopen set μ in Y, is a fuzzy preopen set in the fuzzy product space.*

Theorem 3.8. *Let X and Y be a fuzzy topological spaces such that X is product related to Y. Then the product $\lambda \times \mu$ of a fuzzy semiopen set λ in X and a fuzzy semiopen set μ in Y, is a fuzzy semiopen set in the fuzzy product space.*

Proof. Similar to the proof of Theorem 3.7. □

Theorem 3.9. *If $f_i : (X_i, \tau_i) \to (Y_i, \sigma_i)$ $(i = 1, 2)$ are fuzzy strongly precontinuous and Y_1 is product related to Y_2, then $f_i : X_1 \times X_2 \to Y_1 \times Y_2$ is fuzzy strongly precontinuous.*

Proof. Let $F = \vee(A_i \times B_i)$, where A_i's and B_i's are fuzzy semiopen sets of Y_1 and Y_2, respectively. Since Y_1 is product related to Y_2, then by Theorem 3.8, F is fuzzy semiopen in $Y_1 \times Y_2$. Then we have,

$$\begin{aligned}
(f_1 \times f_2)^{-1}(F) &= \vee(f_1^{-1}(A_i) \times f_2^{-1}(B_i)) \\
&\leq \vee(\mathrm{Int}(\mathrm{Cl}(f_1^{-1}(A_i))) \times \mathrm{Int}(\mathrm{Cl}(f_2^{-1}(B_i)))) \\
&\leq \vee(\mathrm{Int}(\mathrm{Cl}(f_1^{-1}(A_i) \times f_2^{-1}(B_i)))^2) \\
&\leq \vee(\mathrm{Int}(\mathrm{Cl}(f_1^{-1}(A_i) \times f_2^{-1}(B_i)))) \\
&\leq \mathrm{Int}(\mathrm{Cl}(\vee(f_1 \times f_2)^{-1}(A_i \times B_i))) \\
&\leq \mathrm{Int}(\mathrm{Cl}(f_1 \times f_2)^{-1}(\vee(A_i \times B_i))) \\
&\leq \mathrm{Int}(\mathrm{Cl}(f_1 \times f_2)^{-1}(F)).
\end{aligned}$$

Hence $f_1 \times f_2$ is fuzzy strongly precontinuous. □

Theorem 3.10. *Let $f : X \to Y$ be a function, where X is product related to Y, and let $g : X \to X \times Y$ given by $g(x) = (x, f(x))$ be its graph mapping. Then f is fuzzy strongly precontinuous if g is fuzzy strongly precontinuous.*

Proof. Suppose that g is a fuzzy strongly precontinuous function and A is a fuzzy semiopen set in Y. Then $f^{-1}(A) = 1 \wedge f^{-1}(A) = g^{-1}(1 \times A) \leq \mathrm{Int}(\mathrm{Cl}(g^{-1}(1 \times A))) = \mathrm{Int}(\mathrm{Cl}(1 \wedge f^{-1}(A)))$, by Lemma 2.15. It follows that $f^{-1}(A) \leq \mathrm{Int}(\mathrm{Cl}(f^{-1}(A)))$. That is, $f^{-1}(A)$ is fuzzy preopen in X. This shows that f is fuzzy strongly precontinuous. □

Theorem 3.11. *If a function $f : X \to \Pi Y_i$ is fuzzy strongly precontinuous, then $P_i \circ f : X \to Y_i$ is fuzzy strongly precontinuous, where P_i is the projection of ΠY_i onto Y_i.*

Proof. Let λ_i be an arbitrary fuzzy semiopen set of Y_i. Since P_i is fuzzy continuous and fuzzy open, it is fuzzy irresolute [8] and hence $P_i^{-1}(V_i)$ is a fuzzy semiopen in ΠY_i. Since f is fuzzy strongly precontinuous, then $f^{-1}(P_i^{-1}(V_i)) = (P_i \circ f)^{-1}(V_i)$ is fuzzy preopen in X. Hence, $P_i \circ f$ is fuzzy strongly precontinuous for each $i \in \wedge$. □

Theorem 3.12. *If the product function $f : \Pi X_i \to \Pi Y_i$ is fuzzy strongly precontinuous, then $f_i : X_i \to Y_i$ is fuzzy strongly precontinuous for each $i \in \wedge$.*

Proof. Let $i_0 \in \wedge$ be an arbitrary fixed integer and λ_{i_0} be any fuzzy semiopen set of Y_{i_0} Then, $\Pi_{\beta \neq i_0} Y_\beta \times \lambda_{i_0}$ is fuzzy semiopen in ΠY_i by Theorem 3.7. Since f is fuzzy strongly precontinuous, then $f^{-1}(\lambda_{i_0} \times \Pi_{\beta \neq i_0} Y_\beta) = f_{i_0}^{-1}(\lambda_{i_0}) \times f_{i_0}^{-1}(X_\beta)$ is fuzzy preopen in ΠX_i and hence, by Theorem 3.8, $f_{i_0}^{-1}(\lambda_{i_0})$ is fuzzy preopen set in X_{i_0}. This shows that f_{i_0} is fuzzy strongly precontinuous. □

Theorem 3.13. *Let $f : X \to Y$ and $g : Y \to Z$ be functions. Then the composition $g \circ f : X \to Z$ is fuzzy strongly precontinuous if f is fuzzy strongly precontinuous and g is fuzzy irresolute.*

Proof. Let λ be any fuzzy semiopen subset of Z. Since g is fuzzy irresolute, $g^{-1}(\lambda)$ is fuzzy semiopen in Y. Since f is fuzzy strongly precontinuous, then $(g \circ f)^{-1}(\lambda) = f^{-1}(g^{-1}(\lambda))$ is fuzzy preopen in X and hence $g \circ f$ is fuzzy strongly precontinuous. □

Definition 3.14. *A filter base Λ is said to be fuzzy pre-convergent to a fuzzy point x_α in X if for any fuzzy preopen set λ in X containing x_α, there exists a fuzzy set $B \in \Lambda$ such that $B \leq \lambda$.*

Definition 3.15. *A filter base Λ is said to be fuzzy semi-convergent to a fuzzy point x_α in X if for any fuzzy semiopen set λ in X containing x_α, there exists a fuzzy set $B \in \Lambda$ such that $B \leq \lambda$.*

Theorem 3.16. *If a function $f : X \to Y$ is fuzzy strongly pre-continuous, then for each fuzzy point $x_\alpha \in X$ and each filter base Λ in X fuzzy-converging to x_α, the filter base $f(\Lambda)$ is fuzzy semi-convergent to $f(x_\alpha)$.*

Proof. Let $x \in X$ and Λ be any filter base in X fuzzy-converging to x_α. Since f is fuzzy strongly pre-continuous, then for any fuzzy semiopen set λ in Y containing $f(x_\alpha)$, there exists a fuzzy preopen set B in X containing x_α such that $f(B) \leq \lambda$. Since Λ is fuzzy pre-converging to x_α, there exists a fuzzy set $\rho \in \Lambda$ such that $\rho \leq B$. This means that $f(\rho) \leq \lambda$ and therefore the fuzzy filter base $f(\Lambda)$ is fuzzy semi-convergent to $f(x_\alpha)$. □

Definition 3.17. *A collection B of fuzzy sets in a fuzzy space X is said to be cover* [5] *of a fuzzy set η of X if and only if $(\bigcup_{A\in B} A)(x) = 1$, for every $x \in S(\eta)$. A fuzzy cover B of a fuzzy set η in a fuzzy space X is said to have a finite subcover if and only if there exists a finite subcollection $\rho = \{A_1, A_2, ...A_n\}$ of B such that $(\bigvee_{j=1}^{n} A_j)\ (x) \geq \eta(x)$, for every $x \in s(\eta)$, where $s(\eta)$ denotes the support of a fuzzy set η.*

Definition 3.18. *A fuzzy space X is said to be:*

(*i*) *fuzzy pre-compact* (*resp. fuzzy semi-compact*) *if every fuzzy preopen* (*resp. fuzzy semiopen*) *cover of X has a finite subcover;*

(*ii*) *fuzzy countably pre-compact* (*resp. fuzzy countably semi-compact*) *if every fuzzy preopen* (*resp. fuzzy semiopen*) *countably cover of X has a finite subcover;*

(*iii*) *fuzzy pre-Lindelof* (*resp. fuzzy semi-Lindelof*) *if every cover of X by fuzzy preopen* (*resp. fuzzy semiopen*) *sets has a countable subcover.*

Theorem 3.19. *Let $f : X \to Y$ be a fuzzy strongly pre-continuous surjective function. Then the following statements hold:*

(*i*) *If X is fuzzy pre-compact, then Y is fuzzy semi-compact;*

(*ii*) *If X is fuzzy pre-Lindelof, then Y is fuzzy semi-Lindelof;*

(*iii*) *If X is fuzzy countably pre-compact, then Y is fuzzy countably semi-compact.*

Proof. (i) Let $\{\lambda_\alpha : \alpha \in I\}$ be any fuzzy semiopen cover of Y. Since f is fuzzy strongly pre-continuous, then $\{f^{-1}(\lambda_\alpha) : \alpha \in I\}$ is fuzzy preopen cover of X. Since X is fuzzy pre-compact, there exists a finite subset I_0 of I such that $\vee\{f^{-1}(\lambda_\alpha) : \alpha \in I_0\} = 1$. Thus, we have $\{\lambda_\alpha : \alpha \in I_0\} = 1$ and Y is fuzzy semi-compact.

The other proofs are similar. □

Recall that fuzzy sets λ_1 and λ_2 in a fuzzy space X are said to be fuzzy pre-separated [7] (resp. fuzzy semi-separated [4]) if $p\,\mathrm{Cl}(A) + B \leq 1$ and $A + p\,\mathrm{Cl}(B) \leq 1$ (resp. $s\,\mathrm{Cl}(A) + B \leq 1$ and $A + p\,\mathrm{Cl}(B) \leq 1$). A fuzzy set S in a fuzzy space X is said to be fuzzy preconnected (resp. fuzzy semiconnected) if and only if S cannot be expressed as the union of two fuzzy preseparated (resp. fuzzy semiseparated) sets.

Theorem 3.20. *Let $f : X \to Y$ be a fuzzy strongly precontinuous surjective mapping. If B is a fuzzy preconnected subset of X, then $f(B)$ is a fuzzy semiconnected subset of Y.*

Proof. Suppose that $f(B)$ is not fuzzy semiconnected in Y. Then there exist fuzzy semiseparated sets A and B in Y such that $f(B) = A \vee B$, that is, there exist fuzzy semiopen sets A_1 and B_1 such that $A \leq A_1$ and $B \leq B_1$, $A\bar{q}B_1$ and $B\bar{q}A_1$. Since f is fuzzy strongly precontinuous, $f^{-1}(A)$ and $f^{-1}(B)$ are fuzzy preopen sets of X and $B = f^{-1}(f(B)) = f^{-1}(A \vee B) = f^{-1}(A) \vee f^{-1}(B)$. It is clear that $f^{-1}(A)$ and $f^{-1}(B)$ are fuzzy preseparated sets in X. Therefore, B is not fuzzy preconnected in X. □

Theorem 3.21. *If $f : X \to Y$ is fuzzy strongly precontinuous mapping and X is fuzzy preconnected, then Y is fuzzy semiconnected.*

Proof. Follows from the definitions. □

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