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ON STRONGLY PRE-IRRESOLUTENESS VIA TOPOLOGICAL IDEALS

BY

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Abstract. Yuksel et. al. introduced and studied pre-$\mathcal{I}$-irresolute functions. The aim of this paper is to give a new class of functions called strongly pre-$\mathcal{I}$-irresolute functions in ideal topological space which is stronger than the class of pre-$\mathcal{I}$-irresolute functions. Some characterizations and several basic properties of this class of functions are obtained.

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1. Introduction

In 1990, Jankovic and Hamlett [5] have defined the concept of $\mathcal{I}$-open set via local function which was given by Vaidyanathaswamy [7]. The latter concept was also established utilizing the concept of an ideal whose topic in general topological spaces was treated in the classical text by Kuratowski [6]. In 1992, Abd El-Monsef et. al. [1] studied a number of properties of $\mathcal{I}$-open sets, $\mathcal{I}$-closed sets, $\mathcal{I}$-continuous functions and investigated several of their properties. In 1999, Dontchev [2] has introduced the notion of pre-$\mathcal{I}$-open sets which are weaker than that of $\mathcal{I}$-open sets. In this paper, a new class of functions called strongly pre-$\mathcal{I}$-irresolute functions in ideal topological spaces by utilizing pre-$\mathcal{I}$-open sets is introduced and some characterizations and several basic properties are obtained.

2. Preliminaries

Throughtout this paper, for a subset A of a topological space (X, τ), the closure and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. An ideal topological space is a topological space (X, τ) with an ideal $\mathcal{I}$ on X, and is denoted by $(X, \tau, \mathcal{I})$, where the ideal is defined as a nonempty collection of subsets of X satisfying the following two conditions. (i) If $A \in \mathcal{I}$ and $B \subset A$, then $B \in \mathcal{I}$; (ii) If $A \in \mathcal{I}$ and $B \in \mathcal{I}$, then $A \cup B \in \mathcal{I}$. For a subset $A \subset X$, $A^*(\mathcal{I}) = \{x \in X | \ U \cap A \notin \mathcal{I}$ for each neighbourhood U of $x\}$ is called the local function of A with respect to $\mathcal{I}$ and τ [5]. Where there is no chance of confusion, $A^*(\mathcal{I})$ is denoted by A^*. Note that often X^* is a proper subset of X. For every ideal topological space $(X, \tau, \mathcal{I})$, there exists a topology $\tau^*(\mathcal{I})$, finer than τ, generated by the base $\beta(\tau, \mathcal{I}) = \{U \backslash \mathcal{I} \mid U \in \tau$ and $\mathcal{I} \in \mathcal{I}\}$. But in general $\beta(\mathcal{I}, \tau)$ is not always a topology [5]. Observe additionally that $\mathrm{Cl}^*(A) = A^* \cup A$ defines a Kuratowski closure operator for $\tau^*(\mathcal{I})$. A subset S of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be pre-$\mathcal{I}$-open [2] if $S \subset \mathrm{Int}(\mathrm{Cl}^*(S))$. The complement of a pre-$\mathcal{I}$-open set is called pre-$\mathcal{I}$-closed [2]. The intersection of all pre-$\mathcal{I}$-closed sets containing S is called the pre-$\mathcal{I}$-closure [8] of S and is denoted by ${}_{P\mathcal{I}}\mathrm{Cl}(S)$. A set S is pre-$\mathcal{I}$-closed if and only if ${}_{P\mathcal{I}}\mathrm{Cl}(A) = A$. The pre-$\mathcal{I}$-interior [8] of S is defined by the union of all pre-$\mathcal{I}$-open sets of $(X, \tau, \mathcal{I})$ contained in S and is denoted by ${}_{P\mathcal{I}}\mathrm{Int}(S)$. The family of all pre-$\mathcal{I}$-open (resp. pre-$\mathcal{I}$-closed) sets of $(X, \tau, \mathcal{I})$ is denoted by $P\mathcal{I}O(X)$ [8] (resp. $P\mathcal{I}C(X)$). The family of all pre-$\mathcal{I}$-open (resp. pre-$\mathcal{I}$-closed) sets of $(X, \tau, \mathcal{I})$ containing a point $x \in X$ is denoted by $P\mathcal{I}O(X, x)$ (resp. $P\mathcal{I}C(X, x)$).

3. Strongly pre-$\mathcal{I}$-irresolute functions

We have introduced the following definition

Definition 3.1. A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is said to be strongly pre-$\mathcal{I}$-irresolute if for every $x \in X$ and every $V \in P\mathcal{J}O(Y, f(x))$ there exists $U \in P\mathcal{I}O(X, x)$ such that ${}_{P\mathcal{J}}\mathrm{Cl}(f(U)) \subset V$.

Example 3.2. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ and $\mathcal{I} = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$. The class of all pre-$\mathcal{I}$-open sets of $(X, \tau, \mathcal{I})$ is $\emptyset$, $\{a\}$, $\{b, c\}$ and X. Then the function $f : (X, \tau, \mathcal{I}) \to (X, \tau, \mathcal{I})$ defined by $f(a) = f(c) = b$ and $f(b) = c$ is strongly pre-$\mathcal{I}$-irresolute.

Definition 3.3. A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is said to be pre-$\mathcal{I}$-irresolute [8] if $f^{-1}(V)$ is pre-$\mathcal{I}$-open set in $(X, \tau, \mathcal{I})$ for each pre-$\mathcal{J}$-open set V of $(Y, \sigma, \mathcal{J})$.

Remark 3.4. From the above two definitions, we have that every strongly pre-$\mathcal{I}$-irresolute function is pre-$\mathcal{I}$-irresolute. But the converse is not true.

Example 3.5. Let $X = \{a, b, c\}$ with topologies $\tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. The class of all pre-$\mathcal{I}$-open sets of $(X, \tau, \mathcal{I})$ and $(X, \sigma, \mathcal{I})$ is $\emptyset$, $\{a\}$, $\{a, b\}$, $\{a, c\}$ and X. Then the identity function $f : (X, \tau, \mathcal{I}) \to (X, \sigma, \mathcal{I})$ is pre-$\mathcal{I}$-irresolute but not strongly pre-$\mathcal{I}$-irresolute.

Theorem 3.6 ([8])**.** *A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is pre-$\mathcal{I}$-irresolute if and only if $f^{-1}(V)$ is pre-$\mathcal{I}$-closed in $(X, \tau, \mathcal{I})$ for each pre-$\mathcal{J}$-closed set V of $(Y, \sigma, \mathcal{J})$.*

Lemma 3.7. *A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is pre-$\mathcal{I}$-irresolute if and only if for every subset A of X, $f({}_{P\mathcal{I}}\mathrm{Cl}(A)) \subset {}_{P\mathcal{J}}\mathrm{Cl}(f(A))$.*

Proof. *Necessity.* If $A \subset X$, then ${}_{P\mathcal{J}}\mathrm{Cl}(f(A))$ is pre-$\mathcal{J}$-closed in $(Y, \sigma, \mathcal{J})$. Thus, by Theorem 3.6 $f^{-1}({}_{P\mathcal{J}}\mathrm{Cl}(f(A)))$ is pre-$\mathcal{J}$-closed in $(X, \tau, \mathcal{I})$. Furthermore, $A \subset f^{-1}(f(A)) \subset f^{-1}({}_{P\mathcal{J}}\mathrm{Cl}(f(A)))$. Therefore, by the definition of pre-$\mathcal{I}$-closure of a set, ${}_{P\mathcal{I}}\mathrm{Cl}(A) \subset f^{-1}({}_{P\mathcal{J}}\mathrm{Cl}(f(A)))$, and hence $f({}_{P\mathcal{I}}\mathrm{Cl}(A)) \subset {}_{P\mathcal{J}}\mathrm{Cl}(f(A))$.

Sufficiency. Let V be pre-$\mathcal{J}$-closed in $(Y, \sigma, \mathcal{J})$. Then $f({}_{P\mathcal{I}}\mathrm{Cl}(f^{-1}(V)))$ $\subset {}_{P\mathcal{J}}\mathrm{Cl}(f(f^{-1}(V))) = V$. Hence ${}_{P\mathcal{I}}\mathrm{Cl}(f^{-1}(V)) \subset f^{-1}(V)$ and hence $f^{-1}(V)$ is pre-$\mathcal{I}$-closed in $(X, \tau, \mathcal{I})$. Thus, f is pre-$\mathcal{I}$-irresolute. □

Theorem 3.8. *For a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ the following statements are equivalent:*

(*i*) *f is strongly pre-$\mathcal{I}$-irresolute;*

(*ii*) *For every $x \in X$ and every $V \in P\mathcal{J}O(Y, f(x))$, there exists $U \in P\mathcal{I}O(X, x)$ such that ${}_{P\mathcal{I}}\mathrm{Cl}(f({}_{P\mathcal{J}}\mathrm{Cl}(U))) \subset V$;*

(*iii*) *For every $x \in X$ and every pre-$\mathcal{J}$-closed subset F of Y with $f(x) \notin F$, there exist $U \in P\mathcal{I}O(X, x)$ and $V \in P\mathcal{J}O(Y)$ with $F \subset V$ for which $f({}_{P\mathcal{I}}\mathrm{Cl}(U)) \cap V = \emptyset$;*

(*iv*) *For every* $x \in X$ *and every pre-*$\mathcal{J}$*-closed subset* F *with* $f(x) \notin F$, *there exist* $U \in P\mathcal{I}O(X,x)$ *and* $V \in P\mathcal{J}O(Y)$ *with* $F \subset V$ *for which* $f(U) \cap V = \emptyset$.

Proof. (i)⇒(ii): Let $x \in X$ and $V \in P\mathcal{J}O(Y, f(x))$. Then there exists $U \in P\mathcal{I}O(X,x)$ such that ${}_{P\mathcal{J}}\operatorname{Cl}(f(U)) \subset V$. Since f is pre-$\mathcal{I}$-irresolute, by Lemma 3.7, $f({}_{P\mathcal{I}}\operatorname{Cl}(U)) \subset {}_{P\mathcal{J}}\operatorname{Cl}(f(U))$ and therefore ${}_{P\mathcal{J}}\operatorname{Cl}(f({}_{P\mathcal{I}}\operatorname{Cl}(U))) \subset {}_{P\mathcal{J}}\operatorname{Cl}(f(U)) \subset V$.

(ii)⇒(iii): Let $x \in X$ and F be a pre-$\mathcal{J}$-closed subset of Y with $f(x) \notin F$. Then $f(x) \in Y\backslash F$ which is pre-$\mathcal{J}$-open. Hence there exists $U \in P\mathcal{I}O(X,x)$ for which ${}_{P\mathcal{J}}\operatorname{Cl}(f({}_{P\mathcal{I}}\operatorname{Cl}(U))) \subset Y\backslash F$. Then for $V = X\backslash {}_{P\mathcal{J}}\operatorname{Cl}(f({}_{P\mathcal{I}}\operatorname{Cl}(U)))$ we have $F \subset V$ and, since ${}_{P\mathcal{J}}\operatorname{Cl}(f({}_{P\mathcal{I}}\operatorname{Cl}(U)))$ is pre-$\mathcal{J}$-closed, V is pre-$\mathcal{I}$-open. Also, $f({}_{P\mathcal{I}}\operatorname{Cl}(U)) \cap V = \emptyset$.

(iii)⇒(iv): Clear.

(iv)⇒(i): Let $x \in X$ and $V \in P\mathcal{J}O(Y, f(x))$. Then, $Y\backslash V$ is pre-$\mathcal{J}$-closed and $f(x) \notin Y\backslash V$. Therefore, there exist $U \in P\mathcal{I}O(X,x)$ and $W \in P\mathcal{J}O(Y)$ with $Y\backslash V \subset W$ for which $f(U) \cap W = \emptyset$. Since $f(U) \subset Y\backslash W$ which is pre-$\mathcal{J}$-closed, it follows that ${}_{P\mathcal{J}}\operatorname{Cl}(f(U)) \subset Y\backslash W \subset V$ which shows that f is strongly pre-$\mathcal{I}$-irresolute. □

Definition 3.9. An ideal topological space $(X, \tau, \mathcal{I})$ is said to be pre-$\mathcal{I}$-regular [8] if for every pre-$\mathcal{I}$-closed set A and every $x \in X\backslash A$ there exist disjoint sets U, $V \in PIO(X)$ with $x \in U$ and $A \subset V$.

Theorem 3.10 ([8])**.** *An ideal topological space* $(X, \tau, \mathcal{I})$ *is pre-*$\mathcal{I}$*-regular if and only if for every* $x \in X$ *and every* $U \in P\mathcal{I}O(X,x)$ *there exists* $V \in P\mathcal{I}O(X,x)$ *such that* $x \in V \subset {}_{P\mathcal{I}}\operatorname{Cl}(U) \subset U$.

Theorem 3.11. *If* $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ *is pre-*$\mathcal{I}$*-irresolute and* $(Y, \sigma, \mathcal{J})$ *is pre-*$\mathcal{J}$*-regular, then* f *is strongly pre-*$\mathcal{I}$*-irresolute.*

Proof. Let $x \in X$ and $V \in P\mathcal{J}O(Y, f(x))$. Since $(Y, \sigma, \mathcal{J})$ is pre-$\mathcal{J}$-regular, there exists $W \in P\mathcal{J}O(Y, f(x))$ such that $f(x) \in W \subset {}_{P\mathcal{J}}\operatorname{Cl}(W) \subset V$. Since f is pre-$\mathcal{I}$-irresolute, there exists $U \in P\mathcal{I}O(X,x)$ such that $f(U) \subset W$. Then ${}_{P\mathcal{J}}\operatorname{Cl}(f(U)) \subset {}_{P\mathcal{J}}\operatorname{Cl}(W) \subset V$, which proves that f is strongly pre-$\mathcal{I}$-irresolute. □

Theorem 3.12. *The following statements are equivalent for a function* $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$:

(*i*) $(X, \tau, \mathcal{I})$ *is pre-*$\mathcal{I}$*-regular;*

(*ii*) *The identity mapping* $i : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ *is strongly pre-*$\mathcal{I}$*-irresolute;*

(*iii*) *For every space* $(X, \tau, \mathcal{I})$*, every pre-*$\mathcal{I}$*-irresolute function* $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ *is strongly pre-*$\mathcal{I}$*-irresolute.*

Proof. (i)⇒(ii): Clear.
(ii)⇒(iii): Follows from Theorem 3.11.
(iii)⇒(i): Since the identity mapping is obviously pre-$\mathcal{I}$-irresolute, the proof follows immediately. □

Definition 3.13. A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is said to be quasi pre-$\mathcal{I}$-irresolute if for each point $x \in X$ and each $V \in P\mathcal{I}O(Y, f(x))$, there exists $U \in P\mathcal{I}O(X, x)$ such that $f({}_{P\mathcal{I}}\mathrm{Cl}(U)) \subset V$.

Theorem 3.14. *If* $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ *has the property that* ${}_{P\mathcal{J}}\,\mathrm{Cl}(f(U)) \subset f({}_{P\mathcal{I}}\mathrm{Cl}(U))$ *for every* $U \in P\mathcal{I}O(X)$ *and quasi pre-*$\mathcal{I}$*-irresolute, then* f *is strongly pre-*$\mathcal{I}$*-irresolute.*

Proof. Let $x \in X$ and $V \in P\mathcal{J}O(Y, f(x))$. Since f is quasi pre-$\mathcal{I}$-irresolute, there exists $U \in P\mathcal{I}O(X, x)$ for which $f({}_{P\mathcal{I}}\mathrm{Cl}(U)) \subset V$. By the assumption of f, ${}_{P\mathcal{J}}\,\mathrm{Cl}(f(U)) \subset f({}_{P\mathcal{I}}\mathrm{Cl}(U)) \subset V$, which shows that f is strongly pre-$\mathcal{I}$-irresolute. □

Theorem 3.15. *If* $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ *has the property that* $f(U)$ *is pre-*$\mathcal{J}$*-closed for every* $U \in P\mathcal{I}O(X)$ *and pre-*$\mathcal{I}$*-irresolute, then* f *is strongly pre-*$\mathcal{I}$*-irresolute.*

Proof. Let $x \in X$ and $V \in P\mathcal{J}O(Y, f(x))$. Then $f^{-1}(V) \in P\mathcal{I}O(X, x)$ and, since $f(f^{-1}(V))$ is pre-$\mathcal{J}$-closed, ${}_{P\mathcal{J}}\,\mathrm{Cl}(f(f^{-1}(V))) = f(f^{-1}(V)) \subset V$. □

Definition 3.16. A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is said to be $\mathcal{I}$-preopen if for every $U \in P\mathcal{I}O(X)$, $f(U) \subset {}_{P\mathcal{J}}\,\mathrm{Int}({}_{P\mathcal{J}}\mathrm{Cl}(f(U)))$.

Theorem 3.17. *If* $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ *is a strongly pre-*$\mathcal{I}$*-irresolute,* $\mathcal{I}$*-preopen surjection, then* $(Y, \sigma, \mathcal{J})$ *is pre-*$\mathcal{J}$*-regular.*

Proof. Let $y \in Y$ and $V \in P\mathcal{J}O(Y, y)$. Then $x \in X$ for which $y = f(x)$. Since f is strongly pre-$\mathcal{I}$-irresolute, there exists $U \in P\mathcal{I}O(X, x)$ such that ${}_{P\mathcal{J}}\,\mathrm{Cl}(f(U)) \subset V$. Since f is $\mathcal{I}$-preopen, we have

$$\begin{aligned} f(U) &\subset_{P\mathcal{J}} \mathrm{Int}({}_{P\mathcal{J}}\mathrm{Cl}(f(U))) \subset_{P\mathcal{J}} \mathrm{Cl}({}_{P\mathcal{J}}\mathrm{Int}({}_{P\mathcal{J}}\mathrm{Cl}(f(U)))) \\ &\subset_{P\mathcal{J}} \mathrm{Cl}(f(U)) \subset V. \end{aligned}$$

Thus, if $W = {}_{P\mathcal{J}}\operatorname{Int}({}_{P\mathcal{J}}\operatorname{Cl}(f(U)))$, then W is pre-$\mathcal{J}$-open and $y \in W \subset {}_{P\mathcal{J}}\operatorname{Cl}(W) \subset V$. This implies that $(Y, \sigma, \mathcal{J})$ is pre-$\mathcal{J}$-regular. □

Now, Theorems 3.11 and 3.17 yield the following Theorem.

Theorem 3.18. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be a pre-$\mathcal{I}$-irresolute $\mathcal{I}$-preopen surjection. Then f is strongly pre-$\mathcal{I}$-irresolute if and only if $(Y, \sigma, \mathcal{J})$ is pre-$\mathcal{J}$-regular.*

Definition 3.19. An ideal topological space $(X, \tau, \mathcal{I})$ is said to be pre-$\mathcal{I}$-R_0 [4] provided that for every $x \in X$ and every $U \in P\mathcal{I}O(X, x)$, ${}_{P\mathcal{I}}\operatorname{Cl}(\{x\}) \subset U$.

Example 3.20. Let $X = \{a, b, c\}$ and $\tau = \{X, \{a\}, \emptyset\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Observe that $(X, \tau, \mathcal{I})$ is a pre-$\mathcal{I}$-R_0 space but it is not an R_0 space.

Theorem 3.21. *If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is strongly pre-$\mathcal{I}$-irreolute surjection, then $(Y, \sigma, \mathcal{J})$ is pre-$\mathcal{J}$-R_0.*

Proof. Let $y \in Y$ and $V \in P\mathcal{J}O(Y, y)$. By the surjectivity of f, there is a point $x \in X$ for which $y = f(x)$. Since f is strongly pre-$\mathcal{I}$-irresolute, there exists $U \in P\mathcal{I}O(X, x)$ for which ${}_{P\mathcal{J}}\operatorname{Cl}(f(U)) \subset V$. Then we have ${}_{P\mathcal{J}}\operatorname{Cl}(\{y\}) = {}_{P\mathcal{J}}\operatorname{Cl}(\{f(x)\}) \subset {}_{P\mathcal{J}}\operatorname{Cl}(f(U)) \subset V$ and hence $(Y, \sigma, \mathcal{J})$ is pre-$\mathcal{J}$-R_0. □

Theorem 3.22. *If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is pre-$\mathcal{I}$-irresolute and $g : (Y, \sigma, \mathcal{J}) \to (Z, \eta, \mathcal{K})$ is strongly pre-$\mathcal{J}$-irresolute, then their composition $g \circ f$: $(X, \tau, \mathcal{I}) \to (Z, \eta, \mathcal{K})$ is strongly pre-$\mathcal{I}$-irresolute.*

Proof. Straightforward. □

Corollary 3.23. *If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is strongly pre-$\mathcal{I}$-irresolute and $g : (Y, \sigma, \mathcal{J}) \to (Z, \eta, \mathcal{K})$ is strongly pre-$\mathcal{J}$-irresolute, then their composition $g \circ f$: $(X, \tau, \mathcal{I}) \to (Z, \eta, \mathcal{K})$ is strongly pre-$\mathcal{I}$-irresolute.*

Theorem 3.24. *Strongly pre-$\mathcal{I}$-irresoluteness is an equivalence relation in the collection of all ideal topological spaces.*

Proof. Reflexivity and symmetry are immediate and transitivity follows from Corollary 3.23. □

Definition 3.25. A subset S of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be semi-$\mathcal{I}$-open [3] if $S \subset \operatorname{Cl}^*(\operatorname{Int}(S))$.

Lemma 3.26 ([8]). *Let A and X_0 be subsets of an ideal topological space $(X, \tau, \mathcal{I})$. Then,*

(i) If $A \in P\mathcal{I}O(X)$ and X_0 is semi-$\mathcal{I}$-open in $(X, \tau, \mathcal{I})$, then $A \cap X_0 \in P\mathcal{I}O(X_0)$;

(ii) If $A \in P\mathcal{I}O(X_0)$ and $X_0 \in P\mathcal{I}O(X)$, then $A \in P\mathcal{I}O(X)$.

Lemma 3.27 ([8]). *Let A and X_0 be subsets of an ideal topological space $(X, \tau, \mathcal{I})$ such that $A \subseteq X_0 \subseteq X$. Let ${}_{P\mathcal{I}}\mathrm{Cl}_{X_0}(A)$ denote the pre-$\mathcal{I}$-closure of A in the subspace X_0.*

(i) If X_0 is semi-$\mathcal{I}$-open in $(X, \tau, \mathcal{I})$, then ${}_{P\mathcal{I}}\mathrm{Cl}_{X_0}(A) \subset {}_{P\mathcal{I}}\mathrm{Cl}(A)$.

(ii) If $A \in P\mathcal{I}O(X_0)$ and $X_0 \in P\mathcal{I}O(X)$, then ${}_{P\mathcal{I}}\mathrm{Cl}(A) \subset {}_{P\mathcal{I}}\mathrm{Cl}_{X_0}(A)$.

Theorem 3.28. *If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is strongly pre-$\mathcal{I}$-irresolute and X_0 is a semi-$\mathcal{I}$-open subset of $(X, \tau, \mathcal{I})$, then the restriction*

$$f_{|X_0} : (X_0, \tau_{|X_0}, \mathcal{I}_{|X_0}) \to (Y, \sigma, \mathcal{J})$$

is strongly pre-$\mathcal{I}_{|X_0}$-irresolute.

Proof. For any $x \in X_0$ and any $V \in P\mathcal{I}O(Y, f(x))$, there exists $U \in P\mathcal{I}O(X, x)$ such that ${}_{P\mathcal{I}}\mathrm{Cl}(f(U)) \subset V$ since f is strongly pre-$\mathcal{I}$-irresolute. Put $U_0 = U \cap X_0$, then by Lemmas 3.26 and 3.27, $U_0 \in P\mathcal{I}O(X_0, x)$ and ${}_{P\mathcal{I}}\mathrm{Cl}_{X_0}(f(U)) \subset {}_{P\mathcal{I}}\mathrm{Cl}(f(U_0))$. Therefore, we obtain ${}_{P\mathcal{I}}\mathrm{Cl}_{x_0}(f_{|X_0}(U_0)) = {}_{P\mathcal{I}}\mathrm{Cl}_{X_0}(f(U_0)) \subset {}_{P\mathcal{I}}\mathrm{Cl}(f(U_0)) \subset V$. This shows that $f_{|X_0}$ is strongly pre-$\mathcal{I}_{|X_0}$-irresolute. □

Corollary 3.29. *If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is strongly pre-$\mathcal{I}$-irresolute and X_0 is open in X, then the restriction $f_{|X_0}$: $(X_0, \tau_{|X_0}, \mathcal{I}_{|X_0}) \to (Y, \sigma, \mathcal{J})$ is strongly pre-$\mathcal{I}_{|X_0}$-irresolute.*

Theorem 3.30. *A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{I})$ is strongly pre-$\mathcal{I}$-irresolute if for each $x \in X$ there exists $X_0 \in P\mathcal{I}O(X, x)$ such that the restriction $f_{|X_0}$: $(X_0, \tau_{|X_0}, \mathcal{I}_{|X_0}) \to (Y, \sigma, \mathcal{J})$ is strongly pre-$\mathcal{I}_{|X_0}$-irresolute.*

Proof. Let $x \in X$ and $V \in P\mathcal{I}O(Y, f(x))$. There exists $X_0 \in P\mathcal{I}O(X, x)$ such that $f_{|X_0}$ is strongly pre-$\mathcal{I}$-irresolute. Thus, there exists $U \in P\mathcal{I}O(X_0, x)$ such that ${}_{P\mathcal{I}}\mathrm{Cl}_{X_0}(f_{|X_0}(U)) \subset V$. By Lemmas 3.26 and 3.27, $U \in P\mathcal{I}O(X, x)$ and ${}_{P\mathcal{I}}\mathrm{Cl}(U) \subset {}_{P\mathcal{I}}\mathrm{Cl}_{X_0}(U)$. Hence we have ${}_{P\mathcal{I}}\mathrm{Cl}(f(U)) = {}_{P\mathcal{I}}\mathrm{Cl}(f_{|X_0}(U)) \subset {}_{P\mathcal{I}}\mathrm{Cl}_{X_0}(f_{|X_0}(U)) \subset V$. This shows that f is strongly pre-$\mathcal{I}_{|X_0}$-irresolute. □

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