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ON APPROXIMATE SOLUTIONS OF CERTAIN MIXED TYPE INTEGRODIFFERENTIAL EQUATIONS

BY

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Abstract. The aim of the present paper is to study approximate solutions of mixed Volterra-Fredholm type integrodifferential equations in one and two independent variables. The analysis used in the proofs is based on the applications of new integral inequalities with explicit estimates.

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1. Introduction

In studying mathematical models of various dynamic phenomena, it is often desirable not only to prove the existence of a solution satisfying the given initial or boundary conditions, but also ensure that the solution in question posses certain qualitative properties. The method of approximations to the solutions is one of the most powerful and far-reaching tool available for advancing our understanding of a great variety of interesting and important classes of differential and integral equations. In this paper we shall be concerned with the approximate solutions of the initial value problem for a Volterra-Fredholm type integrodifferential equation

$$y'(t) = f(t, y(t), (Ay)(t), (By)(t)), y(0) = y_0, \tag{1.1}$$

with

$$(Ay)(t) = \int_0^t g(t, \sigma, y(\sigma))\, d\sigma,\ (By)(t) = \int_0^a h(t, \sigma, y(\sigma)) d\sigma, \tag{1.2}$$

for $t \in [0,a]$, where f, g, h are given functions and y is the unknown function to be found. Let R denotes the set of real numbers, $R_+ = [0,\infty), I_a = [0,a]$ $(a > 0)$ be given subsets of R, $D = \{(t,s) \in I_a^2 : s \le t\}$ and denote by $C(S_1, S_2)$ the class of continuous functions from the set S_1 to the set S_2. We assume that $f \in C\left(I_a \times R^3, R\right), g \in C(D \times R, R), h \in C\left(I_a^2 \times R, R\right)$.

In earlier works [3,6] the authors studied the problem of existence of solutions of certain special and general forms of equation (1.1) in different space settings. The present paper is motivated by the desire to extend the method of approximations of solutions to study the equation (1.1). Here, we investigate the conditions for the approximation of solutions to equation (1.1) by establishing some new bounds and convergence properties on solutions of approximate problems. Our proofs rely on the application of a certain integral inequality with explicit estimate, recently established by the present author in [6] (see also [4, p. 47]). The results on the two independent variable generalization of equation (1.1) are also given.

2. Main results

Let $y : I_a \to R$ be a continuous function on I_a, differentiable on $(0,a)$ and satisfies the inequality

$$\left|y'(t) - f(t, y(t), (Ay)(t), (By)(t))\right| \le \varepsilon,$$

for a given constant ε, where $y(0) = y_0$. Then we call $y(t)$ the $\varepsilon-$approximate solution with respect to the equation (1.1).

We need the following new integral inequality recently established by the present author in [6] (see also [4, p. 47]).

Lemma 1. *Let $u, p, q, r \in C(I_a, R_+)$ and*

$$u(t) \le c + \int_0^t p(s)\left[u(s) + \int_0^s q(\sigma)\, u(\sigma)\, d\sigma + \int_0^a r(\sigma)\, u(\sigma)\, d\sigma\right] ds, \tag{2.1}$$

for $t \in I_a$, where $c \ge 0$ is a constant. If

$$d = \int_0^a r(\sigma) \exp\left(\int_0^\sigma [p(\tau) + q(\tau)]\, d\tau\right) d\sigma < 1, \tag{2.2}$$

then

$$u(t) \le \frac{c}{1-d} \exp\left(\int_0^t [p(\tau) + q(\tau)]\, d\tau\right), \tag{2.3}$$

for $t \in I_a$.

The following theorem deals with the estimate on the difference between the two approximate solutions of equation (1.1).

Theorem 1. *Suppose that the functions* f, g, h *in equation* (1.1) *satisfy the conditions*

$$|f(t,y,u,v) - f(t,\bar{y},\bar{u},\bar{v})| \le k(t)\left[|y-\bar{y}| + |u-\bar{u}| + |v-\bar{v}|\right], \tag{2.4}$$

$$|g(t,\sigma,y) - g(t,\sigma,\bar{y})| \le e(t)\, q(\sigma)\, |y-\bar{y}|, \tag{2.5}$$

$$|h(t,\sigma,y) - h(t,\sigma,\bar{y})| \le m(t)\, r(\sigma)\, |y-\bar{y}|, \tag{2.6}$$

where $k, e, q, m, r \in C(I_a, R_+)$. *For* $t \in I_a$, *let* $p(t) = \max\{k(t), k(t)e(t), k(t)m(t)\}$ *and* d *be as in* (2.2). *For* $i = 1, 2$, *let* $y_i(t)$ *be respectively* ε_i-*approximate solutions of equation* (1.1) *on* I_a *with* $y_i(0) = \bar{y}_i$ *such that*

$$|\bar{y}_1 - \bar{y}_2| \le \delta, \tag{2.7}$$

where $\delta \ge 0$ *is a constant, then*

$$|y_1(t) - y_2(t)| \le \frac{(\varepsilon a + \delta)}{1-d} \exp\left(\int_0^t [p(\tau) + q(\tau)]\, d\tau\right), \tag{2.8}$$

for $t \in I_a$, *where* $\varepsilon = \varepsilon_1 + \varepsilon_2$.

Proof. Since $y_i(t)$, $t \in I_a$ are respectively, ε_i-approximate solutions of equation (1.1) with $y_i(0) = \bar{y}_i$, we have

$$\left|y_i'(t) - f(t, y_i(t), (Ay_i)(t), (By_i)(t))\right| \le \varepsilon_i, \tag{2.9}$$

for $i = 1, 2$. Integrating both sides of (2.9) from 0 to t over I_a, we have

$$\begin{aligned} \varepsilon_i t &\ge \int_0^t \left|y_i'(s) - f(s, y_i(s), (Ay_i)(s), (By_i)(s))\right| ds \\ &\ge \left|\int_0^t \left\{y_i'(s) - f(s, y_i(s), (Ay_i)(s), (By_i)(s))\right\} ds\right| \\ &= \left|\left\{y_i(t) - \bar{y}_i - \int_0^t f(s, y_i(s), (Ay_i)(s), (By_i)(s))\, ds\right\}\right|, \end{aligned} \tag{2.10}$$

for $i = 1, 2$. From (2.10) and using the elementary inequalities

$$|v - z| \le |v| + |z|, \quad |v| - |z| \le |v - z|, \tag{2.11}$$

we observe that

$$
\begin{aligned}
(\varepsilon_1+\varepsilon_2)\,t \geq & \left|\left\{y_1(t)-\bar{y}_1-\int_0^t f\left(s,y_1(s),(Ay_1)(s),(By_1)(s)\right)ds\right\}\right| \\
& +\left|\left\{y_2(t)-\bar{y}_2-\int_0^t f\left(s,y_2(s),(Ay_2)(s),(By_2)(s)\right)ds\right\}\right| \\
& \geq \left|\left\{y_1(t)-\bar{y}_1-\int_0^t f\left(s,y_1(s),(Ay_1)(s),(By_1)(s)\right)ds\right\}\right. \\
& \left.-\left\{y_2(t)-\bar{y}_2-\int_0^t f\left(s,y_2(s),(Ay_2)(s),(By_2)(s)\right)ds\right\}\right| \qquad (2.12) \\
& \geq |y_1(t)-y_2(t)|-|\bar{y}_1-\bar{y}_2| \\
& -\left|\int_0^t f\left(s,y_1(s),(Ay_1)(s),(By_1)(s)\right)ds\right. \\
& \left.-\int_0^t f\left(s,y_2(s),(Ay_2)(s),(By_2)(s)\right)ds\right|.
\end{aligned}
$$

Let $u(t)=|y_1(t)-y_2(t)|$, $t\in I_a$ and $\varepsilon_1+\varepsilon_2=\varepsilon$. From (2.12) and using the hypotheses, we observe that

$$
\begin{aligned}
u(t) \leq & \varepsilon t+|\bar{y}_1-\bar{y}_2|+\int_0^t \left|f\left(s,y_1(s),(Ay_1)(s),(By_1)(s)\right)\right. \\
& \left.-f\left(s,y_2(s),(Ay_2)(s),(By_2)(s)\right)\right|ds \leq \varepsilon a+\delta \qquad (2.13) \\
& +\int_0^t k(s)\left[u(s)+\int_0^s e(s)\,q(\sigma)\,u(\sigma)d\sigma+\int_0^a m(s)r(\sigma)\,u(\sigma)\,d\sigma\right]ds \\
& \leq (\varepsilon a+\delta)+\int_0^t p(s)\left[u(s)+\int_0^s q(\sigma)u(\sigma)d\sigma+\int_0^a r(\sigma)u(\sigma)d\sigma\right]ds.
\end{aligned}
$$

Now an application of Lemma 1 to (2.13) yields (2.8). □

Remark 1. In case if $y_1(t)$ is a solution of equation (1.1), then we have $\varepsilon_1=0$ and from (2.8) we see that $y_2(t)\to y_1(t)$ as $\varepsilon_2\to 0$ and $\delta\to 0$. Furthermore, if we put $\varepsilon_1=\varepsilon_2=0, \bar{y}_1=\bar{y}_2$ in (2.8), then the uniqueness of solutions of equation (1.1) is established. We note that the study of equation (1.1) contains in the appropriate special cases the study of corresponding initial value problems for ordinary differential equations, Volterra type integrodifferential equations and Fredholm type integrodifferential equations.

Consider the equation (1.1) together with the following Volterra-Fredholm type integrodifferential equation

$$z'(t) = \bar{f}(t, z(t), (Az)(t), (Bz)(t)), \quad z(0) = z_0, \tag{2.14}$$

where A, B are defined as in (1.2) and $\bar{f} \in C\left(I_a \times R^3, R\right)$.

Next, we shall prove the following theorem concerning the closeness of the solutions of equations (1.1) and (2.14).

Theorem 2. *Suppose that the functions f, g, h in equation* (1.1) *satisfy the conditions* (2.4)-(2.6) *and there exist constants $\bar{\varepsilon} \geq 0, \bar{\delta} \geq 0$ such that*

$$\left|f(t, u, v, w) - \bar{f}(t, u, v, w)\right| \leq \bar{\varepsilon}, \tag{2.15}$$

$$|y_0 - z_0| \leq \bar{\delta}, \tag{2.16}$$

where f, y_0 and $\bar{f}, z_0$ are as in equations (1.1) *and* (2.14). *Let $p(t)$ and d be as in Theorem 1. Let $y(t)$ and $z(t)$ be respectively, solutions of equations* (1.1) *and* (2.14) *on I_a. Then*

$$|y(t) - z(t)| \leq \frac{\left(\bar{\varepsilon}a + \bar{\delta}\right)}{1 - d} \exp\left(\int_0^t [p(\tau) + q(\tau)]\, d\tau\right), \tag{2.17}$$

for $t \in I_a$.

Proof. Let $u(t) = |y(t) - z(t)|$, $t \in I_a$. Using the facts that $y(t), z(t)$ are the solutions of equations (1.1), (2.14) and hypotheses, we observe that

$$\begin{aligned}
u(t) &\leq |y_0 - z_0| + \int_0^t |f(s, y(s), (Ay)(s), (By)(s)) \\
&\quad - f(s, z(s), (Az)(s), (Bz)(s))|ds \\
&\quad + \int_0^t |f(s, z(s), (Az)(s), (Bz)(s)) \\
&\quad - \bar{f}(s, z(s), (Az)(s), (Bz)(s))|ds \\
&\leq \bar{\delta} + \bar{\varepsilon}t + \int_0^t k(s) \left[u(s) + \int_0^s e(s)q(\sigma)u(\sigma)d\sigma \right. \\
&\quad \left. + \int_0^a m(s)r(\sigma)u(\sigma)d\sigma\right] ds \leq (\bar{\varepsilon}a + \bar{\delta}) \\
&\quad + \int_0^t p(s) \left[u(s) + \int_0^s q(\sigma)u(\sigma)d\sigma + \int_0^a r(\sigma)u(\sigma)d\sigma\right] ds.
\end{aligned} \tag{2.18}$$

Now an application of Lemma 1 to (2.18) yields (2.17). □

Remark 2. We note that the result given in Theorem 2 relates the solutions of equations (1.1) and (2.14) in the sense that if f is close to $\bar{f}$ and if y_0 is close to z_0, then the solutions of equations (1.1) and (2.14) are also close together.

Now, we consider the equation (1.1) together with the following Volterra-Fredholm type integrodifferential equations

$$y'(t) = f_k(t, y(t), (Ay)(t), (By)(t)), y(0) = \alpha_k, \tag{2.19}$$

for $k = 1, 2, ...$, where A, B are as defined in (1.2), α_k is a seqence in R and $f_k \in C\left(I_a \times R^3, R\right)$.

As an immediate consequence of Theorem 2 we have the following corollary that ensures solutions to (2.19) will converge to solutions to (1.1).

Corollary 1. *Suppose that the functions f, g, h in equation* (1.1) *satisfy the conditions* (2.4)-(2.6) *and there exist constants $\varepsilon_k \geq 0, \delta_k \geq 0$, $(k = 1, 2, ...)$ such that*

$$|f(t, u, v, w) - f_k(t, u, v, w)| \leq \varepsilon_k, \tag{2.20}$$

$$|y_0 - \alpha_k| \leq \delta_k, \tag{2.21}$$

with $\varepsilon_k \to 0$ and $\delta_k \to 0$ as $k \to \infty$, where f, y_0 and f_k, α_k are as in (1.1) *and* (2.19). *Let $p(t)$ and d be as in Theorem* 1. *If $y_k(t)$, $(k = 1, 2, ...)$ and $y(t)$ are respectively the solutions of equations* (2.19) *and* (1.1) *on I_a, then as $k \to \infty$, $y_k(t) \to y(t)$ on I_a.*

Proof. For $k = 1, 2, ...$, the conditions of Theorem 2 hold. As an application of Theorem 2 yields

$$|y_k(t) - y(t)| \leq \frac{(\varepsilon_k a + \delta_k)}{1 - d} \exp\left(\int_0^t [p(\tau) + q(\tau)]\, d\tau\right), \tag{2.22}$$

for $k = 1, 2, ...$, $t \in I_a$. The required result follows from (2.22). □

Remark 3. We note that the result obtained in Corollary 1 provides sufficient conditions that ensures solutions to (2.19) will converge to solutions to (1.1). For some other qualitative properties of solutions of more general version of equation (1.1), we refer the interested readers to [6].

3. Two independent variable generalization

Let $\Delta = I_a \times I_b$ and denote by $\Omega = \left\{(x,y,s,t) \in \Delta^2 : s \leq x, t \leq y\right\}$. The partial derivatives of a function $z(x,y)$ for $(x,y) \in \Delta$ with respect to x, y and xy are denoted respectively by $D_1 z(x,y), D_2 z(x,y)$ and $D_2 D_1 z(x,y) = D_1 D_2 z(x,y)$. In this section we shall be concerned with the approximate solutions of the following Volterra-Fredholm type integrodifferential equation

$$D_2 D_1 u(x,y) = F(x,y,u(x,y),(Lu)(x,y),(Mu)(x,y)), \tag{3.1}$$

with the given initial boundary conditions

$$u(x,0) = \alpha(x),\ u(0,y) = \beta(y),\ u(0,0) = 0, \tag{3.2}$$

for $x \in I_a, y \in I_b$, where

$$(Lu)(x,y) = \int_0^x \int_0^y G(x,y,s,t,u(s,t))\,dtds, \tag{3.3}$$

$$(Mu)(x,y) = \int_0^a \int_0^b H(x,y,s,t,u(s,t))\,dtds, \tag{3.4}$$

F, G, H are given functions and u is the unknown function to be found. We assume that $F \in C\left(\Delta \times R^3, R\right), G \in C(\Omega \times R, R), H \in C(\Delta \times R, R)$, $\alpha \in C(I_a, R), \beta \in C(I_b, R)$. The problem of existence of solutions for equations of the form (3.1)-(3.2) can be dealt with the method employed in [7]. Here, we formulate in brief the results corresponding to Theorems 1 and 2 and Corollary 1 related to the approximate solutions of equations (3.1)-(3.2).

Let $u : \Delta \to R$ be a continuous function on Δ, $D_2 D_1 u(x,y)$ exists on $(0,a) \times (0,b)$ and satisfies the inequality

$$|D_2 D_1 u(x,y) - F(x,y,u(x,y),(Lu)(x,y),(Mu)(x,y))| \leq \varepsilon,$$

for a given constant ε, where it is supposed that (3.2) holds. Then we call $u(x,y)$ the $\varepsilon-$approximate solution with respect to the equations (3.1)-(3.2).

We require the following special form of the integral inequality established by the present author in [5] (see also [4, p. 96]).

Lemma 2. *Let $u, p, q, r \in C(\Delta, R_+)$ and*

$$u(x,y) \leq c + \int_0^x \int_0^y p(s,t)\left[u(s,t) + \int_0^s \int_0^t q(\sigma,\tau)\,u(\sigma,\tau)\,d\tau d\sigma\right.$$

$$+\int_0^a \int_0^b r(\sigma,\tau)\, u(\sigma,\tau)\, d\tau d\sigma \Bigg] dtds, \tag{3.5}$$

for $(x,y) \in \Delta$, *where* $c \geq 0$ *is a constant. If*

$$\bar{d} = \int_0^a \int_0^b r(\sigma,\tau) \exp\left(\int_0^\sigma \int_0^\tau [p(s,t) + q(s,t)]\, dtds\right) d\tau d\sigma < 1, \tag{3.6}$$

then

$$u(x,y) \leq \frac{c}{1-\bar{d}} \exp\left(\int_0^x \int_0^y [p(s,t) + q(s,t)]\, dtds\right), \tag{3.7}$$

for $(x,y) \in \Delta$.

Our results corresponding to the approximate solutions of equations (3.1)-(3.2) are given as follows.

Theorem 3. *Suppose that the functions* F, G, H *in equation* (3.1) *satisfy the conditions*

$$\begin{aligned} &|F(x,y,u,v,w) - F(x,y,\bar{u},\bar{v},\bar{w})| \leq k(x,y)\,[|u-\bar{u}| \\ &\qquad + |v-\bar{v}| + |w-\bar{w}|], \end{aligned} \tag{3.8}$$

$$|G(x,y,\sigma,\tau,u) - G(x,y,\sigma,\tau,\bar{u})| \leq e(x,y)\, q(\sigma,\tau)\, |u-\bar{u}|, \tag{3.9}$$

$$|H(x,y,\sigma,\tau,u) - H(x,y,\sigma,\tau,\bar{u})| \leq m(x,y)\, r(\sigma,\tau)\, |u-\bar{u}|, \tag{3.10}$$

where $k, e, q, m, r \in C(\Delta, R_+)$. *For* $(x,y) \in \Delta$, *let* $p(x,y) = \max\{k(x,y), k(x,y)e(x,y), k(x,y)m(x,y)\}$ *and* $\bar{d}$ *be as in* (3.6). *For* $i = 1,2$, *let* $u_i(x,y)$ *be respectively* ε_i-*approximate solutions of equation* (3.1) *on* Δ *with*

$$u_i(x,0) = \alpha_i(x),\ u_i(0,y) = \beta_i(y),\ u_i(0,0) = 0, \tag{3.11}$$

where $\alpha_i \in C(I_a, R), \beta_i \in C(I_b, R)$ *such that*

$$|\alpha_1(x) - \alpha_2(x) + \beta_1(y) - \beta_2(y)| \leq \delta, \tag{3.12}$$

where $\delta \geq 0$ *is a constant. Then*

$$|u_1(x,y) - u_2(x,y)| \leq \frac{(\varepsilon ab + \delta)}{1-\bar{d}} \exp\left(\int_0^x \int_0^y [p(s,t)+q(s,t)]dtds\right), \tag{3.13}$$

for $(x,y) \in \Delta$, *where* $\varepsilon = \varepsilon_1 + \varepsilon_2$.

Now consider the equations (3.1)-(3.2) together with the following equation

$$D_2D_1v(x,y)=\bar{F}(x,y,v(x,y),(Lv)(x,y),(Mv)(x,y)), \tag{3.14}$$

with the given initial boundary conditions

$$v(x,0)=\bar{\alpha}(x),\ v(0,y)=\bar{\beta}(y),\ v(0,0)=0, \tag{3.15}$$

where L,M are as defined in (3.3), (3.4) and $\bar{F}\in C\left(\Delta\times R^3,R\right)$, $\bar{\alpha}\in C(I_a,R)$, $\bar{\beta}\in C(I_b,R)$.

The following theorem holds.

Theorem 4. *Suppose that the functions F,G,H in equation* (3.1) *satisfy the conditions* (3.8)-(3.10) *and there exist constants $\bar{\varepsilon}\geq 0$ and $\bar{\delta}\geq 0$ such that*

$$\left|F(x,y,u,v,w)-\bar{F}(x,y,u,v,w)\right|\leq\bar{\varepsilon}, \tag{3.16}$$

$$\left|\alpha(x)-\bar{\alpha}(x)+\beta(y)-\bar{\beta}(y)\right|\leq\bar{\delta}, \tag{3.17}$$

where F,α,β and $\bar{F},\bar{\alpha},\bar{\beta}$ are as in equations (3.1)-(3.2) *and* (3.14)-(3.15). *Let $p(x,y)$ and $\bar{d}$ be as in Theorem* 3. *Let $u(x,y)$ and $v(x,y)$ be respectively the solutions of equations* (3.1)-(3.2) *and* (3.14)-(3.15) *for $(x,y)\in\Delta$. Then*

$$|u(x,y)-v(x,y)|\leq\frac{\left(\bar{\varepsilon}ab+\bar{\delta}\right)}{1-\bar{d}}\exp\left(\int_0^x\int_0^y\left[p(s,t)+q(s,t)\right]dtds\right), \tag{3.18}$$

for $(x,y)\in\Delta$.

Next, we consider the equations (1.1)-(1.2) and sequence of equations

$$D_2D_1u(x,y)=F_k(x,y,u(x,y),(Lu)(x,y),(Mu)(x,y)), \tag{3.19}$$

with the given initial boundary conditions

$$u(x,0)=\alpha_k(x),\ u(0,y)=\beta_k(y),\ u(0,0)=0, \tag{3.20}$$

for $x\in I_a,y\in I_b$, where L,M are as defined in (3.3), (3.4) and for $k=1,2,...$, $F_k\in C\left(\Delta\times R^3,R\right),\alpha_k\in C(I_a,R),\beta_k\in C(I_b,R)$.

As a consequence of Theorem 4, the following corollary holds.

Corollary 2. *Suppose that the functions F, G, H in equation* (3.1) *satisfy the conditions* (3.8)-(3.10) *and*

$$|F(x,y,u,v,w) - F_k(x,y,u,v,w)| \leq \bar{\varepsilon}_k, \tag{3.21}$$

$$|\alpha(x) - \alpha_k(x) + \beta(y) - \beta_k(y)| \leq \bar{\delta}_k, \tag{3.22}$$

with $\bar{\varepsilon}_k \to 0$ and $\bar{\delta}_k \to 0$ as $k \to \infty$, where F, α, β and F_k, α_k, β_k are as in equations (1.1)-(1.2) *and* (3.19)-(3.20). *Let $p(x,y)$ and $\bar{d}$ be as in Theorem* 3. *If $u_k(x,y)$ $(k = 1, 2, ...)$ and $u(x,y)$ are respectively the solutions of equations* (3.19)-(3.20) *and* (3.1)-(3.2) *on Δ, then $u_k(x,y) \to u(x,y)$ on Δ as $k \to \infty$.*

The proofs of Theorems 3, 4 and Corollary 2 are straightforward in view of the results given in section 2. Here, we omit the details.

Remark 4. We note that the results given in this section can be extended very easily to study the approximate solutions of the non-self-adjoint hyperbolic partial Volterra-Fredholm type integrodifferential equations

$$D_2D_1u(x,y) = D_2(p(x,y)u(x,y)) + F(x,y,(Lu)(x,y),(Mu)(x,y)), \tag{3.23}$$

$$D_2D_1u(x,y) = D_1(p(x,y)u(x,y)) + F(x,y,(Lu)(x,y),(Mu)(x,y)), \tag{3.24}$$

with the given initial boundary conditions (3.2), under some suitable conditions on the functions involved in (3.23), (3.24) and (3.2) by making use of the integral inequality established by the present author in [5] (see also [4, p. 96]) or its suitable variant, in place of Lemma 2. For the study of approximate solutions of certain dynamic equations, see [1,2,8] and some of the references given therein.

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