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Sunlet decomposition of tensor product graphs

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Abstract For any integer $r \geq 3$, the sunlet graph of order $2r$ is a graph consisting of a cycle of length r with each vertex of the cycle adjacent to a pendant vertex. In this present article, we shall obtain the necessary and sufficient conditions for decomposing the tensor product of complete graphs and complete graph minus a 1-factor with complete graph K_m (that is, $K_n \times K_m$ and $K_n - I \times K_m$ respectively) into sunlet graphs.

Keywords graph decomposition · tensor product graph · sunlet graph · complete graph · regular graphs

Mathematics Subject Classification (2010) 05C51

1 Introduction

In this paper, we consider simple and finite graphs. Let C_r,K_m, $K_m - I$ denotes a cyle of length r, the complete graph with m vertices and a complete graph on m vertices minus a 1-factor respectively. Furthermore, we shall make use of the following definitions in the work.

Definition 1.1 *Sunlet graph denoted by L_{2r} is a graph that consists of a cycle C_r with pendant vertex (vertex of degree one) attached to each vertex of the cycle C_r, as shown in Figure 1.*

Definition 1.2 *A graph is decomposable into graph S or S decomposes G, if G can be written as the union of edge-disjoint copies of S, so that all edge in G belongs to only one copy of S. Then $G = S \oplus S \oplus ... \oplus S$, or simply write it as $S|G$, if G is decomposable into S. In addition, $S|G$ also means decomposition of G into copies of S.*

Definition 1.3 *Let G and S be two graphs, the tensor product of G and S written as $G \times S$ has vertex set $V(G) \times V(S)$ in which two vertices (g_1, s_1) and (g_2, s_2) are adjacent whenever $g_1g_2 \in E(G)$ and $s_1s_2 \in E(S)$. See Figure 2 for an example.*

Note that the tensor product of graphs is distributive over edge disjoint union of graphs, that is, if $G = S_1 \oplus S_2 \oplus ... \oplus S_k$, then $G \times S = (S_1 \times S) \oplus ... \oplus (S_k \times S)$. The graph $C_r \times K_m$ is an equipartite graph, with the degree of any vertex being $2(n-1)$ and the total number of edges is $rm(m-1)$. The graph $K_m \times K_n$ is isomorphic to the complete n-partite graph in which each partite set contains m vertices. Also, the graph $(K_m - I) \times K_n$ is an equipartite graph.
The necessary conditions for the existence of a r-cycle decomposition of a simple graph G is that G has at least k vertices, the degree of every vertex of G is even and the total number of edges in G must be divisible by k. These conditions have been shown to be sufficient in the case that G is complete graph or the complete graph minus a 1-factor, $K_n - I$ [[1], [12]].
The study of cycle decomposition of $K_m \times K_n$ have been investigated . In [[5], [6] and [8]], the necessary and the sufficient conditions for the existence of C_p (p, prime) decomposition of $K_m \times K_n$ was given. It was shown in [7] that Hamilton cycles decomposes the tensor product of two regular complete graphs. Also, in [11], it was proved that the necessary conditions are sufficient for the decomposition of tensor product of complete graphs into a resolvable k-cycles, when k is even. In the recent work, the necessary and sufficient conditions for the decomposition of $K_m \times K_n$ into cycles of length 4 and 10 was given [see [9], [10]].
In [[2], [3]], the necessary and sufficient conditions for the decomposition of certain equipartite graphs into sunlet graphs and sunlet graphs of order $2p$ was given. Decomposition of product graphs into sunlet graphs of order eight was given, in [13].
In this paper, we consider the decomposition of $C_r \times K_n$, $K_m \times K_n$ $(m, n$ odd) and $K_m - I \times K_n$ (m, even and n odd) into L_{2r}, where r is a positive integer. Among others, we prove the following results:

1. For positive odd integers n and m, the graph $K_m \times K_n$ is decomposable into sunlet graphs L_{2r} if and only if $mn(m-1)(n-1) \equiv 0(mod \;\; 2r)$.
2. For positive even integer m and odd integer n, the graph $K_m - I \times K_n$ is decomposable into sunlet graphs L_{2r} if and only if $mn(m-2)(n-1) \equiv 0(mod \;\; 2r)$.

2 Main results

We begin this section with the following Lemma.

Lemma 2.1 *For* $r \geq 3, L_{2r} | C_r \times C_3$.

Proof. Each vertex x_i in C_r is replaced by 3 independent vertices $x_{i,1}, x_{i,2}, x_{i,3}$ and each edge $x_i x_j, 1 \leq i, j \leq r$ is replaced by $K_{3,3} - I$ in $C_r \times C_3$ which follows from the definition of tensor product graph. Next, we split the problem into the following 3 cases.

Case 1: when $r \equiv 0 (mod \;\; 3)$.
First construct the base cycles C_r^1, C_r^2, C_r^3 as follows:

$$C_r^{s+1} = x_{1,s+1}, x_{2,s+2}, x_{3,s+3}, x_{4,s+1} ..., x_{r,s+3}$$

where $s = 0, 1, 2$. Also, note that the sum on the second suffix is calculated modulo 3. Therefore, we have 3 cycles C_r^1, C_r^2, C_r^3. Define a mapping ρ by $\rho(x_{i,1}) = x_{i+1,3}$, $\rho(x_{i,2}) = x_{i+1,1}$,$\rho(x_{i,3}) = x_{i+1,2}$, where $1 \leq i \leq r$. Note that the sum on the first suffix is calculated modulo r. The vertices $\rho(x_{i,1}), \rho(x_{i,2})$ and $\rho(x_{i,3})$ are pendant vertices. For $1 \leq t \leq 3$, attach each pendant vertex $\rho(x_{i,t})$ to each vertex $x_{i,t}$ in C_r^1, C_r^2, C_r^3 . The cycles C_r^1, C_r^2, C_r^3 together with pendant vertices $\rho(x_{i,t})$ attached gives sunlet graphs of order $2r$.
Case 2: when $r \equiv 1 (mod \; 3)$.
First, we construct three base cycles C_r^1, C_r^2, C_r^3 as follows:

$$C_r^{s+1} = x_{1,s+1}, x_{2,s+2}, x_{3,s+3}, x_{4,s+1} ..., x_{r-1,s+3}, x_{r,s+2}$$

where $s = 0, 1, 2$ and the sum on the second suffix is calculated modulo 3. Hence, we have three cycles C_r^1, C_r^2, C_r^3. Next, define mapping ρ by $\rho(x_{i,1}) = x_{i+1,3}$, $\rho(x_{i,2}) = x_{i+1,1}$,$\rho(x_{i,3}) = x_{i+1,2}$, where $1 \leq i \leq r-1$. Note that the sum on the first suffix is calculted modulo r.
Also, $\rho(x_{r,1}) = x_{1,2}$, $\rho(x_{r,2}) = x_{1,3}$,$\rho(x_{r,3}) = x_{1,1}$. The vertices $\rho(x_{i,1}), \rho(x_{i,2})$, $\rho(x_{i,3})$ are pendant vertices. For $t = 1, 2, 3$, attach each pendant vertex $\rho(x_{i,t})$ to each vertex $x_{i,t}$ in the three base cycles . The base cycles with pendant vertices attached gives the sunlet graph L_{2r}.
Case 3: when $r \equiv 2 (mod \; 3)$.
Construct three base cycles C_r^1, C_r^2, C_r^3 as follows:

$$C_r^{s+1} = x_{1,s+1}, x_{2,s+2}, x_{3,s+3}, x_{4,s+1} ..., x_{r-1,s+1}, x_{r,s+2}$$

where $s = 0, 1, 2$ and the sum on the second suffix is calculated modulo 3. Therefore, we have three cycles C_r^1, C_r^2, C_r^3. Applying mapping ρ define in case 2 above to get the pendant vertices $\rho(x, t), t = 1, 2, 3$. Attach each pendant vertex $\rho(x_{i,t})$ to each vertex $x_{i,t}$ in the cycles C_r^1, C_r^2, C_r^3 to get three sunlet graphs with $2r$ vertices. Hence, $L_{2r} | C_r \times C_3$.

□

Remark 2.1 For $n \geq 3, L_{2n} | C_n \times C_n$. This follows from the previous lemma.

Lemma 2.2 *For $r \geq 3$, $L_{2r} | L_{2r} \times C_3$.*

Proof. Let the vertices of the sunlet graphs on $2r$ vertices be $\{1, 2, ..., r, 1^*, 2^*, ..., r^*\}$ where i^* are pendant vertices adjacent to vertex i of the cycle contained in the sunlet graph L_{2r}. Then $L_{2r} \times C_3$ have the set of vertices $\{(p, c), (p^*, c) | p, p^* = 1, 2, ..., r \text{ and } c = 1, 2, 3\}$. Next, form the sunlet graphs $L_{2r}^1, ..., L_{2r}^{2c}$ from $L_{2r} \times C_3$ as follows:
If $r \equiv 0 (mod \;\; 3)$, C_r is of the form:

$$C_r^{s+1} = x_{1,s+1}, x_{2,s+2}, x_{3,s+3}, x_{4,s+1}, ..., x_{r-1,s+2}, x_{r,s+3}$$

and

$$^*C_r^{s+1} = x_{1,s+1}, x_{2,s+3}, x_{3,s+2}, x_{4,s+1}, ..., x_{r-1,s+3}, x_{r,s+2}.$$

If $r \equiv 1(mod\ \ 3)$, C_r is of the form:

$$C_r^{s+1} = x_{1,s+1}, x_{2,s+2}, x_{3,s+3}, x_{4,s+1}, ..., x_{r-1,s+3}, x_{r,s+2}$$

and

$$^*C_r^{s+1} = x_{1,s+1}, x_{2,s+3}, x_{3,s+2}, x_{4,s+1}, ..., x_{r-1,s+2}, x_{r,s+3}.$$

If $r \equiv 2(mod\ \ 3)$, cycle C_r is of the form:

$$C_r^{s+1} = x_{1,s+1}, x_{2,s+2}, x_{3,s+3}, x_{4,s+1}, ..., x_{r-1,s+1}, x_{r,s+2}$$

and

$$^*C_r^{s+1} = x_{1,s+1}, x_{2,s+3}, x_{3,s+2}, x_{4,s+1}, ..., x_{r-1,s+1}, x_{r,s+3}.$$

Where $s = 0, 1, 2$ and the sum on the second suffix is calculated modulo 3. Now, we define the pendant vertices as the mapping ρ as follows: For each vertex $x_{i,c}$ in C_r^{s+1}, we have
$\rho(x_{i,1}) = x_{i+r,2}$, $\rho(x_{i,2}) = x_{i+r,3}$, $\rho(x_{i,3}) = x_{i+r,1}$. Also, for each vertex $x_{i,c}$ in $^*C_r^{s+1}$, we have
$\rho(x_{i,1}) = x_{i+r,3}$, $\rho(x_{i,2}) = x_{i+r,1}$, $\rho(x_{i,3}) = x_{i+r,2}$. The vertices $\rho(x_{i,c}), i = 1, 2, ..., r$ and $c = 1, 2, 3$ are the pendant vertices. Join each pendant vertex $\rho(x_{i,c})$ to each vertex $x_{i,c}$ in the cycles C_r^{s+1} and $^*C_r^{s+1}$ to give the sunlet graphs L_{2r} which is the desired result. Therefore $L_{2r} \times C_3 = L_{2r}^1 \oplus L_{2r}^2 \oplus ... \oplus L_{2r}^6$.

□

Next, we give the following existing result on cycle decomposition.

Theorem 2.3 *[1] For any odd integer $t \geq 3$, if $n \equiv 1\ \ or\ \ t(mod\ \ 2t)$, then $C_t|K_n$.*

Now, we consider the sunlet decomposition of tensor product of sunlet graph with complete graph.

Theorem 2.4 *The graph $L_{2r}|L_{2r} \times K_n$ for any positive integer $n \equiv 1\ \ or\ \ 3(mod\ \ 6)$.*

Proof. Note that for any graphs G, H and T, if $H|G$, then $H \times T|G \times T$. Therefore by Theorem 2.3, $C_3|K_n$, then we have $L_{2r} \times (C_3 \oplus C_3 \oplus ... \oplus C_3) \cong$ $(Ł_{2r} \times C_3) \oplus (Ł_{2r} \times C_3) \oplus ... \oplus (Ł_{2r} \times C_3)$. Applying Lemma 2.2 give the desired result.

□

Next, we give the following results on the sunlet decomposition of cycle and complete graphs.

Theorem 2.5 *The graph $C_r \times K_n$ is decpomposable into sunlet graph L_{2r} for any positive odd integer $n \equiv 1\ \ or\ \ 3(mod\ \ 6)$.*

Proof. The graph K_n is decomposable into cycles C_3 by Theorem 2.3. Therefore, combining Theorem 2.3 and Lemma 2.1 gives the result.

□

Theorem 2.6 *[4] If $m \equiv 5(mod \ \ 6)$, then K_m can be decomposed into $\frac{m(m-1)-20}{6}$ 3-cycles and 5-cycles.*

Lemma 2.7 *For positive integers r and n such that $r > n \geq 3$ and $r \equiv 0 \ \ or \ \ 2(mod \ \ n)$, the graph L_{2r} decomposes $C_r \times C_n$*

Proof. We split the problem into the following two cases.
Case 1: when $r \equiv 0(mod\ n)$.
First, construct the base cycles $C_r^1, C_r^2, ..., C_r^n$ by appying Lemma 2.1 (case 1) as follows:

$$C_r^{s+1} = x_{1,s+1}, x_{2,s+2}, x_{3,s+3}, ..., x_{r,s+r}$$

where $s = 0, 1, 2, ..., n-1$. Also, note that the sum on the second suffix is calculated modulo n. Therefore, we have n cycles $C_r^1, C_r^2, ..., C_r^n$.
Define the mapping ρ by $\rho(x_{i,j}) = x_{i+1,j-1}, 1 \leq i \leq r$ and $1 \leq j \leq n$. Note that the sum on the first and second suffixes are calculated modulo r and n respectively. The vertices $\rho(x_{i,j})$ are pendant vertices . Attach each pendant vertex $\rho(x_{i,j})$ to vertex $x_{i,j}$ in the base cycles to give the sunlet graphs L_{2r}.
Case 2: when $r \equiv 2(mod \ \ n)$.
Apply Lemma 2.1 (case 3) to construct n base cycles:

$$C_r^{s+1} = x_{1,s+1}, x_{2,s+2}, x_{3,s+3}, ..., x_{r-2,s+n}, x_{r-1,s+1}, x_{r,s+2}$$

where $s = 0, 1, 2, ..., n-1$. The sum on the second suffix is calculated modulo n.
Next, define mapping ρ on vertices $x_{i,j}, 1 \leq i \leq r, 1 \leq j \leq n$ to get pendant vertices as follows:
$\rho(x_{i,j}) = x_{i+1,j-1}$ for $1 \leq i \leq r-1$ and for $i = r$, define $\rho(x_{i,j}) = x_{1,j+1}$. Note that the sum on the first and second suffixes are calculated modulo r and n respectively. Attach each pendant vertex $\rho(x_{i,j})$ to each vertex in the base cycles to give the sunlet graph L_{2r}. Hence the proof.

□

Theorem 2.8 *Let $n \equiv 5(mod \ \ 6)$, the graph $C_3 \times K_n$ is decomposable into sunlet graphs L_{2r}, for $r = 3, 5$ and n.*

Proof. We split the problem into the following two cases:
Case 1: when $r = 3, 5$.
Recall by Theorem 2.6 that K_n is decomposable into copies of cycles C_3 and C_5. Therefore, we have $C_3 \times (C_3 \oplus C_3 \oplus ...C_5 \oplus ... \oplus C_5) = (C_3 \times C_3) \oplus ... \oplus (C_3 \times C_3) \oplus (C_3 \times C_5) \oplus ...(C_3 \times C_5)$. Now, apply Lemma 2.1 to $C_3 \times C_3$ and Lemma 2.7 to $C_3 \times C_5$, this gives the desired result.
Case 2: when $r = n$.
The graph K_n is decomposable into Hamilton cycles C_n. Therefore we have

$C_3 \times K_n = C_3 \times (C_n \oplus C_n \oplus ... \oplus C_n)$. By Lemma 2.1, we have $L_{2r}|C_3 \times K_n$. Hence the proof.

□

Corollary 2.9 *Let $n \equiv 5(mod\ \ 6)$ and $r \equiv 0\ \ or\ \ 2(mod\ \ 5)$, then the graph $C_r \times K_n$ is decomposable into sunlet graphs L_{2r}*

Proof. Recall that K_n is decomposable into copies of cycles C_3 and C_5 by Theorem 2.6. Apply Lemma 2.1 to each $Cr \times C_3$ and Lemma 2.7 to each $C_r \times C_5$ gives the result.

□

Lemma 2.10 *Let m be a positive even integer and n a positive integer, the graph $C_m \times C_n$ is decomposable into sunlet graphs L_{2m}.*

Proof. First, we construct the base cycles C_m from $C_m \times C_n$ as follows:

$C_m^{s+1} = x_{1,s+1}, x_{2,s+2}, x_{3,s+3}, ..., x_{\frac{m}{2},s+\frac{m}{2}}, x_{\frac{m}{2}+1,s+\frac{m}{2}+1}, x_{t,s+\frac{m}{2}}, x_{t+1,s+\frac{m}{2}-1}, ..., x_{m,s+2},$
where $t = \frac{m}{2} + 2 \leq n$ and $s = 0, 1, 2, ..., n-1$. The sum on the second suffix is calculated modulo n. Hence, we have n m-cycles.
Next, we define the pendant vertices as the mapping ρ as follows:

$$\rho(x_{i,j}) = x_{i+1,j-1}, \quad for \ \ 1 \leq i \leq \frac{m}{2}$$
$$\rho(x_{i,j}) = x_{i+1,j+1}, \quad for \ \ \frac{m}{2} + 1 \leq i \leq \frac{m}{2}.$$

Note that the sum on the first and second suffixes are calculated modulo m and n respectively. Attach each pendant vertex $\rho(x_{i,j})$ to vertex $x_{i,j}$ to give the sunlet graph L_{2m}.

□

Theorem 2.11 *For any odd integer n and positive integer m, the graph $C_m \times K_n$ is decomposable into sunlet graphs L_{2r} if and only if $mn(n-1) \equiv 0(mod\ \ 2r)$.*

Proof. First, we prove the necessary condition. Assume that the graph $C_m \times K_n$ is decomposable into sunlet graph L_{2r}. This implies that the number of edges in $C_m \times K_n$ is divisible by $2r$, i. e. $mn(n-1) \equiv 0(mod\ \ 2r)$.
Next, we prove the sufficient condition. Assume that $mn(n-1) \equiv 0(mod\ \ 2r)$. Then we split the problem into the following 2 cases:
Case 1: $r|m$:
We only prove the case for when $r = m$. By Theorem 2.3, $C_3|K_n$, for $n \equiv 1\ \ or\ \ 3(mod\ \ 6)$. Next, apply Lemma 2.1 gives the desired result. Also, whenever $n \equiv 5(mod\ \ 6)$ and $r \equiv 0\ \ or\ \ 2(mod\ \ 5)$. Combining Theorem 2.6, Lemma 2.1 and Theorem 2.7 gives the result.
Case 2: $r|n(n-1)$
The graph K_n is decomposable into cycle C_r by Theorem 2.3. Therefore, we have $C_m \times K_n = (C_m \times C_r) \oplus ... \oplus (C_m \times C_r)$.

We split the problem into the following two subcases:
Subcase 1: when r is even and m a positive intetger. Note that $C_m \times C_r \equiv C_r \times C_m$. Applying Lemma 2.10 gives the result.
Subcase 2: when r is odd and m a positive integer. If $m = 3$, apply Lemma 2.1 to give the result. Also, if $r > m$ and $r \equiv 0 \;\; or \;\; 2(mod \;\; m)$, apply Lemma 2.7 to have L_{2r}. Hence the proof.

□

Next, we give the previous results on cycle decomposition of complete graphs.

Theorem 2.12 *[12] Let n be an even integer and m be an odd integer with $3 \leq m \leq n$. The graph $K_n - I$ can be decomposed into cycles of length m whenever m divides the number of edges in $K_n - I$.*

Theorem 2.13 *[1] For positive even integers m and n with $4 \leq m \leq n$, the graph $K_n - I$ can be decomposed into cycles of length m if and only if the number of edges in $K_n - I$ is a multiple of m.*

Theorem 2.14 *[12] Let n be an odd integer and m be an even integer with $3 \leq m \leq n$. The graph K_n can be decomposed into cycles of length m whenever m divides the number of edges in K_n.*

The next theorem gives the necessary and sufficient conditions for sunlet decomposition of tensor product of complete graphs.

Theorem 2.15 *For positive odd integers n and m, the graph $K_m \times K_n$ is decomposable into sunlet graphs L_{2r} if and only if $mn(m-1)(n-1) \equiv 0(mod \;\; 2r)$.*

Proof. The necessary condition is obvious, we will only prove the sufficient condition. Next, we split the problem into the following two cases:
Case 1: $r|m(m-1)$
By Theorem 2.3 and 2.14, $C_r|K_m$ whenever r divides the number of edges in K_m. Therefore, we have

$$K_m \times K_n = (C_r \times K_n) \oplus (C_r \times K_n) \oplus ... \oplus (C_r \times K_n).$$

Each $C_r \times K_n$ decomposes into sunlet graphs L_{2r} by Theorem 2.11. Hence the result.
Case 2: $r|n(n-1)$
Using the same argument in case 1 above and interchanging m and n gives the desired result.

□

Theorem 2.16 *For positive even integer m and odd integer n, the graph $K_m - I \times K_n$ is decomposable into sunlet graphs L_{2r} if and only if $mn(m-2)(n-1) \equiv 0(mod \;\; 2r)$.*

Proof. The necessary condition is obvious since the number of edges in $K_m - I \times K_n$ is $\frac{mn(m-2)(n-1)}{2}$. We only prove the sufficient condition. Next, we split the problem into the following two cases:
Case 1: $r|m(m-2)$
By Theorems 2.12 and 2.13, $K_m - I$ is decomposable into cycles C_r whenever r divides the number of edges in $K_m - I$. Therefore, we have

$$K_m - I \times K_n = (C_r \oplus C_r \oplus ... \oplus C_r) \times K_n \cong (C_r \times K_n) \oplus ... \oplus (C_r \times K_n).$$

Applying Theorem 2.11 to each $C_r \times K_n$ gives the result.
Case 2: $r|n(n-1)$
By Theorems 2.3, 2.6 and 2.14, K_n decomposes into cycle C_r. Also, by Theorems 2.13 and 2.12, $C_t|K_m - I$, for any positive integer $t \leq m$. Therefore, we have $C_t \times C_r$. Note that $C_t \times C_r \cong C_r \times C_t$. Apply Lemma 2.10 to $C_r \times C_t$ to have L_{2r} decomposition, for any positive even integer r. If r is odd, apply Lemma 2.1 and Lemma 2.7 to have the result. Hence L_{2r} decomposes $K_m - I \times K_n$.

□

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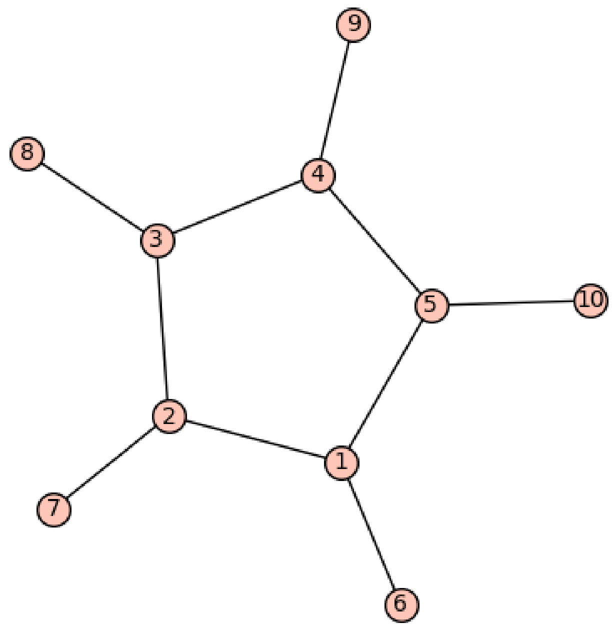

Fig. 1 Sunlet graph on 10 vertices

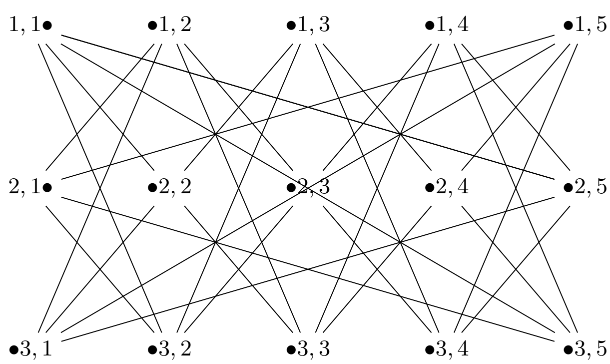

Fig. 2 $C_3 \times C_5$