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The order divisor-power graph of finite groups

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Abstract Let G be a finite group. In this paper, we introduce the order divisor-power graph $\Gamma_{odp}(G)$ associated with G as the simple undirected graph whose vertices are the elements of G and such that two vertices a, b $a \neq b$ are adjacent if one is a power of the other and their orders are different. We investigate some algebraic properties and combinatorial structures of the order divisor-power graph $\Gamma_{odp}(G)$ and obtain the conditions under which the order divisor-power graph $\Gamma_{odp}(G)$ can be a star graph. Also, we exhibit some connection between the order divisor-power graph and the power graph of dihedral groups up to an isomorphism. Furthermore, we prove that the order divisor-power graphs of some classes of dihedral groups are neither bipartite nor tripartite, but it is a complete multipartite graph if the group is a cyclic group.

Keywords graph of finite groups · order-divisor graph · power graph · order divisor-power graph · star graph · isomorphic graph

Mathematics Subject Classification (2010) MSC 20 · D60

1 Introduction

Graph theory is a branch of mathematics that provides a way of visualizing complex mathematical structures. According to R.J. Wilson [35], graph theory has established itself as an important mathematical tool in a wide variety of subjects, ranging from operational research and chemistry to genetics and linguistics, and from electrical engineering and geography to sociology and architecture. This powerful combinatorial tool has also been used by many researchers to show the relationship between groups and graphs. Y.F. Zakariya [37] noted that graphs associated to finite groups have a long history starting from Cayley graphs that date back to 1878. Cayley [10] used the generators of a finite group G to define a graphical structure called the Cayley graph of finite groups. More recently, the interaction between groups and graphs has raised number of other graphs from groups and they are extensively studied, see [2, 4–6, 9, 13, 21–26, 28, 29, 32, 34]. Graph theory

has also played vital roles to provide answer to questions and proofs to significant results in finite groups, see [3,9,27,31,33].

S.U. Rehman et al. [32] introduced and studied the order divisor graph of finite groups; the graphs whose vertex set is G and two distinct vertices a and b are adjacent if their orders are different and $o(a)$ divides $o(b)$ or $o(b)$ divides $o(a)$. Besides, Kelarev and Quinn [22] began the idea of the directed power graph of a semigroup and later in 2002, in another work, defined the undirected power graph [23]. In fact, the undirected power graph of a finite group G is the simple undirected graph whose vertex set is the group itself, and two distinct vertices a and b are adjacent if and only if either $a = b^n$ or $b = a^m$ for some positive integers n and m. Chattopadhyay and Panigrahi [12] motivated by an open problem of Abawajy et al. [1], discovered some relations between power graphs and Cayley graphs of finite cyclic groups. They proved that the vertex deleted sub graphs of some power graphs are spanning sub graphs or equal to the complement of vertex deleted sub graphs of some unitary Cayley graphs. They also proved that some Cayley graphs can be expressed as direct product of power graphs.

In the same vein, [11] defined the order divisor graph of subgroups of a finite group. In their work, they defined the graph with vertex set as subgroups of the finite group and two different subgroups H and K are adjacent if the $o(H)$ divides $o(K)$ or $o(K)$ divides $o(H)$. They further obtained the density, girth and diameter of the graph. Similarly, [18] studied the vertex relations of order divisor graphs of subgroups of finite groups. In this paper, we introduce a new graph by name the order divisor-power graph of finite groups and explore some of its properties.

2 Preliminaries

For the reader's convenience, we state some useful concepts and known results which will be needed in the proof of our main results and understanding of this paper. For the definition of the basic terms and results given in this section (see [7,15,19,20,25,26,32]). Let G be a finite group. The order of an element $a \in G$ denoted by $o(a)$, is the smallest positive integer n such that $a^n = e$, where e is the identity element in G. For a prime number p, a group G is called a *p-group* if order of every element in G is a power p. The exponent of a group G is the smallest positive integer m such that $g^m = e$ for all $g \in G$. We say that a group G is finitely generated if there is a finite subset S of G such that every element of G is a product of elements of S and their inverses.

The group of symmetries of the regular n-gon ($n \geq 3$) is called a dihedral group. It has order $2n$, contains a cyclic subgroup of order n consisting of rotations and every element outside this cyclic subgroup is a reflection of order 2. We denote this group by D_{2n}. In fact $D_{2n} = \langle a, b \mid a^n = e, b^2 = e, ba = a^{-1}b \rangle$ [14]. An action of a group G on a set Ω is a homomorphism from G to the symmetric group $Sym(\Omega)$. In other words, to each group

element we associate a permutation, and the product of group elements is associated with the composition of the corresponding permutations. We will always have in mind a fixed action Θ; so $g\Theta$ is a permutation of Ω. For any positive integer $n \geq 1$, the Euler *totient function* is defined to be the number of positive integers which are less or equal to n and are relatively prime to the integer [16].

Throughout this paper Γ is a finite simple undirected graph with vertex set V and edge set E. A path in Γ is said to be a *Hamiltonian path* if it contains each vertex exactly once. Also, a *Hamiltonian circuit* is a circuit that contains each vertex exactly once except for the first vertex which is equal to the last vertex. A graph Γ is called bipartite if its vertex set can be partitioned into two disjoint subsets V_1 and V_2 of V, such that every edge e has one end in V_1 and another end in V_2. A complete bipartite graph is a bipartite graph with every vertex in V_1 is adjacent to every vertex in V_2. The diameter of a graph is defined as the maximum distance $d(a, b)$ over all pairs of vertices $a, b \in \Gamma$. A star graph is a graph consisting of a vertex which is adjacent to all other vertices. The neighbourhood of a vertex $a \in V(\Gamma)$ denoted by $N_\Gamma(a)$, is the set $\{b \in V(\Gamma) \mid ab \in E(\Gamma)\}$. The following are referred in this paper.

Theorem 2.1 *[7] Suppose that the prime p divides the order of the group G. Then G contains an element of order p.*

Theorem 2.2 *[20] Let Γ be a graph with n vertices. If, for any two vertices a and b of Γ; deg a + deg $b \geq n$, then Γ contains a Hamiltonian path*

Corollary 2.3 *[20] Let Γ be a graph with $n \geq 3$ vertices. If each vertex has degree greater than or equal to $\frac{n}{2}$, then Γ has a Hamiltonian circuit.*

Theorem 2.4 *[17, 36] Let G be a finite abelian group and let m be the largest order of an element in G. Then, for any $a \in G$, the $o(a)$ divides m.*

3 Main results

Let us recall the definition of the order divisor-power graph of a finite group G.

Definition 3.1 *Let G be a finite group. Then $\Gamma_{odp}(G)$ denotes the order divisor-power graph of G. It is a simple undirected graph with vertex set G and such that two distinct veritices $a, b \in G$ are adjacent if one is a power of the other and their orders are different.*

Example 3.1 Consider the symmetric group

$$S_3 = \left\{ e = \begin{pmatrix} 1\,2\,3 \\ 1\,2\,3 \end{pmatrix},\ I = \begin{pmatrix} 1\,2\,3 \\ 1\,3\,2 \end{pmatrix},\ J = \begin{pmatrix} 1\,2\,3 \\ 3\,1\,2 \end{pmatrix} \right.$$

$$K = \begin{pmatrix} 1\,2\,3 \\ 3\,2\,1 \end{pmatrix},\ L = \begin{pmatrix} 1\,2\,3 \\ 2\,1\,3 \end{pmatrix},\ M = \begin{pmatrix} 1\,2\,3 \\ 2\,3\,1 \end{pmatrix}\Big\}.$$

The order divisor-power graph $\Gamma_{odp}(S_3)$ is given in Figure 1.

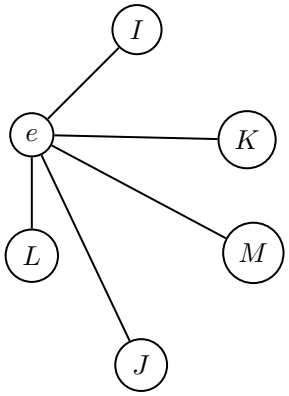

Figure 1. $\Gamma_{odp}(S_3)$

Remark 3.1 (a). Let G be a finite group of order n. The identity element of G is adjacent to all vertices in $\Gamma_{odp}(G)$ and hence $\Gamma_{odp}(G)$ is connected. Further the diameter of $\Gamma_{odp}(G)$ is 2.

(b). Let $a, b \in G$ be two conjugate elements in G. Then a and b are not adjacent $\Gamma_{odp}(G)$.

(c). The order divisor-power graph $\Gamma_{odp}(G)$ is a sub graph of the order divisor graph $\Gamma_{od}(G)$.

(d). The girth of an order divisor-power graph $\Gamma_{odp}(G)$ is greater than or equal to 4.

(e) The order divisor-power graph $\Gamma_{odp}(G)$ is not necessarily planar. Its planarity depends on the specific divisibility and power relationships in G.

(f) The order divisor-power graph $\Gamma_{odp}(G)$ does not always contain Hamiltonian paths or cycles. It depends on the group's structure.

Theorem 3.2 *Let G Cyclic Group, then the $\Gamma_{odp}(G)$ is a complete multipartite graph.*

Proof. Let $G = \langle g \rangle$ be a cyclic group of order n, generated by an element g. The order of each element in G divides n, and each element of G can be written as a power of g. Since G is cyclic, for any two elements $a = g^i$ and $b = g^j$ in G, one element is a power of the other because they are both powers of the generator g. Thus, the first adjacency condition of the graph is satisfied. Also, the second condition for adjacency, that $o(a) \neq o(b)$, will hold for any two distinct elements a and b unless they have the same order, which occurs only within specific order classes. Thus, all elements in G can be partitioned into distinct sets based on their order, and within

each set, the elements are not adjacent (since they have the same order). However, elements from different sets are adjacent because they satisfy both adjacency conditions (one is a power of the other, and their orders are different). Therefore, $\Gamma_{\mathrm{odp}}(G)$ is a complete multipartite graph, where the parts correspond to the sets of elements of the same order.

□

Theorem 3.3 *Let G is a finite group, then $\Gamma_{odp}(G)$ is a regular graph if and only if*

1. *(i) G is cyclic of prime order.*
2. *(ii) All non-identity elements of G have the same order, and the adjacency conditions uniformly apply.*

Proof. If G is cyclic of prime order, then the order divisor-power graph $\Gamma_{odp}(G)$ consist of one identity element and $p-1$ non-identity elements, all of the same order and the following imply

(i) The identity element has degree 0 or another consistent degree based on adjacency rules and all non-identity elements have identical adjacency property, leading to regularity.

(ii) Adjecency depends unformily on being powers of each other and differing from the identity. Thus, regularity follows from uniform adjacency rules across all the elements.

Conversely, if $\Gamma_{odp}(G)$ is regular, the degrees imply uniformity in adjacency conditions and the uniformity is achievable only if the order of the vertices and their power relations are consistent across G.

□

Corollary 3.4 *If $\Gamma_{odp}(G)$ is not regular, then G must contain elements of differing orders or varying adjacency relationships based on power relation.*

Proof. Since vertex degrees in $\Gamma_{\mathrm{odp}}(G)$ are influenced by element orders and power relationships, variation in degrees indicate variation in these properties.

□

Theorem 3.5 *Let G be a finite group.*

a. *The order divisor-power graph $\Gamma_{odp}(G)$ determines the power graph $\Phi(G)$ uniquely.*
b. *The power graph $\Phi(G)$ determines the order divisor-power graph $\Gamma_{odp}(G)$ up to isomorphism.*

Proof. (*a.*) Suppose the order divisor-power graph $\Gamma_{odp}(G)$ of G is known. Then we can claim that every pair of vertices a and b with varying orders that are adjacent on the order-divisor power graph of G are either adjacent

on the power graph of G or are of equal neighbourhoods, but on the other hand suppose these vertices are not adjacent of the order-divisor power graph but are joined on the power graphs then they are of the same orders; this shows that the order-divisor power graph of G determines the power graph uniquely.
(b.) By [8], the power graph determines the directed power graph up to isomorphism. Now two vertices a and b are joined in the order divisor-power graph if and only if there is an arc in one direction but not the other.

□

Theorem 3.6 *Let G be a finite group such that the order of every non-identity is a prime number. Then the order divisor-power graph $\Gamma_{odp}(G)$ is star. G*

Proof. Assume that $o(a) = p$, a prime for every $e \neq a \in G$. If $o(G) = n$, by Remark 3.1, $K_{1,n-1}$ is a subgraph of $\Gamma_{odp}(G)$. To conclude the proof, it is enough to prove that no two non-identity elements are adjacent in $\Gamma_{odp}(G)$. Let $a, b \in G$ be two distinct non-identity elements. Since orders of these elements $a, b \in G$ are primes, we get that two distinct primes p and q such that $o(a) = p$ and $o(b) = q$. Now, trivially neither $o(a)$ divides $o(b)$ and $o(b)$ divides $O(a)$. Thus they are not adjacent in $\Gamma_{odp}(G)$. Hence $\Gamma_{odp}(G)$ is a star graph.

□

Recall that an elementary abelian group is an abelian group in which every non-trivial element has prime order. Thus we have the following corollary from Theorem 3.6.

Corollary 3.7 *Let G be an elementary abelian group. Then $\Gamma_{odp}(G)$ is a star graph.*

Lemma 3.8 *Let $n \geq 3$ be a composite number and n is not square free. Then the order divisor-power graph $\Gamma_{odp}((Z_n))$ is not a star graph.*

Proof. By assumption n has at least one prime divisor p such that p^k divides n and $k \geq 2$. Then there exist non-identity elements $a, b \in (Z_n)$ with $a \neq b$, $o(a) = p$ and $o(b) = p^2$. Cleary a and b are adjacent in $\Gamma_{odp}((Z_n))$ and hence $\Gamma_{odp}((Z_n))$ is not a star graph.

□

Now, we obtain a characterization for $\Gamma_{odp}((Z_n))$ to be a star graph.

Theorem 3.9 *Let $n \geq 2$ and $\mathbb{Z}_n$ is a p-group. Then the order divisor-power graph $\Gamma_{odp}((Z_n))$ is a star graph.*

Proof. If $\mathbb{Z}_n$ is a p-group, then every non-identity element of $\mathbb{Z}_n$ is of order p^k for $k \geq 2$ and p^k divides n. Clearly different orders of non-identity elements can not divide each other. Since the identity vertex is adjacent to all other vertices, $\Gamma_{odp}((Z_n))$ is a star graph.

□

In view of Example 3.1, the converse of Theorem 3.9 is not true. The following can be easily observed from the definition.

Lemma 3.10 *Let G be a finite cyclic group generated by a. Then a is adjacent to all other vertices in $\Gamma_{odp}(G)$.*

Remark 3.2 Let G be a finite abelian group of order n and let $a \in G$ be such that $o(a) = p$ is a prime number. Then the number of vertices adjacent to a in $\Gamma_{odp}(G)$ is not a multiple of $p-1$.

Proof. Since a is adjancent of $a, \ldots, a^{p-1}, e$ in $\Gamma_{odp}(G)$, the number of vertices adjacent to a is p. Hence the result is true.

□

Theorem 3.11 *Let G be ab elementary abelian group $G \simeq (\mathbf{Z}_p)^n$ for some prime p and integer $n \geq 1$, the automorphism group $Aut(\Gamma_{odp}(G))$ is isomorphic to the general linear group $GL(n, p)$.*

Proof. Since $G \simeq (\mathbf{Z}_p)^n$ is an elementary abelian group, then every non-identity element has order p; and G is a vector over the finite field $(\mathbf{Z}_p)^n$. The non-identity vertices on $(\Gamma_{odp}(G))$ have the same order p while the identity element has order 1. This implies adjacency occurs only between the identity and elements of order p that are powers of others. Thus, the automophisms correspond to invertible linear transformation and it preserves the vector space structure, hence, preserving the adjacency of $\Gamma_{odp}(G))$.

□

4 The order divisor-power graph of dihedral groups

The dihedral group $D_n = \{e, r, r^2, ..., r^{n-1}, s, sr, sr^2, ..., sr^{n-1}\}$, $n \geq 3$ is a rigid motion or isometry of the regular n-gon, r is a rotation of $\frac{2\pi}{n}$ radians and as a transformation of the $n-$gon, it has order n ($\mid r \mid = n$). Also, s is a reflection through vertex e and the origin. In addition, a rotation by multiple $\frac{2\pi}{n}$ radians around the center takes the polygon back to itself.

Theorem 4.1 *Let $n \geq 3$, n be odd and let D_n be a dihedral group of order $2n$. Then the order divisor graph $\Gamma_{od}(D_n)$ is isomorphic to the power graph $\Phi(D_n)$.*

Proof. From definition 3.1, it is clear that the vertex sets of two graphs $\gamma_{od}(D_n)$ and $\Phi(D_n)$ are the same. Further let a and b be adjacent vertices in $\gamma_{od}(D_n)$. Then $a \neq b$ and either $o(a)$ divides $o(b)$ or $o(b)$ divides $o(a)$. This implies that either $a = b^n$ or $b = a^m$ where n and m are positive integers. From this a and b are also adjacent in the power graph $\Phi(D_n)$. Similarly, one can prove that non-adjacency is also preserved. This completes the proof.

□

Remark 4.1 Let $n \geq 3$ and n be even. Then the order divisor graph $\gamma_{od}(D_n)$ is not isomorphic to the the power graph $\Phi(D_n)$.

Suppose D_p is an arbitrary Dihedral group, such that p is a prime greater than 2, the following theorem is from the results as given by [[30], D. Nanci, 2018], (1.1.1: that one element in the group has order 1, at least p elements have order 2 while $p-1$ elements have order $p-1$).

Theorem 4.2 *Let $p \geq 3$ be a prime number. Then the following are true regarding the order divisor-power graph $\Gamma_{odp}(D_p)$ of the dihedral group $D_p = \{e, r, r^2, \ldots, r^{p-1}, s, sr, sr^2, \ldots, sr^{p-1}\}$ and $i = \{0, 1, ...(p-1)\}$.*

(1) $deg(a) = \begin{cases} p & \text{if } a = sr^i \in D_p; \\ 2p-1 & \text{if } a = e \in D_p; \\ p+1 & \text{if } a = r^i \in D_p. \end{cases}$

(2) *The graph $\Gamma_{odp}(D_p)$ is a tree graph.*

(3) *The graph $\Gamma_{odp}(D_p)$ is acyclic.*

Proof. (1) As observed previously the degree of the vertex e in $\Gamma_{odp}(D_p)$ is $2p-1$. Similarly, let $A \neq e \in D_p$, then, it is either a reflection or a rotation.

If it is a reflection then, by ([30], D. Nanci, 2018), they are at least p of such elements of order 2, and 2 is a divisor of 1 and also a divisor of $p-1$, the order of the rotation elements of D_p.

(2) The proof follows from (1); the $\mathrm{V}(\Gamma(D_p))$ is divided into three sets of vertices, the identity element, reflection elements of order p and the rotation elements of order $p-1$, clearly the orders of the reflection elements and the rotation elements are different but they can not be written as product of each other.

(3) Proofs follows from (2) above.

□

Example 4.1 Let D_5 be a dihedral group, then the order divisor-power graph of D_5, $\Gamma_{odp}(D_5)$ is given below:

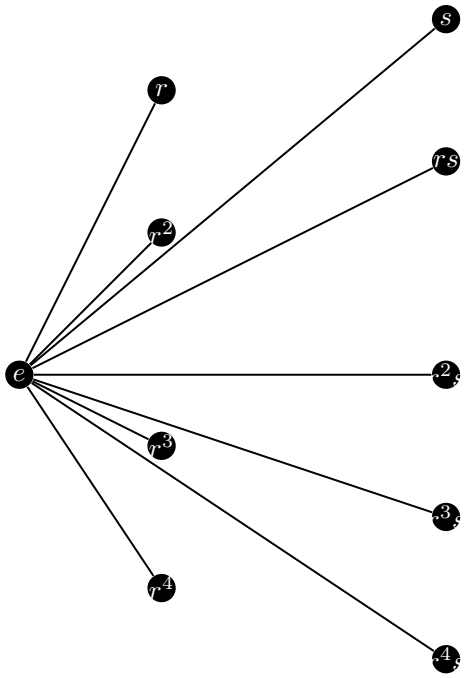

Figure 2: Order divisor-power graph of D_5.

Corollary 4.3 *Let $\Gamma(D_n)$ be the order divisor-power graph of a dihedral group, such that n is greater than 2, then the graph $\Gamma(D_n)$ is neither bipartite or tripartite.*

Proof. The proof follows from Theorem 4.2.

□

Theorem 4.4 *Let $\Gamma_{odp}(S_n)$ be an order divisor-power graph of the symmetric group S_n. Then $\alpha, \rho \in V(\Gamma(S_n))$ are adjacent in $\Gamma_{dop}(S_n)$ if α and ρ lies in the same cycle.*

Proof. Let $\Gamma(S_n)$ such that α, $\rho \in V(\Gamma(S_n))$ then either α or ρ is a power of the other and both lies in the cyclic group generated by either of them. Certainly by Langrange's Theorem $o(\alpha)$ divides $o(\rho)$ or $o(\rho)$ divides $o(\alpha)$.

□

Corollary 4.5 *Let $\Gamma(S_n)$ be an order divisor-power graph of the symmetric group, S_n, then $(\alpha, \rho) \in E(\Gamma(S_n))$ if α and ρ are not transposition.*

Proof: Let $\Gamma(S_n)$ be an order divisor-power graph of the symmetric group, S_n. Suppose $(\alpha, \rho) \in E(\Gamma(S_n))$ then either α or ρ is a power of the other and either $o(\alpha)$ divides $o(\rho)$ or $o(\rho)$ divides $o(\alpha)$, but α and ρ are conjugates since they are transposition. Clearly it implies they have the same order and it contradict Definition 3.1. □

5 Conclusion

This study looks at a modified order divisor and power graph called the order divisor-power graph of finite groups. Clearly we show that the order divisor-power graph of G determines the power graph uniquely while the power graph determines the order divisor-power graph up to isomorphism. We further establish that if every non identity element of a finite group G has a prime order, then the order divisor-power graph $\Gamma(G)$ is a star graph. However, the order divisor-power graph $\Gamma(Z_n)$ of group of integer modulo n such that $n \geq 3$ is not a star graph if n is even. Moreover, we show that if p is a prime greater or equal to 3, the order divisor-power graph is acyclic and a star graph.

References

1. Abawajy, A.; Kelarev, A.; Chowdhury, M. – *Power graphs: a survey*, Electron. J. Graph Theory Appl., **1** (2013), no. 2, 125-147.
2. Ambrozie, C.; Bračič, J.; Kuzma, B.; Mülller, V. – *The commuting graph of bounded linear operators on a Hilbert space*, J. Funct. Anal., **264** (2013), no. 4, 1068–1087.
3. Ballester-Bolinches, A.; Cossey, J.; Esteban-Romero, R. – *Graphs and classes of finite groups*, Note Mat., **33** (2013), no. 1, 89-94.
4. Bertram, E.A.; Herzog, M.; Mann, A. – *On a graph related to conjugacy classes of groups*, Bull. London Math. Soc., **22** (1990), no. 6, 569–575.
5. Bianchi, M.; Chillag, D.; Mauri, A.G.B.; Herzog, M.; Scoppola, C.M. – *Applications of a graph related to conjugacy classes in finite groups*, Arch. Math. (Basel), **58** (1992), no. 2, 126–132.
6. Brauer, R.; Fowler, K.A. – *On groups of even order*, Ann. of Math., **62** (1955), no. 2, 565–583.
7. Cameron, P.J. – *Notes on finite group theory*, (2013), https://cameroncounts.wordpress.com/2013/11/01/notes-on-finite-groups/.
8. Cameron, P.J. – *The power graph of a finite group, II*, J. Group Theory, **13** (2010), no. 6, 779–783.
9. Cameron, P.J.; Ghosh, S. – *The power graph of a finite group*, Discrete Math., **311** (2011), no. 13, 1220–1222.
10. Cayley, A. – *Desiderata and suggestions: no. 2. The theory of groups: graphical representation*, Amer. J. Math., **1** (1878), no. 2, 174-176.
11. Chalapathi, T.; V M S S Kiran Kumar, R. – *Order divisor graphs of finite groups*, Malaya Journal of Matematik, **5** (2017), no. 2, 464-474.
12. Chattopadhyay, S.; Panigrahi, P. – *Some relations between power graphs and Cayley graphs*, J. Egyptian Math. Soc., **23** (2015), no. 3, 457-462.
13. Chudnovsky, M.; Cizek, M.; Crew, L.; Miná̌č, J.; Nguyen, T.T.; Spirkl, S.; Duy Tân, N. – *On prime Cayley graphs*, https://arxiv.org/abs/2401.06062, (2024).
14. Conrad, K. – *Dihedral groups II*, lecture note (2009), https://kconrad.math.uconn.edu/blurbs/grouptheory/dihedral.pdf.
15. Dummit, D.S.; Foote R.M. – *Abstract algebra*, Third Edition, John Wiley & Sons, Inc., Hoboken, NJ, (2004).
16. Dineva, R. – *The Euler totient, the möbius and the divisor functions*, Mount Holyoke College, (2005).
17. Gallian, J.A. – *Contemporary Abstract Algebra*, Brooks/Cole Publishing Company, Cengage learning, **8** (2013), 118.
18. Garba, A.I.; Ejima, O.; Aremu, K.O. – *Vertex relations of order divisor graphs of subgroups of finite groups*, J. Nigerian Math. Soc., **39** (2020), no. 3, 315-322.
19. Godsil, C.; Royle, G. – *Algebraic graph theory*, Graduate Texts in Mathematics, 207, Springer-Verlag, New York, (2001).
20. Gupta, A. – *Discrete Mathematics*, S.K. Kataria & Sons, (2002), 258-310.

21. Kayacan, S.; Yaraneri, E. – *Finite groups whose intersection graphs are planar*, J. Korean Math. Soc., **52** (2015), no. 1, 81-96.
22. Kelarev, A.V.; Quinn, S.J. – *A combinatorial property and power graphs of groups*, Proceedings of the Vienna Conference (Vienna University of Technology) Springer Verlag, **12** (2000), no. 58, 229-235.
23. Kelarev, A.V.; Quinn, S.J. – *Directed graphs and combinatorial properties of semigroups*, J. Algebra, **251** (2002), no. 1, 16-26.
24. Kelarev, A.V.; Ryan, J.; Yearwood, J. – *Cayley graphs as classifiers for data mining: the influence of asymmetries*, Discrete Math., **309** (2009), no. 17, 5360-5369.
25. Kelarev, A.V. – *Ring constructions and applications*, Series in Algebra, **9**, World Scientific Publishing Co., Inc., River Edge, NJ, (2002).
26. Kelarev, A.V. – *Graph algebras and automata*, Monographs and Textbooks in Pure and Applied Mathematics, **257**, Marcel Dekker, Inc., New York, (2003).
27. Kumar, J.; Panda, R.P.; Parveen, P. – *On the difference graph of power graphs of finite groups*, Quaest. Math., **47** (2024), no. 5, 997–1018.
28. Lewis, M.L. – *An overview of graphs associated with character degrees and conjugacy class sizes in finite groups*, Rocky Mountain J. Math., **38** (2008), no. 1, 175–211.
29. Mináč, J.; Nguyen, T.T.; Duy Tân, N. – *A complete classification of perfect unitary Cayley graphs*, https://arxiv.org/abs/2409.01922, (2024).
30. Nacin, D. – *The applied algebra workbook*, William Paterson University, (2018).
31. Raza, Z.; Faizi, S. – *Commuting graphs of dihedral type groups*, Appl. Math. E-Notes, **13** (2013), 221-227.
32. Rehman, S.U.; Baig, A.Q.; Imran, M.; Khan, Z.U. – *Order divisor graphs of finite groups*, An. Ştiinţ. Univ. "Ovidius" Constanţa Ser. Mat., **26** (2018), no. 3, 29-40.
33. Rehman, S.U. – *Comaximal factorization graphs in integral domains*, J. Prime Res. Math., **9** (2013), 65-71.
34. Shaker, A.A.; Shelash, H.B. – *The Cayley graph of semi-direct product of finite groups: interrelationships and construction*, Journal of Kufa for Mathematics and Computer, **11** (2024), no. 2, 18-25.
35. Wilson, R.J. – *Introduction to graph theory*, Fourth Edition, Longman, Harlow, **4** (1996).
36. Yashinski, A. – *Additional topics in group theory*, https://math.hawaii.edu, **3** (2015).
37. Zakariya, Y.F. – *Graphs from finite groups: an overview*, Proceedings of the 53rd Mathematical Association of Nigeria Annual Conference at Ahmadu Bello University, Samaru Zaria, (2016).

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