où $\bar{f}$ et $\bar{g}$ sont les extensions de f, resp. g, aux groupes de cochaînes. En passant aux groupes de cohomologie, de (8) nous obtenons le diagramme commutatif

$$
\begin{tikzcd}
H^n(J, P) \arrow[r, "\bar{f}^n"] \arrow[d, "\bar{g}^n"'] & H^n(C(J, P^*)) \arrow[d, "\bar{g}^n"] \\
H^n(J, Q) \arrow[r, "\bar{f}^n"] & H^n(C(J, Q^*))
\end{tikzcd}
\tag{9}
$$

D'autre part est commutatif le diagramme

$$
\begin{tikzcd}
H^n(\varinjlim_{i\in I} (P^i)) \arrow[r, "\bar{j}^n"] \arrow[d, "H^n(\varinjlim_{i\in I} g^i)"'] & H^n(C(J, P^*)) \arrow[d, "\bar{g}^n"] \\
H^n(\varprojlim_{i\in I} (Q^i)) \arrow[r, "\bar{j}^n"] & H^n(C(J, Q^*))
\end{tikzcd}
\tag{10}
$$

$\bar{j}^n$ étant isomorphisme (proposition 3.2), des diagrammes (9) et (10) nous obtenons le résultat.

BIBLIOGRAPHIE

1. Godement, R. — *Topologie algébrique et théorie des faisceaux*. Hermann, Paris, 1958.
2. Miron, R., Pop, I. — *Topologie algebrică — omologie, omotopie, spaţii de acoperire*. Editura Academiei R.S.R., Bucureşti, 1974.
3. Miron, R., Pop, I. — *On the abstract Čech homology and cohomology*. *Proceedings of the Institute of Mathematics*, Iaşi, (1974), 57—60.
4. Pitiş, Gh. — *Une méthode de calcul des groupes de cohomologie de Čech abstraite*. Memoriile Academiei R.S.R., tom. 11(1979), 87—96.
5. Pitiş, Gh. — *Suites spectrales en cohomologie de Čech abstraite*. Bull. de la Soc. des Sci. Math. de la R.S.R., (à paraître).

Received 15.V. 1982

Université de Braşov
Str. Karl Marx, 50
2200 — Braşov, România

ANALELE ŞTIINŢIFICE ALE UNIVERSITĂŢII „AL. I. CUZA" DIN IAŞI
Tomul XXX, s.I a. Matematică, 1984—3

A GEOMETRICAL CHARACTERIZATION OF NORMAL FINSLER CONNECTION

BY

EMIL STOICA

1. In the paper [4], R. Miron defines the notion of Finsler connection on vectorial bundles and establishes that locally, these are characterized by four coeficients $\overset{1}{F}, \overset{2}{F}, \overset{1}{C}, \overset{2}{C}$.

In the particular case of the tangent bundle the Finsler connection which is so obtained is more general then that known so far. The classical connections are obtained if the conditions $\overset{1}{F}=\overset{2}{F}$ and $\overset{1}{C}=\overset{2}{C}$, are satisfied.

In this paper, by means of the natural almost complex structure of tangent bundle endowed with a nonlinear connection, we give a geometrical characterization of these connections.

The terminology and notations are those of the paper [4].

The author wishes to express his sincer gratitude to Prof. R. Miron for his kindness and invaluable suggestions.

2. Let M be an n-dimensional differentiable manifold, $T(M)=(TM, \Pi, M)$ the tangent bundle, N a nonlinear connection, VTM and HTM the vertical and horizontal subbundles of TTM.

R. Miron defines a Finsler connections on TM as a linear connection ∇ on TM, which preserves by parallelism the horizontal N and vertical TM^V distributions.

It is known that a Finsler connection ∇ on TM is then characterized by conditions: $(\nabla_X Y^H)^V=0$ and $(\nabla_X Y^V)^H=0$. Putting $\nabla_X Y=\nabla^H_X Y+\nabla^V_X Y$, where $\nabla^H_X Y=\nabla_{X^H} Y$ and $\nabla^V_X Y=\nabla_{X^V} Y$, two covariant derivative operators ∇^H and ∇^V in the algebra of Finsler tensors fields on TM, are obtained.

In canonical coordinates, those covariant derivatives are expressed as:

$$
\begin{aligned}
&\nabla^H_{\frac{\delta}{\delta x^k}} \frac{\delta}{\delta x^j} = \overset{1}{F}{}^i_{jk} \frac{\delta}{\delta x^i}, \quad \nabla^H_{\frac{\delta}{\delta x^k}} \frac{\partial}{\partial y^j} = \overset{2}{F}{}^i_{jk} \frac{\partial}{\partial y^i} \\
&\nabla^V_{\frac{\partial}{\partial y^k}} \frac{\delta}{\delta x^j} = \overset{1}{C}{}^i_{jk} \frac{\delta}{\delta x^i}, \quad \nabla^V_{\frac{\partial}{\partial y^k}} \frac{\partial}{\partial y^j} = \overset{2}{C}{}^i_{jk} \frac{\partial}{\partial y^i}
\end{aligned}
\tag{1}
$$

where $\delta/\delta x^i=\partial/\partial x^i-N^j_i\partial/\partial y^j$, $i, j=1, 2, \ldots, n$ is a local base of horizontal distribution N and $\{\partial/\partial y^i\}$, $i=1, 2, \ldots, n$ is a local base of vertical distribution TM^V.

Definition. *We call a normal Finsler connection on TM, a Finsler connection ∇ on TM whose coefficients of connection, given by* (1), *have the property:*

$$
\overset{1}{F}{}^i_{jk}(x, y)=\overset{2}{F}{}^i_{jk}(x, y), \quad \overset{1}{C}{}^i_{jk}(x, y)=\overset{2}{C}{}^i_{jk}(x, y).
\tag{2}
$$

The above definition being analitical, a geometrical characterization of normal. Finsler connections, is claimed for. To this aim, let F be the almost complex canonical structure on TM determined by the two distributions N and TM^V, i.e.

$$F\left(\frac{\delta}{\delta x^i}\right)=\frac{\partial}{\partial y^i}, \quad F\left(\frac{\partial}{\partial y^i}\right)=-\frac{\delta}{\delta x^i}. \tag{3}$$

We obtain :

Theorem. *Let* ∇ *be a Finsler connection on* TM. *The following conditions are equivalent* :

1°. ∇ *is a normal Finsler connection*

2°. *The almost complex canonical structure* F *is absolutely parallel with respect to* ∇.

Proof. We suppose that the coeficients of Finsler connection ∇ in adapted frames $(\delta/\delta x^i, \partial/\partial y^i)$, satisfy :

$$\overset{1}{F}{}^i_{jk}(x, y)=\overset{2}{F}{}^i_{jk}(x, y) \text{ and } \overset{1}{C}{}^i_{jk}(x, y)=\overset{2}{C}{}^i_{jk}(x, y) \text{ Since}$$

$$(\nabla^H_X F)(y)=\nabla^H_X(F(y))-F(\nabla^H_X y) \tag{4}$$

and F is the almost complex canonical structure on TM, it follows :

$$(\nabla^H_{\frac{\delta}{\delta x^k}} F)\left(\frac{\delta}{\delta x^j}\right)=0, \quad (\nabla^H_{\frac{\delta}{\delta x^k}} F)\left(\frac{\partial}{\partial y^j}\right)=0,$$

In conclusion $\nabla^H_X F=0$. Analogous by we prove that $\nabla^V F=0$ and so we have $\nabla_X F=0$. Conversely, if $\nabla_X F=0$, then $\nabla^H_X F=\nabla^V_X F=0$ and (4) ; in adapted frames, we have :

$$(\nabla^H_{\frac{\delta}{\delta x^k}} F)\left(\frac{\delta}{\delta x^j}\right)=\nabla^H_{\frac{\delta}{\delta x^k}}\left(F\left(\frac{\delta}{\delta x^j}\right)\right)-F\left(\nabla^H_{\frac{\delta}{\delta x^k}}\frac{\delta}{\delta x^j}\right)=$$

$$=\nabla^H_{\frac{\delta}{\delta x^k}}\frac{\partial}{\partial y^j}-F\left(\overset{1}{F}{}^i_{jk}\frac{\delta}{\delta x^i}\right)=(\overset{2}{F}{}^i_{jk}-\overset{1}{F}{}^i_{jk})\frac{\partial}{\partial y^i}$$

which implies $\overset{1}{F}{}^i_{jk}(x, y)=\overset{2}{F}{}^i_{jk}(x, y)$. Analogous by $\nabla^V F=0$ implies $\overset{1}{C}{}^i_{jk}(x, y)=$ $=\overset{2}{C}{}^i_{jk}(x, y)$.

Corollary. *If on* TM, *there is a normal symmetrical Finsler connection, then the almost complex canonic structure* F *on* TM *is integrable.*

BIBLIOGRAPHY

1. Atanasiu, Gh. — *Structures et connexions Finsler presque complex et presque hermitiennes.* The Proc. of the Nat. Sem on Finsler spaces, Brașov, 1980
2. Ichijio, Y. — *Almost complex structures of tangent bundles and Finsler metrics,* J. Math. Kyoto, Univ., 6, 1967
3. Miron, R. — *Introduction to the theory of Finsler spaces.* The Proc. of the Nat. Sem. on Finsler spaces, Brasov, 1980
4. Miron, R. — *Vector bundles Finsler geometry.* The Proc. of the Nat. Sem. on Finsler spaces, vol II, Brașov, 1982

Received 1.XI. 1983

Faculty of Mathematics
University of Brașov
Str. Karl Marx 50
220 — Brașov, Romania

ANALELE ŞTIINŢIFICE ALE UNIVERSITĂŢII „AL. I. CUZA" DIN IAŞI
Tomul XXX, s.I a. Matematică, 1984—3

A SIMPLE RANDOM WALK WITH ONE RANDOM AND ONE NON RANDOM ABSORBING BARRIER

BY

P. C. G. VASSILIOU* and K. X. KARAKOSTAS

1. Introduction. The classical two-player gambler's ruin problem is well known and has attracted a great deal of attention in the literature in various forms. Extensions of these two player problems have also been discussed in [1]. An interesting extension of the two player gambler's ruin problem appeared in [2]. In the present we assume that a gambler who plays against someone with a very large economical capacity has possibilities to increase his capital for example by borrowing or to decrease his capital for example by investing it to something else. It is obvious that this is a more realistic convention. In presenting the problem we will use the picturesque language of the particle performing a simple random walk in the (x, y) axis in discrete time, i.e. unit steps at unit intervals ; this is done to simplify and enliven the formulations. It should be noted however that this convention merely lends colour to the description and that a continuous time stochastic process is easily obtained from it see [3]. In this context our problem consists of a particle performing a simple random walk with the presence of a random absorbing barrier performing also a simple random walk and a non-random absorbing barrier.

In section 2 of the present paper the notation used is given. Also the probability of absorption of the particle by the random variable at point k at time n and the probability of absorption at the non-random variable are evaluated. The evaluations are done in two ways: (i) by numerically solving suitable difference equations with the use of an algorithm given in the present paper and (ii) by providing analytic expressions for the required probabilities in the form of theorems. Both ways have advantages of their own and are useful and important.

In section 3 we revert to the problem where we have the particle and one random barrier and we study the distributions of the maximum distances upwards run by the particle and the barrier. Thus important questions concerning the present problem are answered.

2. Probabilities of the various possible absorptions. We suppose with no loss of generality that a particle is initially at the origin and that

---

* Part of this work was done while the author was an Academic Visitor at Imperial College, University of London.