The above definition being analitical, a geometrical characterization of normal. Finsler connections, is claimed for. To this aim, let F be the almost complex canonical structure on TM determined by the two distributions N and TM^V, i.e.

$$F\left(\frac{\delta}{\delta x^i}\right) = \frac{\partial}{\partial y^i}, \quad F\left(\frac{\partial}{\partial y^i}\right) = -\frac{\delta}{\delta x^i}. \tag{3}$$

We obtain :

Theorem. *Let ∇ be a Finsler connection on TM. The following conditions are equivalent :*

1°. ∇ is a normal Finsler connection

2°. The almost complex canonical structure F is absolutely parallel with respect to ∇.

Proof. We suppose that the coeficients of Finsler connection ∇ in adapted frames $(\delta/\delta x^i, \partial/\partial y^i)$, satisfy :

$$\overset{1}{F}{}^i_{jk}(x, y) = \overset{2}{F}{}^i_{jk}(x, y) \text{ and } \overset{1}{C}{}^i_{jk}(x, y) = \overset{2}{C}{}^i_{jk}(x, y) \text{ Since}$$

$$(\nabla^H_X F)(y) = \nabla^H_X(F(y)) - F(\nabla^H_X y) \tag{4}$$

and F is the almost complex canonical structure on TM, it follows :

$$(\nabla^H_{\frac{\delta}{\delta x^k}} F)\left(\frac{\delta}{\delta x^j}\right) = 0, \quad (\nabla^H_{\frac{\delta}{\delta x^k}} F)\left(\frac{\partial}{\partial y^j}\right) = 0,$$

In conclusion $\nabla^H_X F = 0$. Analogous by we prove that $\nabla^V F = 0$ and so we have $\nabla_X F = 0$. Conversely, if $\nabla_X F = 0$, then $\nabla^H_X F = \nabla^V_X F = 0$ and (4) ; in adapted frames, we have :

$$(\nabla^H_{\frac{\delta}{\delta x^k}} F)\left(\frac{\delta}{\delta x^j}\right) = \nabla^H_{\frac{\delta}{\delta x^k}}\left(F\left(\frac{\delta}{\delta x^j}\right)\right) - F\left(\nabla^H_{\frac{\delta}{\delta x^k}} \frac{\delta}{\delta x^j}\right) =$$

$$= \nabla^H_{\frac{\delta}{\delta x^k}} \frac{\partial}{\partial y^j} - F\left(\overset{1}{F}{}^i_{jk} \frac{\delta}{\delta x^i}\right) = (\overset{2}{F}{}^i_{jk} - \overset{1}{F}{}^i_{jk}) \frac{\partial}{\partial y^i}$$

which implies $\overset{1}{F}{}^i_{jk}(x, y) = \overset{2}{F}{}^i_{jk}(x, y)$. Analogous by $\nabla^V F = 0$ implies $\overset{1}{C}{}^i_{jk}(x, y) = \overset{2}{C}{}^i_{jk}(x, y)$.

Corollary. *If on TM, there is a normal symmetrical Finsler connection, then the almost complex canonic structure F on TM is integrable.*

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A SIMPLE RANDOM WALK WITH ONE RANDOM AND ONE NON RANDOM ABSORBING BARRIER

BY

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1. Introduction. The classical two-player gambler's ruin problem is well known and has attracted a great deal of attention in the literature in various forms. Extensions of these two player problems have also been discussed in [1]. An interesting extension of the two player gambler's ruin problem appeared in [2]. In the present we assume that a gambler who plays against someone with a very large economical capacity has possibilities to increase his capital for example by borrowing or to decrease his capital for example by investing it to something else. It is obvious that this is a more realistic convention. In presenting the problem we will use the picturesque language of the particle performing a simple random walk in the (x, y) axis in discrete time, i.e. unit steps at unit intervals ; this is done to simplify and enliven the formulations. It should be noted however that this convention merely lends colour to the description and that a continuous time stochastic process is easily obtained from it see [3]. In this context our problem consists of a particle performing a simple random walk with the presence of a random absorbing barrier performing also a simple random walk and a non-random absorbing barrier.

In section 2 of the present paper the notation used is given. Also the probability of absorption of the particle by the random variable at point k at time n and the probability of absorption at the non-random variable are evaluated. The evaluations are done in two ways: (i) by numerically solving suitable difference equations with the use of an algorithm given in the present paper and (ii) by providing analytic expressions for the required probabilities in the form of theorems. Both ways have advantages of their own and are useful and important.

In section 3 we revert to the problem where we have the particle and one random barrier and we study the distributions of the maximum distances upwards run by the particle and the barrier. Thus important questions concerning the present problem are answered.

2. Probabilities of the various possible absorptions. We suppose with no loss of generality that a particle is initially at the origin and that

* Part of this work was done while the author was an Academic Visitor at Imperial College, University of London.

it undergoes a simple random walk with independent increments $Z_1, Z_2, \dots$ where for $i=1, 2, \dots$

$$\Pr(Z_i=1)=P_{1e},\ \Pr(Z_i=0)=P_{0e},\ \Pr(Z_i=-1)=P_{-1e} \text{ and } \tag{1}$$
$$P_{1e}+P_{0e}+P_{-1e}=1.$$

Let X_n be the state of the particle at time n then obviously $X_0=0$ and $X_n=\sum_{i=1}^{n} Z_i$.

Let also that a barrier is at $a>0$ at time 0 and that it undergoes an independent of the particle simple random walk with independent increments $R_1, R_2, \dots$ where for $i=1, 2, \dots$

$$\Pr(R_i=1)=p_{1b},\quad \Pr(R_i=0)=p_{0b},\quad \Pr(R_i=-1)=p_{-1b} \text{ and } \tag{2}$$
$$p_{1b}+p_{0b}+p_{-1b}=1.$$

Denote by Y_n the state of the barrier at time n; then $Y_0=a$ and $Y_n=a+\sum_{i=1}^{n} R_i$.

If all the states (Y_n, ∞) are absorbing we say that the barrier is a random absorbing barrier. Analogously if $Y_0<X_0$ we define the barrier as a random absorbing barrier if it also undergoes a simple random walk and all the states $(-\infty, Y_n)$ are absorbing.

Let also that $D_i=X_i-Y_i$ for $i=1, 2, \dots$ Moreover let us assume that a non-random absorbing barrier is at point $-b$ $(b>0)$.

We will be interested in finding the probability of absorption by the random barrier. Define the following

$$F^{(n)}(a, 0, b, K)=\Pr\{D_1>0,\ D_2>0, \dots, D_{n-1}>0,\ D_n=0,\ X_1>-b,$$
$$X_2>-b,\ X_{n-1}>-b,\ X_n=K \mid,\ Y_0=a,\ X_0=0\}.$$

Let E_n the event

$$E_n=\{D_1>0,\ D_2>0, \dots, D_{n-1}>0,\ D_n=0,\ X_1>-b,\ X_2>-b, \dots,$$
$$X_{n-1}>-b,\ X_n=K\}.$$

then we have that

$$F^{(n)}(a, 0, b, K)=\Pr\{E_n \mid Y_0=a,\ X_0=0\}$$

If the first step of the particle is $+1$ and the first step of the random barrier is $+1$ then the distance between them becomes a and the distance between the particle and the non-random barrier becomes $b+1$. For E_n now to occur E_{n-1} must occur conditiona' on $(a, b+1)$. Thus we have

$$\Pr(E_n \mid Y_0=a,\ X_0=1)=P_{1e}\,P_{1b}\,F^{(n-1)}(a, 0, b+1, K)$$

taking all the possible steps and using the fact that they are mutually exclusive events we arrive at

$$F^{(n)}(a, 0, b, K)=\sum_{x=-1}^{1}\sum_{y=-1}^{1} P_{xe}P_{yb}F^{(n-1)}(a+y-x, 0, b+x, K),\ (n, a, b=$$
$$=1, 2, \dots;K=-b+1,\ -b+2, \dots),$$

with initial and boundary conditions

$$F^{(0)}(a, 0, b, K)=\begin{cases} 1 \text{ for } a=0,\ k=0, \\ 0 \text{ otherwise}\end{cases} \text{ and } \begin{array}{l} F^{(n)}(a, 0, b, K)=0 \text{ for } n<|a-K| \\ F^{(n)}(a, 0, b, K)=0 \text{ for } n<|K| \end{array}$$

$$F^{(n)}(0, 0, b, K)=0,\ F^{(n)}(-1, 0, b, K)=0 \text{ and } F^{(n)}(a, 0, 0, K)=0 \text{ for } n=1,2, \dots$$

Equation (3) could not be solved but numerically with the following sketched Algorithm. Let us suppose that we want to calculate $F^{(n)}(a, 0, b, K)$ for given values of (n, a, b, K). In order to do this it is obvious from equation (3) that we have to calculate a number of analogous probabilities for various values of the parameters n, a, b. So there is a need to find out which are these probabilities and in what way to calculate them. Let $n-n_0$ $(n>n_0)$ a value of the time parameter and let in the space R^2 $(a, b-n_0)$, $(a, b+n_0)$, $(b-n_0, a+2n_0)$, $(b+n_0, a-2n_0)$ be the parallelogram and consider its intersection, say I, with the space $A_1=\{(x, y)\in R: x>0, y>0\}$. Then if we consider the mesh of unit size on space I then the coordinates of the points of the mesh determines the probabilities $F^{(n-n_0)}(x, 0, y, K)$ needed for the calculation of the probabilities $F^{(n)}(a, 0, b, K)$. Now the algorithm is rather obvious, we start with the initial conditions and start evaluating the needed probabilities for the values of the time parameter in the order $n=1, 2, \dots, n_0-1, n_0$.

Let that

$$F^{(n)}(a, -1, b, K)=\Pr\{D_1>0, D_2>0, \dots, D_{n-1}>0, D_n=-1, X_1>-b, X_2>-$$
$$-b, \dots, X_{n-1}>-b\ X_n=K \mid Y_0=a,\ X_0=0\}.$$

It is soon apparent that probability $F^{(n)}(a, -1, b, K)$ could be calculated in the same way as $F^{(n)}(a, 0, b, K)$. The probability of absorption of the particle by the random barrier at point K at time n is $F^{(n)}(a, 0, b, K)+F^{(n)}(a, -1, b, K)$.

We will now provide an analytical solution for the probabilities $F^{(n)}(a, 0, b, K)$. Let the particle as described previously undergo a simple random walk with the presence of a non-random absorbing barrier at state $-b$. Define the probability

$$q^{(n)}(r, K)=\Pr\{X_1>-b, X_2>-b, \dots, X_{n-1}>-b, X_n=K \mid X_0=r\},$$

where r is defined below (see theorem 2.1). Let that the random barrier as described previously undergoes a simple random walk with presence of a non-random absorbing barrier at state $-b$. Define the probability

$$p^{(n)}(a, K)=\Pr\{Y_1>-b,\ Y_2>-b, \dots, Y_{n-1}>-b,\ Y_n=K \mid Y_0=a\},$$

Then for every $0<s\leqslant n$ we define the following quantity which as we shall see later is a probability

$$R(s, r)=F^{(s)}(a, 0, b, r)\,q^{(n-s)}(r, K)\,p^{(n-s)}(r, K)+F^{(s)}(a, -1, b, r)$$
$$q^{(n-s)}(r, K)\,p^{(n-s)}(r-1, K),$$

We also define the following

$n_1=[\frac{1}{2}(n+K)]$, $r_1=\max\{-b+1,\ a-s\}$, $r_2=s$, $n_2=[\frac{1}{2}(a+n-K)]$, $r_3=n+K-s$, $r_4=K+s-n$, where by $[a]$ we mean the integer part of $[a]$. Then the following theorem is true.

Theorem 2.1. *Let $n^*=a/2$ if a even and $n^*=(a+1)/2$ if a odd then*
a) If $K\leqslant n^*$ *then*

$$F^{(n)}(a,0,b,r)=q^{(n)}(0,K)\,p^{(n)}(a,K)-\sum_{s=n^*}^{n_1}\sum_{r=r_1}^{r_2}R(s,r)-\sum_{s=n_1+1}^{n_2}\sum_{r=r_1}^{r_3}R(s,r)-$$
$$-\sum_{s=n_2+1}^{n-1}\sum_{r=r_4}^{r_3}R(s,r);$$

b) *If $K>n^*$ then*

$$F^{(n)}(a,0,b,r)=q^{(n)}(0,K)\,p^{(n)}(a,K)-\sum_{s=n^*}^{n_2}\sum_{r=r_1}^{r_2}R(s,r)-\sum_{s=n_2+1}^{n_1}\sum_{r=r_4}^{r_2}R(s,r)-$$
$$-\sum_{s=n_2+1}^{n-1}\sum_{r=r_4}^{r_3}R(s,r).$$

Proof. Let the probability

$$F_*^{(n)}(a,0,b,K)=\Pr\{X_1>-b,\ X_2>-b,\ \ldots,\ X_n=K,\ Y_1>-b,\ Y_2>-$$
$$-b,\ \ldots,\ Y_n=K\mid Y_0=a,\ X_0=0\};$$

then since the random walks $\{X_n\}$, $\{Y_n\}$ are independent we have $F_*^{(n)}(a,0,b,K)=q^{(n)}(0,K)\,p^{(n)}(a,K)$. Consider the event

$E(s,r)=\{D_1>0,\ D_2>0,\ \ldots,\ D_{s-1}>0,\ D_s=0$ or $-1,\ X_1>-b,\ X_2>-b,\ \ldots,$ $X_s=r,\ Y_{s+1}>-b,\ Y_{s+2}>-b,\ \ldots,\ Y_n=K,\ X_{s+1}>-b,\ X_{s+2}>-b,\ \ldots,\ X_n=K\}$.

Then it could be proved that

$$\Pr\{E(s,r)\mid Y_0=a,\ X_0=0\}=F^{(s)}(a,0,b,r)\,q^{(n-s)}(r,K)\,p^{(n-s)}(r,K)+$$
$$+F^{(s)}(a,-1,b,r)\,q^{(n-s)}(r,K)\,p^{(n-s)}(r-1,K)=R(s,r).$$

Comparing the probabilities $F^{(n)}(a,0,b,K)$ and $F_*^{(n)}(a,0,b,K)$ we get that

$$(4)\qquad F^{(n)}(a,0,b,K)=F_*(a,0,b,K)-\Pr(\{\bigcup_{s,r}E(s,r)\}/\{Y_0=a,\ X_0=0\})$$

where by the symbol $\bigcup_{s,r}$ we mean the union of the events $E(s,r)$ for the appropriate values of s, r. Let that $K\leqslant n^*$ then consider the space S as the intersection of the space $S_0=\{(x,y): x>0,\ y>0\}$ and the parallelogram (n^*,n^*), (n_1,n_1), $(n_2,a-n_2)$, (n,k). Then we consider on the space S the mesh of unit size with lines parallel to the (x,y) axes. The coordinates of the points of the mesh provides us with the appropriate values for s, r, in equation (4). Taking into account that all the events $E(s,r)$ are mutually exclusive and exhaustive possibilities we get a) of theorem 2.1. In a similar manner we arrive at b).

It is easy to see that following the steps of the previous theorem we could arrive at a similar relation for the probability $F^{(n)}(a,-1,b,K)$.

Another important case of absorption is for the particle to be absorbed by the non-random barrier at time n, i.e. we are looking for the probability

$$(5)\qquad \Pr\{D_1>0,\ D_2>0,\ \ldots,\ D_n>0,\ X_1>-b,\ X_2>-b,\ \ldots,\ X_{n-1}>-$$
$$-b,\ X_n=-b\mid Y_0=a,\ X_0=0\}.$$

If we define by

$$F^{(n)}(a,a_n,b,0)=\Pr\{D_1>0,\ D_2>0,\ \ldots,\ D_n=a_n>0,\ X_1>-b,\ X_2>-$$
$$-b,\ \ldots,\ X_{n-1}>-b,\ X_n=-b/Y_0=a,\ X_0=0\}$$

then probability (5) is

$$\sum_{a_n=a_n'}^{a_n''}F^{(n)}(a,a_n,b,0),$$

where $a_n'=\max\{1,\ a-n\}$, $a_n''=\min\{2n,\ a+n+b-1\}$. Now $F^{(n)}(a,a_n,b,0)$ could be evaluated from the following difference equation with the use of an algorithm similar to that sketched previously

$$F^{(n)}(a,a_n,b,0)=\sum_{x=1}^{1}\sum_{y=1}^{1}p_{x0}p_{y0}F^{(n-1)}(a+y-x,\ a_n,\ b+n,\ 0),\quad (n=1,2,\ldots,\ a=$$
$$=1,2,\ldots).$$

with initial and boundary conditions

$$F^{(0)}(a,a_n,b,0)=\begin{cases}1 & \text{for } b=0,\ a=a_n\\ 0 & \text{otherwise}\end{cases}\qquad \begin{array}{l}F^{(n)}(a,a_n,b,0)=0 \text{ for } n<|a+b-a_n|\\ F^{(n)}(a,a_n,b,0)=0 \text{ for } n<|b|\end{array}$$

$$F^{(n)}(0,a_n,b,0)=0,\ F^{(n)}(-1,a_n,b,0)=0,\ F^{(n)}(a,a_n,0,0)=0\ (a\neq a_n)\ \text{for}\ n=$$
$$=1,2,\ldots$$

Let that the particle as described previously undergoes a simple random walk with the presence of a non-random absorbing barrier at state $-b$. Then define the probability

$$f^{(n)}(r,-b)=\Pr\{X_1>-b,\ X_2>-b,\ \ldots,\ X_{n-1}>-b,\ X_n=K/X_0=r\}.$$

Then for every $0<s\leqslant n$ define

$$A^{(s,r)}=F^{(s)}(a,0,b,r)\,f^{(n-s)}(r,-b)\,p^{(n-s)}(r,a_n-b)+F^{(s)}(a,-1,b,r)\,f^{(n-s)}(r,-$$
$$-b)\,p^{(n-s)}(r-1,a_n-b).$$

We also define by

$$n_3=[\tfrac{1}{2}(n-b)],\ n_4=[\tfrac{1}{2}(a+n-a_n+b)],\ r_5=(n-b-s),\ r_6=(a_n-b+s-n),$$

Then in a way which has apparent similarities with the proof of theorem 1 one could prove the following theorem.

Theorem 2.
a) *If $2b\geqslant a_n-a$ then*

$$F^{(n)}(a,a_n,b,0)=f^{(n)}(0,-b)\,p^{(n)}(a,a_n-b)-$$

$$-\sum_{s=n^*}^{n_1}\sum_{r=r_1}^{r_2} A(s,r)-\sum_{s=n_1+1}^{n_2}\sum_{r=r_3}^{r_4} A(s,r)-\sum_{s=n_2+1}^{n_3}\sum_{r=r_6}^{r_5} A(s,r)$$

b) *If* $2b<a_n-a$ *then*

$$F^{(n)}(a, a_n, b, 0)=f^{(n)}(0,-b)\,p^{(n)}(a, a_n-b)-\sum_{s=n^*}^{n_2}\sum_{r=r_1}^{r_7} A(s,r)-$$

$$-\sum_{s=n_2+1}^{n_1}\sum_{r=r_6}^{r_2} A(s,r)-\sum_{s=n_1+1}^{n_3}\sum_{r=r_8}^{r_6} A(s,r).$$

3. The distributions of the maximums. Let us assume that we have only the particle and the random barrier, both as described in section 2. We define by U_n the maximum distance reached upwards by the particle at time n. Then

$$U_n=\max_{0\leqslant r\leqslant n} X_r.$$

An interesting problem is to find the distribution of the random variable U_n, i.e. we are interesting for the prob $\{U_n<m\}$ where $m>0$. Let us assume that at point m we impose a non-random barrier which is absorbing for the particle but not for the random barrier. We define by

$$F_*^{(n)}(a, a_n, m, m_r)=\Pr\{D_1>0, D_2>0, \ldots, D_n=a_n>0, X_1<m, X_2<$$

$$<m, \ldots, X_n=m \mid Y_0=a, X_0=0\}.$$

Then we have that

$$\Pr\{U_n<m\}=\sum_{t=n+1}^{\infty}\sum_{a_t=a_t^*}^{a+t} F_*^{(t)}(a, a_t, m, m).$$

Where $a_t^*=\min\{1, a-t\}$.

Thus the problem has been reduced in finding the probability $F^{*(t)}(a, a_t, m, m)$. The evaluation of this probability is similar withthat of theorem 2 but not the same since the order random barrier, particle, non-random barrier is now changed and moreover the non-random barrier is not an absorbing one for the random barrier. In this respect let that the random barrier as described on section 2 undergoes an unrestricted random walk and let that

$$g^{(n)}(a, y)=\Pr\{Y_n=Y \mid Y_0=a\}.$$

Then for every $0\leqslant s\leqslant n$ define by

$$B(s,x)=F_*^{(n)}(a, 0, m, x)\,f^{(n-s)}(x, m)\,g^{(n-s)}(x, a_n+m)+F_*^{(n)}(a, -$$

$$-1, m, x)\,f^{(n-s)}(x, m)\,g^{(n-s)}(x-1, a_n+m)$$

and

$$t_1=[\tfrac{1}{2}(n+a-a_n-m)],\ x_1=a-s,\ x_2=\min\{s, m-1\},\ t_2=n+a_n-a,$$

$$x_3=\min\{(a_n+m+s-n), m-1\}.$$

then similarly as in theorem 2 one could arrive at the following

$$F_*^{(n)}(a, a_n, m, m)=f^{(n)}(0, m)\,g^{(n)}(a, a_n+m)-\sum_{s=n^*}^{t_1}\sum_{x=x_1}^{x_2} B(s,x)-\sum_{s=t_1+1}^{t_2}\sum_{x=x_3}^{x_3} B(s,x).$$

It is apparent that the probabilities $F_*^{(n)}(a, 0, m, x)$ and $F_*^{(n)}(a, -1, m, x)$ remain to be evaluated. Define by $x_4=n+k-s$, $x_5=k+s-n$ and

$$G(s,x)=F_*^{(s)}(a, 0, m, x)\,q^{(n-s)}(x, k)\,g^{(n-s)}(x, k)+F_*^{(s)}(a, -$$

$$-1, m, x)\,q^{(n-s)}(x, k)\,g^{(n-s)}(x-1, k)$$

Then we have that

a) If $k\leqslant n^*$

$$F_*^{(n)}(a, 0, m, k)=q^{(n)}(0, k)\,g^{(n)}(a, k)-\sum_{s=n^*}^{n_1}\sum_{x=x_1}^{x_2} G(s,x)-$$

$$-\sum_{s=n_1+1}^{n_2}\sum_{x=x_1}^{x_4} G(s,x)-\sum_{s=n_2+1}^{n-1}\sum_{x=x_5}^{x_4} G(s,x).$$

b) If $k>n^*$

$$F_*^{(n)}(a, 0, m, k)=q^{(n)}(0, k)\,g^{(n)}(a, k)-\sum_{s=n^*}^{n_2}\sum_{x=x_1}^{x_2} G(s,x)-$$

$$-\sum_{s=n_2+1}^{n_1}\sum_{x=x_5}^{x_2} G(s,x)-\sum_{s=n_1+1}^{n-1}\sum_{x=x_5}^{x_3} G(s,x)$$

Now let U_n^* is the maximum distance reached by the barrier at time n then we have

$$U_n^*=\max_{0\leqslant r\leqslant n}(Y_r-a)$$

We are looking for the distribution of the random variable U_n^* i.e. for prob $\{U_n^*<b\}$ for every n. This is equal with the probability of no absorption up to the time n if we impose a non-random absorbing barrier at point b which will be absorbing for both the random barrier and the particle. Thus following similar steps as previously we could estimate the distribution of U_n^*.

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