2) (X, F, S) *est S-p-métrique aléatoire si et seulement si la condition*

(12) $$q \in (S_d(p, \varepsilon))^\lambda \Rightarrow p \in (S_d(q, \varepsilon))^\lambda$$

est satisfaite pour tous $p \in X$, $\varepsilon > 0$ *et* $\lambda \in [0,1]$.

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Institut Polytechnique de Jassy
Département de Mathématiques
6600 — Iasi, Roumanie

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CHARACTERIZATION OF H-CLOSED EXTENSIONS

BY

IULIAN HAIMOVICI

0. Preliminaries. As one knows, a profitable method of approaching topological extensions is by using the complete description with the filter trace of all possible extensions of a given space. Our main purpose is to establish a theorem of the same kind for the identifications of H-closed extensions only. Before that, we had to prove a similar description for Hausdorff extensions, some characterizations for H-closed spaces and to study properties of maximal filters with open base, which we use as a tool. Most of the concequences are included in a sequel to this paper devoted to locally H-closed spaces.

Let X denote a Hausdorff space. If $A \subset X$, then cA denotes the complement of A. Let Sp (X) be the set of filters on X. Let $\mathcal{O}$ be the set of those filters on X which have a base of open sets and $\mathcal{H}$ the set of those filters belonging to $\mathcal{O}$ which have no cluster points. If A is an open subset of X, we consider the set $O(A) = \{\mathcal{F} \in \mathrm{Sp}(X) : A \in \mathcal{F}\}$; the sets of this kind form a base of topology on Sp(X) which will be denoted by O_0. The topology induced by O_0 on different subspaces of Sp(X) will also be denoted by O_0, if the underlying set is well specified.

The mapping $i : X \rightarrow \mathrm{Sp}(X)$, where $i(x)$ is the neighbourhood filter of x, is homeomorphism between X and $i(X)$, if we endow $i(X)$ with the topology O_0.

The set $\{i(A) : A \text{ open in } X\} \cup \{i(A) \cup \{\mathcal{F}\} : A \text{ open in } X, A \in \mathcal{F}\}$ is also the base of a topology, denoted by O_1. The same notation will be used for the induced topology on well specified of Sp(X).

Clearly $O_0 < O_1$. If we endow Sp(X) with a topology between O_0 and O_1 and $i(X)$ with the induced topology, then i previously defined is still a homeomorphism, as the topology on $i(X)$ is the same.

Definition 1. *The topological space* (Y, σ) *is an extension of* (Z, τ) *if* $Z \subset Y$, $\sigma|_Z = \tau$, *and* $\bar{Z} = Y$. *If Y is Hausdorff (respectively H-closed) it is called a Hausdorff (respectively an H-closed) extension.*

Definition 2. *An open filter on the topological space Z is a filter in the lattice of the open subsets of Z.*

Replacing in the definition of the topologies O_0 and O_1, Sp(X) by the family of the open filters on X and $i(x)$ by the filter $i(x)$ of the open neighbourhoods of x for any $x \in X$, two correspondent topologies O_0' and O_1' on the family of open filters on X are obtained.

In [3] it is pointed out that any family of open filters which contains $\mathring{i}(X)$, endowed with a topology between O_0' and O_1' is the family of the open filter trace of a suitable extension of X, and conversely, the topology of the open filter trace of a given extension is between O_0' and O_1'.

It is easily seen that a similar theorem takes place for O and the topologies O_0 and O_1.

1. **Hausdorff Extensions. Definition 3.** *A topological space is said to be solvable if it is the disjoint union of two dense subsets* ([4], I, §2, Ex. 11). For instance the space $R=Q\cup cQ$ is solvable.

A topological space is said to be nowhere solvable if it has no open subset which forms, with the induced topology, a solvable space.

Theorem 1. *A topological space is nowhere solvable if and only if all the maximal elements of O are ultrafilters.*

Proof. Let us suppose that there is a maximal element $\mathscr{M}$ of O which is not an ultrafilter; hence there is A such that $A\notin\mathscr{M}$, $cA\notin\mathscr{M}$. If $\mathring{A}=\varnothing$ and $c^{\circ}A=\varnothing$ then A and cA are both dense, hence, the space is even solvable. If $\mathring{A}\neq\varnothing$ and $c^{\circ}A\neq\varnothing$ then we use the fact that $\mathscr{M}\cup\{\mathring{A}\}$ and $\mathscr{M}\cup\{c^{\circ}A\}$ cannot be filter subbases as $\mathscr{M}$ is maximal in O; hence there are the open sets F and G in $\mathscr{M}$ such that $F\cap\mathring{A}=\varnothing$, $G\cap c^{\circ}A=\varnothing$, therefore $F\subset c^{\circ}A\cup \mathrm{Fr}A$, $G\subset\mathring{A}\cup\mathrm{Fr}A$; then $F\cap G\in\mathscr{M}$ and $F\cap G\subset \mathrm{Fr}\mathring{A}$, hence $\mathrm{Fr}\mathring{A}\neq\varnothing$; it follows that $\mathrm{Fr}A$ is solvable, since it is the union of the sets $A\cap\mathrm{Fr}\mathring{A}$, $cA\cap\mathrm{Fr}\mathring{A}$ which are dense in $\mathrm{Fr}A$. Finally, if $\mathring{A}=\varnothing$ and $c^{\circ}A\neq\varnothing$ then again $\mathscr{M}\cup\{c^{\circ}A\}$ is not a filter subbase and there is the open set $G\in\mathscr{M}$, $G\subset\mathring{A}\cup\mathrm{Fr}A$; therefore $G\subset\mathrm{Fr}\mathring{A}$ which is therefore unempty, hence solvable again.

If B is a solvable subspace, there is a set A such that both $B\cap A$ and $B\cap cA$ are dense in B. Let $\mathscr{M}$ be a maximal element of O which contains the neighbourhood filter of B. Neither A nor cA belongs to $\mathscr{M}$, as it would follow that either $B\cap A$ or $B\cap cA$ belongs to $\mathscr{M}$, and this is a contradiction as these sets have empty interiors. Therefore, $\mathscr{M}$ is not an ultrafilter.

Most papers use open filters as an instrument for approaching H-closed spaces. Maximal elements of this class are called open ultrafilters. In order to take advantage of a special class of usual filters, namely O, instead of the open ones, we had to establish whether its maximal elements are ultrafilters in the usual acceptance (not only as maximal elements in O but even as maximal filters which together with a set contain its interior). Theorem 1 is the desired result. Hence we must speak about maximal elements of $\mathscr{H}$, which in the general case are not ultrafilters.

Theorem 2. *Y is a Hausdorff extension of X if and only if it is homeomorphic to a subspace $\mathscr{X}$ of* Sp(X) *such that: $i(X)\subset\mathscr{X}$, $\mathscr{X}\setminus i(X)\subset\mathscr{H}$, any maximal filter in $\mathscr{H}$ has at most a subfilter in $\mathscr{X}$, and the topology of X is between O_0 and O_1.*

Proof. Suppose that $\mathscr{X}$ has the stated properties. $\mathscr{X}$ is on extension of X as follows from the preliminaries. We must prove that $\mathscr{X}$ is Hausdorff, and while doing this we can endow it with the coarsest admissible topology,

i.e. O_0. Let $\mathcal{F}$ and $\mathcal{G}$ be two distinct filters in $\mathscr{X}$. If $\mathcal{F}=i(x)$ and $\mathcal{G}=i(y)$, then $x\neq y$ and therefore x and y have two disjoint open neighbourhoods F and G; then $O(F)\cap\mathscr{X}$ and $O(G)\cap\mathscr{X}$ are open disjoint neighbourhoods of $\mathcal{F}$ and $\mathcal{G}$ in $\mathscr{X}$. If $\mathcal{F}=i(x)$ and $\mathcal{G}\in\mathscr{H}\cap\mathscr{X}$, there exists $G\in\mathcal{G}$ such that $x\notin\bar{G}$, as $\mathcal{G}$ has no cluster point; $\mathring{G}\in\mathcal{G}$ which has an open base, hence $O(c\bar{G})\cap\mathscr{X}$ and $O(\mathring{G})\cap\mathscr{X}$ are open disjoint neighbourhoods of $\mathcal{F}$ and $\mathcal{G}$ $\mathscr{X}$. Finally, if both $\mathcal{F}$ and $\mathcal{G}$ are in $\mathscr{H}\cap\mathscr{X}$, we suppose by reductio ad absurdum that any $F\in\mathcal{F}$ and $G\in\mathcal{G}$ have nonempty intersection. $\mathcal{F}\cup\mathcal{G}$ would then be a filter subbase and would be contained in a maximal element of $\mathscr{H}$. Therefore the assumption is false and there are F and G, with $F\in\mathcal{F}$, $G\in\mathcal{G}$, $F\cap G=\varnothing$. We also have $\mathring{F}\in\mathcal{F}$, $\mathring{G}\in\mathcal{G}$, hence $O(\mathring{F})\cap\mathscr{X}$ and $O(\mathring{G})\cap\mathscr{X}$ are disjoint open neighbourhoods of $\mathcal{F}$ and $\mathcal{G}$ in $\mathscr{X}$.

Conversely, let Y be a Hausdorff extension of X and let $\mathscr{N}_X(y)$ be the trace filter of $y\in Y\setminus X$ on X. For any $x\in X$, x and y must have disjoint open neighbourhoods in Y, hence there is $U\in\mathscr{N}_X(y)$ such that $x\notin\bar{U}$ (closure in X). Hence $\mathscr{N}_X(y)$ has no cluster point. Let N be in $\mathscr{N}_X(y)$; there is a neighbourhood of y whose trace on X is N; the interior of this neighbourhood is also a neighbourhood and its trace on X is contained in $\mathring{N}$, hence $N\in\mathscr{N}_X(y)$ implies $\mathring{N}\in\mathscr{N}_X(y)$. Therefore the filter $\mathscr{N}_X(y)\in\mathscr{H}$.

Let us consider $\mathscr{X}=i(X)\cup(\mathscr{N}_X(y))_{y\in Y\setminus X}$. Since Y is Hausdorff, for any y_1 and y_2 from $Y\setminus X$, $y_1\neq y_2$, we can find $N_1\in\mathscr{N}_X(y_1)$, $\mathscr{N}_2\in N_X(y_2)$ which are traces on X of disjoint open neighbourhoods of y_1 and y_2. No maximal element of $\mathscr{H}$ can be finer than both $\mathscr{N}_X(y_1)$ and $\mathscr{N}_X(y_2)$, because $N_1\cap N_2=\varnothing$.

Let $\bar{i}:Y\to\mathscr{X}$ be defined by $\bar{i}(x)=i(x)$ if $x\in X$ and $\bar{i}(y)=\mathscr{N}_X(y)$ if $y\in Y\setminus X$; as Y is Hausdorff, $\bar{i}$ is bijective [3]. If we endow $\mathscr{X}$ with the topology in which open sets are images through $\bar{i}$ of open sets in Y, $\bar{i}$ is a homeomorphism. The topology of $\mathscr{X}$ is between O_0 and O_1 as remarked in the preliminaries.

2. **Characterizations of H-closed spaces.** H-closed spaces are those topological spaces whose images when embedded in a Hausdorff space are always closed subspaces. They were introduced by Alexandroff and Urysohn in [1].

Theorem 3. *The following conditions are equivalent*: 1) *X is H-closed*; 2) *every open covering of X contains a finite subfamily whose closures cover X*; 3) *every family of open subsets of X whose closures have an empty intersection, contains a finite family with empty intersection*; 4) *every open filter on X has at least one cluster point*; 5) *every open ultrafilter on X converges*; 6) *every covering of X with sets from any base of its topology contains a finite subfamily whose closures cover X*; 7) *every family of subsets of X from any base of its topology whose closures have an empty intersection, contains a finite family with empty intersection*; 8) *every filter with an open base on X has at least one cluster point.*

The equivalence of the first 5 conditions is well-known (see [4], ch. I, §9, Ex. 18). As any family of open sets is contained in a base of the topology of X, it follows that 2) and 6), as well as 3) and 7) are equivalent. The equivalence of 4) and 8) is also straightforward.

Theorem 4. *X is H-closed if and only if is itself contained as an element in any cover which contains $\bar A\cup\bar B$ whenever it contains A and B and which has an open subcover.*

Proof. If X is H-closed, any cover of X with open subcover contains a finite dense family formed of sets of the subcover and then $X=\bar D_1\cup\dots\cup\bar D_n$ is also an element of the cover.

If X is not H-closed, there exists a filter in $\mathscr{H}$, hence a family $(F_i)_{i\in I}$ of sets which contains $\mathring F_i\cap\mathring F_j$ whenever it contains F_i and F_j and with $\bigcap_{i\in I}\bar F_i=\varnothing$, therefore there exists a cover of X with open subcover which contains $\bar A\cup\bar B$ whenever it contains A and B and which does not contain X, namely $(cF_i)_{i\in I}$.

This result was suggested by a theorem established in [7], characterizing compact spaces as being those contained in any of their covers which contains $A\cup B$ whenever it contains A and B.

From Theorem 1 it follows that characterization 5) from Theorem 3 can be replaced with a similar one using ultrafilters with an open base only in the case of nowhere solvable spaces.

Proposition 1. *A Hausdorff nowhere solvable space is H-closed if and only if any ultrafilter with open base converges.*

Proof. In a nowhere solvable space, any open ultrafilter contains the open sets of an ultrafilter with open base and any ultrafilter with open base has as one of its bases an open ultrafilter.

Because H-closed spaces do not have proper Hausdorff extensions they are sometimes called Hausdorff-complete [3]; in [6] H a r r i s defines H-closed spaces by this property. Starting with the usual definition, we can prove this characteristic feature as a corollary of Theorem 2 as from its proof one can notice that X has proper Hausdorff extensions if and only if $\mathscr{H}\neq\varnothing$.

3. H-closed extensions. Let $i(X)\subset\mathscr{X}\subset O$. If $\mathscr{X}$ is endowed with a topology between O_0 and O_1, then its open sets can be described in the following manner: there is a topological extension Y of X which has $\mathscr{X}$ as its filter trace; to any open subset B of Y corresponds the set $i(A)\cup\cup\{\mathcal{U}_X(y)\}_{y\in A'}\subset\mathscr{X}$, where $A=B\cap X$ and $A'=B\setminus A$; we denote this set by $\bar i(A\cup A')$; when B runs over the open subsets of Y, the family of such sets forms the topology of $\mathscr{X}$.

Proposition 2. *The filter $\mathcal{F}\in\mathscr{X}$ is in $\overline{\bar i(A\cup A')}$ if and only if $\mathcal{F}\cup\{A\}$ is a filter subbase.*

Proof. If $\mathcal{F}$ is in the exterior of $\bar i(A\cup A')$, we can find an open subset $\bar i(F\cup F')$ of $\mathscr{X}$ such that $\mathcal{F}\in\bar i(F\cup F')$, i.e. F is an open subset of X and $F\in\mathcal{F}$, and $\bar i(F\cup F')\cap\bar i(A\cup A')=\varnothing$, hence $i(A)\cap i(F)=\varnothing$, hence $A\cap F=\varnothing$.

Conversely, if $F\in\mathcal{F}$ is an open subset of X and $A\cap F=\varnothing$, the neighbourhood $\bar i(F\cup F')$ of $\mathcal{F}$ in $\mathscr{X}$ does not meet $\bar i(A\cup A')$, as no filter can contain both A and F.

Theorem 5. *Y is an H-closed extensions of X if and only if it is homeomorphic to a subspace $\mathscr{X}$ of $Sp(X)$ such that: $i(X)\subset\mathscr{X}$, $\mathscr{X}-i(X)\subset\mathscr{H}$, any maximal element of $\mathscr{H}$ has exactly one subfilter in $\mathscr{X}$ and the topology of $\mathscr{X}$ is between O_0 and O_1.*

Proof. By Theorem 2, $\mathscr{X}$ is Hausdorff and thus by Theorem (3.7) is H-closed if and only if $\bigcap_{j\in J}\overline{\bar i(A_j\cup A'_j)}=\varnothing$ implies the existence of a finite subfamily such that $\bigcap_{m=1}^{n}\bar i(A_m\cup A'_m)=\varnothing$. But $\bigcap_{m=1}^{n}\overline{\bar i(A_m\cup A'_m)}=\varnothing$ implies $\bigcap_{m=1}^{n} i(A_m)=\varnothing$, hence $\bigcap_{m=1}^{n}A_m=\varnothing$ and this is even equivalent with $\bigcap_{m=1}^{n}\bar i(A_m\cup A'_m)=\varnothing$. Similarly, $\bigcap_{j\in J}\overline{\bar i(A_j\cup A'_j)}=\varnothing$ implies $\bigcap_{j\in J}\bar A_j=\varnothing$. By proposition 2, $\bigcap_{j\in J}\overline{\bar i(A_j\cup A'_j)}=\varnothing$ if and only if there is no $\mathcal{F}$ in $\mathscr{X}$ such that $\mathcal{F}\cup\{A_j\}$ be a filter subbase for any $j\in J$. A_j are open by the manner in which the topology of $\mathscr{X}$ has been choosen.

We have proved that $\mathscr{X}$ is H-closed if and only if whenever $(A_j)_{j\in J}$ is a family of open subsets of X (their closures having empty intersection) such that there is no $\mathcal{F}\in\mathscr{X}$ with the property that $F\cup\{A_j\}$ is a filter subbase for any $j\in J$, we can conclude that $(A_j)_{j\in J}$ cannot be a filter subbase. The condition between parentheses can be omitted as it is implied by the non-existence of a filter $\mathcal{F}$ with the above properties, but we emphasized it for subsequent needs.

Equivalently, if $(A_j)_{j\in J}$ is a filter subbase of open sets (which is the same as claiming that it be a subbase for a filter in $\mathscr{H}$, because the closures of the sets have an empty intersection hence the filter has no cluster point), then there is an $\mathcal{F}\in\mathscr{X}$ such that $\mathcal{F}\cup\{A_j\}$ is a filter subbase for any $j\in J$. Equivalently again, any maximal element of $\mathscr{H}$ has a subfilter in $\mathscr{X}$. The last sufficiency holds because any subbase $(A_j)_{j\in J}$ of a filter in $\mathscr{H}$ is contained in a maximal element of $\mathscr{H}$ and the necessity is obtained by taking in particular $(A_j)_{j\in J}$ as the subbase of a maximal element in $\mathscr{H}$.

In [8] K a t e t o v shows that every Hausdorff space X possesses a unique H-closed extension kX such that every continuous function $f: X\to T$, where T is Hausdorff and $\overline{f(X)}=T$ can be extended by continuity to a function of a certain subspace of kX onto T.

We can consider kX with $X\cup X^{\vee}$ as underlying set, where $X^{\vee}$ is the set of the open ultrafilters on X with void adherence, with a topology base formed by the open sets of X and the sets $A\cup\{\mathcal{U}\}$ where $\mathcal{U}\in X^{\vee}$ and $A\in\mathcal{U}$ is open in X. In view of the intimate relation between open ultrafilters with void adherence and maximal elements of $\mathscr{H}$, kX can be obtained by attaching to X the maximal elements of $\mathscr{H}$, this set being endowed with the topology O_1.

In [11] another H-closed extension is obtained on the set $X\cup X^{\vee}$, using the topology O'_0. The proof that this is indeed an H-closed extension is the object of that paper, prepared by two previous papers of the same author. F o m i n also described this extension.

Theorem 5 easily implies that the two extensions above are H-closed extensions. Some results of [5] also follow from Theorem 5.

Definition 4. *If* (Y, σ) *and* (Z, ρ) *are extensions of* (X, τ), *then* (Y, σ) *is called projectively larger than* (Z, ρ) *if there is a continuous function from* Y *onto* Z *which leaves* X *pointwise fixed. If* f *is a homeomorphism, the extensions are called isomorphic.*

Definition 5. *An extension in a set* $\mathcal{S}$ *of Hausdorff extensions of a space* (X, τ) *is called a projective maximum in the class* $\mathcal{S}$ *if it is projectively larger than any other extension in* $\mathcal{S}$.

Taking advantage of Theorem 5, we can give a new proof of a conjecture due to Alexandroff [2], already proved in [10] and [12].

Theorem 6. kX *is a projective maximum among the H-closed extensions of* X.

Proof. If T is another H-closed extension, it is homeomorphic to a certain $\mathcal{X}\subset \mathrm{Sp}(X)$ satisfying the conditions in Theorem 5. The function $f: kX\to(\mathcal{X}, O_1)$ which maps each maximal element of $\mathcal{H}$ into its unique subfilter and leaves $i(X)$ pointwise fixed is obviously continuous and surjective. The identity map $t: (\mathcal{X}, O_1)\to(\mathcal{X}, O)$, where O is the topology of $\mathcal{X}$, has the same qualities and so $t\circ f$ is the required application.

The H-closed extensions of a Hausdorff space which were described with filters used the maximal elements of a certain suitable class of filters Using Theorem 5, we shall easily build an H-closed extension with the minimal elements of a class a filters, a method suggested by Bourbaki ([4], ch. II) who finds the completion of a uniform space on the set of minimal elements in the class of Cauchy filters.

Let $\mathcal{D}$ be the class of the filters on X which belong to $\mathcal{H}$ and are not coarser than a filter which has strictly more subfilters among the minimal filters of $\mathcal{H}$. $\mathcal{D}$ contains all the maximal elements of $\mathcal{H}$ and thus is not empty if X is not already H-closed. If we take for X the set of real numbers with the usual topology we can see that $\mathcal{D}$ contains other filters, besides the maximal element of $\mathcal{H}$.

Theorem 7. *The union* $\mathcal{X}$ *of* $i(X)$ *and the minimal elements of* $\mathcal{D}$, *endowed with a topology between* O_0 *and* O_1 *is an H-closed extension of* X.

Proof. We must prove that each maximal element of $\mathcal{H}$ has exactly a subfilter in $\mathcal{X}$. Maximal elements of $\mathcal{H}$ are maximal elements of $\mathcal{D}$ also, hence any of them has least a subfilter in $\mathcal{X}$. Suppose there is a maximal element $\mathcal{M}$ to contain $\mathcal{F}_1$ and $\mathcal{F}_2$, $\mathcal{F}_1\neq\mathcal{F}_2$ both in $\mathcal{X}$. Then $\mathcal{F}_1$ and $\mathcal{M}$ have the same subfilters among the minimal elements of $\mathcal{H}$ and this also holds true for $\mathcal{F}_2$ and $\mathcal{M}$, hence $\mathcal{F}_1$, $\mathcal{F}_2$ and then $\mathcal{F}_1\cap\mathcal{F}_2$ are finer than the same minimal filters of $\mathcal{H}$. As $\mathcal{F}_1$ and $\mathcal{F}_2$ are minimal elements of $\mathcal{D}$, $\mathcal{F}_1\cap\mathcal{F}_2\notin\mathcal{D}$ hence there is a maximal element $\mathcal{N}$ of $\mathcal{H}$ which contains $\mathcal{F}_1\cap\mathcal{F}_2$ and has strictly more subfilters that are minimal in $\mathcal{H}$ than, $\mathcal{F}_1\cap\mathcal{F}_2$. Hence $\mathcal{N}$ cannot contain $\mathcal{F}_1$ or $\mathcal{F}_2$, therefore we can find the open sets $F_1\in\mathcal{F}_1$, $F_2\in\mathcal{F}_2$, $F_1\neq F_2$ such that $cF_1\in\mathcal{N}$, $cF_2\in\mathcal{N}$, hence $cF_1\cap cF_2=c(F_1\cup F_2)\in\mathcal{N}$. But $F_1\in\mathcal{F}_1$, $F_2\in\mathcal{F}_2$, hence $F_1\cup F_2\in\mathcal{F}_1\cap\mathcal{F}_2\subset\mathcal{N}$ and this contradiction completes the proof

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Faculty of Mathematics
University of Iași
R—6600, Iași, Romania