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ON A TYPE OF CONFORMALLY FLAT NEARLY KÄHLER MANIFOLD

BY

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To Professor Adolf Haimovici

Introduction : Let (M, g) be a connected n-dimensional $(n>3)$ conformally flat nearly Kähler manifold which is not Kählerian, S be its Ricci tensor and A be the symmetric endomorphism of the tangent space at each point corresponding to the Ricci tensor S, i.e., $g(AX, Y)=S(X, Y)$ for tangent vectors X, Y. In this paper it is shown that if such a manifold has commuting curvature and Ricci transformations, then the endomorphism A has two different eigenvalues. It is further shown that if the product of these two eigenvalues is positive then the manifold is six-dimensional.

1. Preliminaries. It is known [1, p. 339] that for a conformally flat n-dimensional $(n>3)$ Riemannian manifold the curvature tensor R is of the form

$$(1.1)\quad R(X,Y)=[1/(n-2)][(AX\wedge Y)+(X\wedge AY)-\mathrm{tr}[A/((n-1)(n-2))](X\wedge Y),$$

where $X\wedge Y$ denotes the skew-symmetric endomorphism of the tangent space at every point defined by

$$(1.2)\qquad (X\wedge Y)Z=g(Y,Z)X-g(X,Z)Y.$$

Let us suppose that the manifold under consideration has commuting curvature and Ricci transformations. Then

$$(1.3)\qquad R(X, Y)\circ A=A\circ R(X, Y).$$

From (1.3) it follows that (X, Y). $A=0$, But this implies that

$$(1.4)\qquad R(X, Y)\,.\,S=0,$$

Now,

$$(1.5)\qquad [R(X, Y),\ S](Z, W)=-S(R(X, Y)Z, W)-S(Z, R(X, Y)W).$$

In virtue of (1.4) we can express (1.5) as

$$(1.6)\qquad S(R(X, Y)Z, W)+S(Z, R(X, Y).\ W)=0$$

Using (1.1), the relation (1.6) can be expressed as follows :

$$g(Y,Z)\,S(AX,W)-g(X,Z)\,S(AY,W)+g(Y,W)\,S(AX,Z)-$$
$$(1.7)\quad -g(X,W)S(AY,Z)-(r/n-1)[g(Y,Z)\,S(X,W)+g(Y,W)\,S(X,Z)-$$
$$-g(Y,W)\,S(X,Z)-g(X,W)\,S(Y,Z)]=0,$$

where $r=\mathrm{tr}A$. Putting $Y=Z$ in (1.7) we get

$$g(Z,Z)\,S(AX,W)-g(X,Z)\,S(AZ,W)+g(Z,W)\,S(AX,Z)-$$
$$(1.8)\quad -g(X,W)S(AZ,Z)-(r/n-1)[g(Z,Z)S(X,W)-g(Z,W)S(X,Z)-$$
$$-g(Z,W)\,S(X,Z)-g(X,W)\,S(Z,Z)]=0.$$

Let $\{e_i\}$ be an orthonormal basis in the tangent space at a point. The sum for $1\leqslant i\leqslant n$ of the above expression for $Z=e_i$ gives

$$S(AX,W)-[r/(n-1)]S(X,W)-1/n\left[\sum_{i=1}^{n} S(Al_{e_i},l_{e_i})-(r^2/n-1)\right]g(X,W)=0 \text{ or,}$$

$$(1.9)\qquad S(AX,W)-(r/n-1)\,S(X,W)+\rho g(X,W)=0$$

where

$$(1.10)\qquad \rho=-(1/n)\left[\sum_{i=1}^{n} S(Al_{e_i},l_{e_i})-(r^2/n-1)\right];$$

We shall use the results (1.9) and (1.10) in the next section.

2. Conformally flat nearly Kähler manifold with commuting Ricci curvature transformations. An almost Hermitian manifold M with metric tensor g, Riemannian connection ∇ and almost complex structure J is said to be a nearly Kähler manifold (or a K-space) if $(\nabla_x J)(X)=0$ for all vector fields X on M. In this section we shall consider a type of conformally flat nearly Kähler manifold which is not Kählerian and satisfies the condition (1.3). It is known [2, p. 438] that in a conformally flat nearly Kähler manifold, the following relation holds:

$$(2.1)\qquad (n-4)\,(n-12)\sum_{i=1}^{n} S(Al_{e_i},l_{e_i})+(6n^2-33n+32)\,r^2/[n-1)^2]=0.$$

Since in a non-Kählerian nearly Kähler manifold $n\neq 4$, 12, from (2.1) we have

$$(2.2)\qquad \sum_{i=1}^{n} S(Al_{e_i},\ l_{e_i})=-\frac{(6n^2-33n+32)}{(n-4)(n-12)(n-1)^2}r^2$$

Hence ρ defined by (1.10), takes the form

$$(2.3)\qquad \rho=r^2\frac{(n^3-11n^2+31n-16)}{n(n-4)(n-12)(n-1)^2}.$$

Taking account of (1.9) and (2.3) we see that if a conformally flat nearly Kähler manifold is not Kählerian and has commuting Ricci and curvature transformations, then

$$(2.4)\qquad S(AX,W)-[r/(n-1)]\,S(X,W)+\rho g(X,W)=0,$$

where ρ is given by (2.3).

Let λ be the eigenvalue of the endomorphism A corresponding to the eigenvector X. Then

$$(2.5)\qquad AX=\lambda X.$$

Using (2.5) from (2.4) we get

$$(\lambda^2-[r/(n-1)]\,\lambda+\rho)\ g(X,W)=0.$$

Hence

$$(2.6)\qquad \lambda^2-[r/(n-1)]\,\lambda+\rho=0.$$

From (2.6) it follows that the endomorphism A has at most two eigenvalues.

If the equation (2.6) has two equal roots, then it is known [3, Theorem Case (a)] that the manifold is flat. Consequently $r=0$. But this is inadmissible, because, then the manifold will become Kählerian. Hence the two roots of (2.6) must be different. Denote this roots by λ_1 and λ_2 and the eigen-subspaces corresponding to λ_1 and λ_2 by V_1 and V_2 respectively. Let the multiplicity of λ_1 be m. Then the multiplicity of λ_2 must be $n-m$. Since A is symmetric the dimensions of V_1 and V_2 are m and $n-m$ respectively. We can write

$$(2.7)\qquad r=m\lambda_1+(n-m)\,\lambda_2 \text{ because } r \text{ is the trace of } A.$$

Again from (2.6) we have

$$(2.8)\qquad \lambda_1+\lambda_2=r/(n-1) \qquad\text{and}\qquad (2.9)\qquad \lambda_1\lambda_2=\rho,$$

Solving (2.7) and (2.8) we get

$$(2.10)\qquad \lambda_1=\frac{m-1}{(n-1)\,(2m-n)}r \quad\text{and}\quad (2.11)\quad \lambda_2=\frac{m-n+1}{(n-1)(2m-n)}r.$$

From (2.10) and (2.11) we have

$$(2.12)\qquad \lambda_1\lambda_2=\frac{-(m-1)\,(m+n-1)}{(n-1)^2\,(2m-n)^2}r^2,$$

If $\lambda_1\lambda_2>0$, then from (2.12) it follows that either $m-1$ is negative and $(-m+n-1)$ is positive or, $m-1$ is positive and $(-m+n-1)$ is negative. Hence if $\lambda_1\lambda_2>0$, then $m=0$ or $n=0$, Therefore from a known result [3, Theorem case (b)], it follows that the manifold is of constant curvature. Hence the manifold is an Einstein manifold and we have therefore

$$(2.13)\qquad \sum_{i=1}^{n} S(Al_{e_i},l_{e_i})=r^2/n.$$

From (2.2) and (2.13) we get $(n-4)\,(n-12)\,(n-1)^2+n\,(6n^2-33n+32)=0$ or, $(n-6)\,(n-2)^3=0$ whence $n=6$ ($\because\ n\neq 2$). Hence we can state the following theorem:

Theorem. *If a connected conformally flat n-dimensional* ($n>3$) *nearly Kähler manifold is not Kählerian and has commuting Ricci and curvature transformations, then the symmetric endomorphism of the tangent space at every point corresponding to the Ricci tensor has two different eigenvalues. Further, if the product of these eigenvalues is positive, then the manifold is sixdimensional.*

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THE EQUIVARIANT STABILITY OF THE DIFFERENTIABLE MAPPINGS BETWEEN G-EQUIVARIANT FOLIATED MANIFOLDS

BY

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Mather's theory of C^∞-stability of smooth (C^∞) proper mappings between smooth manifolds is a natural generalization of the earlier results of Morse and Whitney which described dense sets of mappings between manifolds of specific dimensions. If M and N are two smooth manifolds and $C^\infty(M, N)$ is the space of smooth mappings between M and N (the topology on $C^\infty(M, N)$ is the Whitney topology) then $f \in C^\infty(M, N)$ is C^∞-*stable* if there is a neighborhood $\mathbf{U}_f$ of f in $C^\infty(M, N)$ so that if $g \in \mathbf{U}_f$ then there are diffeomorphisms $h \in \mathrm{Diff}^\infty(M)$, $k \in \mathrm{Diff}^\infty(N)$ such that $f = k \circ g \circ h$.

Mather's work ([4]) gives a complete answer to the principal questions related to the stability of mappings: local classification, local normal forms, density. On the other hand, V. Poenaru in [7] gives some results on C^∞-stability of equivariant maps between compact G-manifolds M and N, where G is a compact Lie group. Namely, denoting by $C^\infty_G(M, N)$ the space of smooth G-equivariant maps between M and N, that is:

$$C^\infty_G(M, N) = \{f \in C^\infty(M, N), f(g \circ x) = g \circ f(x), (\forall) x \in M\},$$

a mapping $f \in C^\infty_G(M, N)$ is C^∞-*G-equivariant stable* (*G-stable*) if there is a neighborhood $\mathbf{U}^G_f$ of f in $C^\infty_G(M, N)$ so that if $g \in \mathbf{U}^G_f$ there are the equivariant diffeomorphisms $H \in \mathrm{Diff}^\infty_G(M) = \mathrm{Diff}^\infty_G(M, M) \cap \mathrm{Diff}^\infty(M)$, $K \in \mathrm{Diff}^\infty_G(N) = C^\infty_G(N, N) \cap \mathrm{Diff}^\infty(N)$ such that $f = K \circ g \circ H$.

L.A. Favaro in [2] and Izumya in [3] deal with C^∞-stability of differentiable mappings between foliated manifolds $(M, \mathbf{F}_M)$ and $(N, \mathbf{F}_N)$. If $f \in C^\infty(M, N)$ let $C^\infty(M, N, f) = \{g \in C^\infty(M, N)$, for each $x \in M$ $f(x)$ and $g(x)$ belong to the same leaf of $\mathbf{F}_N\}$ and $\mathrm{Diff}^\infty_{\mathbf{F}}(M) = C^\infty(M, N, id_M) \cap \mathrm{Diff}^\infty(M)$ (resp. $\mathrm{Diff}^\infty_{\mathbf{F}}(N) = C^\infty(N, N, id_N) \cap \mathrm{Diff}^\infty(N)$) be the group of C^∞-diffeomorphisms on M (resp. N) taking each leaf of $\mathbf{F}_M$ (resp $\mathbf{F}_N$) onto itself. A C^∞-mapping f between the foliated manifolds $(M, \mathbf{F}_M)$, $(N, \mathbf{F}_N)$ is called C^∞-*stable in the tangential sens* (**F**-*stable*) (see [2]) if there exists a neighborhood $\mathbf{U}^{\mathbf{F}}_f \subset C^\infty(M, N, f)$ such that for every $g \in \mathbf{U}^{\mathbf{F}}_f$ there are $\Phi \in \mathrm{Diff}^\infty_{\mathbf{F}}(M)$, $\Psi \in \mathrm{Diff}^\infty_{\mathbf{F}}(N)$ such that $g = \Psi \circ f \circ \Phi$.

We consider now a compact Lie group G, two C^∞-compact G-manifolds M, N and $\mathbf{F}_M$, $\mathbf{F}_N$ the C^∞-regular G-equivariant foliations on M and N.