Theorem. *If a connected conformally flat n-dimensional* ($n > 3$) *nearly Kähler manifold is not Kählerian and has commuting Ricci and curvature transformations, then the symmetric endomorphism of the tangent space at every point corresponding to the Ricci tensor has two different eigenvalues. Further, if the product of these eigenvalues is positive, then the manifold is sixdimensional.*

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Received 21. 1985

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Analele ştiinţifice ale Universităţii „Al. I. Cuza" din Iaşi
Tomul XXXI, s. I a, Matematică, 1985, f. 3.

THE EQUIVARIANT STABILITY OF THE DIFFERENTIABLE MAPPINGS BETWEEN G-EQUIVARIANT FOLIATED MANIFOLDS

BY

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Mather's theory of C^∞-stability of smooth (C^∞) proper mappings between smooth manifolds is a natural generalization of the earlier results of Morse and Whitney which described dense sets of mappings between manifolds of specific dimensions. If M and N are two smooth manifolds and $C^\infty(M, N)$ is the space of smooth mappings between M and N (the topology on $C^\infty(M, N)$ is the Whitney topology) then $f \in C^\infty(M, N)$ is *C^∞-stable* if there is a neighborhood $\mathbf{U}_f$ of f in $C^\infty(M, N)$ so that if $g \in \mathbf{U}_f$ then there are diffeomorphisms $h \in \mathrm{Diff}^\infty(M)$, $k \in \mathrm{Diff}^\infty(N)$ such that $f = k \circ g \circ h$.

Mather's work ([4]) gives a complete answer to the principal questions related to the stability of mappings: local classification, local normal forms, density. On the other hand, V. Poenaru in [7] gives some results on C^∞-stability of equivariant maps between compact G-manifolds M and N, where G is a compact Lie group. Namely, denoting by $C^\infty_G(M, N)$ the space of smooth G-equivariant maps between M and N, that is:

$$C^\infty_G(M, N) = \{f \in C^\infty(M, N), f(g \circ x) = g \circ f(x), (\forall) x \in M\},$$

a mapping $f \in C^\infty_G(M, N)$ is *C^∞-G-equivariant stable* (*G-stable*) if there is a neighborhood $\mathbf{U}^G_f$ of f in $C^\infty_G(M, N)$ so that if $g \in \mathbf{U}^G_f$ there are the equivariant diffeomorphisms $H \in \mathrm{Diff}^\infty_G(M) = \mathrm{Diff}^\infty_G(M, M) \cap \mathrm{Diff}^\infty(M)$, $K \in \mathrm{Diff}^\infty_G(N) = C^\infty_G(N, N) \cap \mathrm{Diff}^\infty(N)$ such that $f = K \circ g \circ H$.

L.A. Favaro in [2] and Izumya in [3] deal with C^∞-stability of differentiable mappings between foliated manifolds $(M, \mathbf{F}_M)$ and $(N, \mathbf{F}_N)$. If $f \in C^\infty(M, N)$ let $C^\infty(M, N, f) = \{g \in C^\infty(M, N)$, for each $x \in M$ $f(x)$ and $g(x)$ belong to the same leaf of $\mathbf{F}_N\}$ and $\mathrm{Diff}^\infty_{\mathbf{F}}(M) = C^\infty(M, N, id_M) \cap \mathrm{Diff}^\infty(M)$ (resp. $\mathrm{Diff}^\infty_{\mathbf{F}}(N) = C^\infty(N, N, id_N) \cap \mathrm{Diff}^\infty(N)$) be the group of C^∞-diffeomorphisms on M (resp. N) taking each leaf of $\mathbf{F}_M$ (resp $\mathbf{F}_N$) onto itself. A C^∞-mapping f between the foliated manifolds $(M, \mathbf{F}_M)$, $(N, \mathbf{F}_N)$ is called *C^∞-stable in the tangential sens* (*$\mathbf{F}$-stable*) (see [2]) if there exists a neighborhood $\mathbf{U}^{\mathbf{F}}_f \subset C^\infty(M, N, f)$ such that for every $g \in \mathbf{U}^{\mathbf{F}}_f$ there are $\Phi \in \mathrm{Diff}^\infty_{\mathbf{F}}(M)$, $\Psi \in \mathrm{Diff}^\infty_{\mathbf{F}}(N)$ such that $g = \Psi \circ f \circ \Phi$.

We consider now a compact Lie group G, two C^∞-compact G-manifolds M, N and $\mathbf{F}_M$, $\mathbf{F}_N$ the C^∞-regular G-equivariant foliations on M and N.

In this paper we will deal with a more stronger concept of C^∞-stability for mappings between such manifolds M and N.

Some notations:

$$C_{\mathbf{F}}^\infty(M, N)=\{f\in C^\infty(M, N), f \text{ takes each leaf of } \mathbf{F}_M \text{ on a leaf of } \mathbf{F}_N\};$$

$$C_{G,\mathbf{F}}^\infty(M, N)=C_G^\infty(M, N)\cap C_{\mathbf{F}}^\infty(M, N);\ \mathrm{Diff}_{\mathbf{F},G}^\infty(M)=\mathrm{Diff}_G^\infty\cap \mathrm{Diff}_{\mathbf{F}}^\infty(M);$$

$$\mathrm{Diff}_{\mathbf{F},G}^\infty(N)=\mathrm{Diff}_G^\infty(N)\cap \mathrm{Diff}_{\mathbf{F}}^\infty(N).$$

For a fixed mapping $f\in C^\infty(M, N)$ let

$$C_G^\infty(M, N, f)=C_G^\infty(M, N)\cap C^\infty(M, N, f).$$

It is clear that if $f\in C_{\mathbf{F}}^\infty(M, N)$ then $C^\infty(M, N, f)\subset C_{\mathbf{F}}^\infty(M, N)$. So, if $f\in C_{G,\mathbf{F}}^\infty(M, N)$ then

$$C_G^\infty(M, N, f)\subset C_{G,\mathbf{F}}^\infty(M, N).$$

Definition 1. *We call a mapping $f\in C^\infty(M, N)$ C^∞-G-equivariant stable in the tangencial sense ($G\mathbf{F}$-stable) if there exists a neighborhood $U_f^{G,\mathbf{F}}$ of f in $C_G^\infty(M, N, f)$ such that for every $g\in \mathbf{U}_f^{G,\mathbf{F}}$, there are the diffeomorphisms $h_1\in \mathrm{Diff}_{G,\mathbf{F}}^\infty(M)$, $h_2\in \mathrm{Diff}_{G,\mathbf{F}}^\infty(N)$ so that $f=h_2 o g o h_1$.*

We begin the study of this concept of stability with reduction of the stability to an infinitesimal condition.

Let

$$t_f: \Gamma^\infty(TM)\to\Gamma^\infty(f^*TN),\quad \omega_f: \Gamma^\infty(TN)\to\Gamma^\infty(f^*TN),$$

be the R-linear mappings given by:

$$t_f(X)=Tf\circ X,\quad \omega_f(Y)=Y\circ f.$$

Let L be the subbundle of TM with fiber L_x the tangent space to the leaf of $\mathbf{F}_M$ at x and J be the subbundle of TN with fiber J_y the tangent space to the leaf of $\mathbf{F}_N$ at y. The equivariance of $\mathbf{F}_M$ and $\mathbf{F}_N$ implies that L and J are G-bundles over M and N respectivelly. So, for $f\in C_G^\infty(M, N)$, we obtain by restriction the mappings:

$$t_f^G: \Gamma^\infty(TM)^G\to\Gamma^\infty(f^*TN)^G,\ \omega_f^G: \Gamma^\infty(TN)^G\to\Gamma^\infty(f^*TN)^G,$$

$$t_f^{G,\mathbf{F}}: \Gamma^\infty(L)^G\to\Gamma^\infty(f^*TN)^G,\quad \omega_f^{G,\mathbf{F}}: \Gamma^\infty(J)^G\to\Gamma^\infty(f^*TN)^G,$$

where by $\Gamma^\infty(E)^G$ it is denoted the set of G-equivariant C^∞-cross-sections of a G-bundle E over M or N.

A mapping $f\in C^\infty(M, N)$ is called *infinitesimally stable* in Mather's sens (*I-stable*) if $t_f+\omega_f$ is a surjective mapping and is called *infinitesimally stable in the tangential sens* (Favaro). (*IF-stable*) if $t_f(\Gamma^\infty(L))+\omega_f(\Gamma^\infty(J))\supseteq \Gamma^\infty(f^*J)$.

A mapping $f\in C_{\mathbf{F}}^\infty(M, N)$ is infinitesimally stable in the tangential sens (Izumya) iff $t_f(\Gamma^\infty(L))+\omega_f(\Gamma^\infty(J))=\Gamma^\infty(f^*J)$.

A mapping $f\in C_G^\infty(M, N)$ is *infinitesimally stable in the equivariant sens* (Poenaru) (*IG-stable*) if $t_f^G(\Gamma^\infty(TM)^G)+\omega_f^G(\Gamma^\infty(TN)^G)=\Gamma^\infty(f^*TN)^G$.

We define now the corresponding infinitesimal concept of our stability concept:

Definition 2. *Let $f\in C_G^\infty(M, N)$. We call f infinitesimally stable in the equivariant tangential sens (IGF-stable) if $t_f^{G,\mathbf{F}}(\Gamma^\infty(L)^G)+\omega_f^{G,\mathbf{F}}(\Gamma^\infty(J)^G)\supseteq$ $\subseteq\Gamma^\infty(f^*J)^G$.*

If $f\in C_{G,\mathbf{F}}^\infty(M, N)$ then f is $IG\mathbf{F}$-stable iff the mapping $t_f^{G,\mathbf{F}}+\omega_f^{G,\mathbf{F}}$ is surjective on $\Gamma^\infty(f^*J)^G$.

Remark. J. Mather in [4] and Von Essen in [1] (the case of manifolds with boundary) prove that a proper mapping $f\in C^\infty(M, N)$ is stable iff it is I-stable.

V. Poenaru in [7] proves that if $f\in C_G^\infty(M, N)$ with M, N the compact G-manifolds is IG-stable then it is G-stable.

L.A. Favaro in [2] proves that if a proper mapping $f\in C^\infty(M, N)$ is $I\mathbf{F}$-stable, then it is $\mathbf{F}$-stable.

Recently Izumya in [3] proves that a proper mapping $f\in C_{\mathbf{F}}^\infty(M, N)$ is $I\mathbf{F}$-stable iff f is $\mathbf{F}$-stable.

The main result of this paper is the following:

Theorem. *If M, N are C^∞-compact G-manifolds without boundary with G a compact Lie group and $f\in C_{G,\mathbf{F}}^\infty(M, N)$ is IGF-stable then it is $G\mathbf{F}$-stable.*

Hence, for a mapping $f\in C_{\mathbf{F},G}^\infty(M, N)$, $IG\mathbf{F}$-stability$\Rightarrow G\mathbf{F}$-stability.

Proof. First, we remark that if we denote by:

$$\Phi_f: \mathrm{Diff}_{G,\mathbf{F}}^\infty(M)\times \mathrm{Diff}_{G,\mathbf{F}}^\infty(N)\to C_G^\infty(M, N, f),$$

$\Phi_f(h_1, h_2)=h_2\circ f\circ h_1$, the orbite map, $G\mathbf{F}$-stability of $f\in C_{G,\mathbf{F}}^\infty(M, N)$ is equivalent with the existence of a neighborhood $U_f^{G,\mathbf{F}}$ of f in $C_G^\infty(M, N, f)$ and of a mapping γ_U such that the following diagram is commutative:

$$\begin{tikzcd} & \mathrm{Diff}_{G,\mathbf{F}}^\infty(M)\times \mathrm{Diff}_{G,\mathbf{F}}^\infty(N) \\ \mathbf{U}_f^{G,\mathbf{F}} \arrow[ur, "\gamma_U"] \arrow[dr, "\psi_U"'] & \\ & C_G^\infty(M, N, \psi_f) \end{tikzcd}$$

with Ψ_U the canonical inclusion.

Let $f\in C_{G,\mathbf{F}}^\infty(M, N)$ be the given mapping, $Z_f=C_G^\infty(M, N, f)$ be the subspace of the metrisable space $C_G(M, N)$, $\Psi: Z_f\to Z_f$ be the identity map of Z_f and $\Psi_f\in C_f^0(Z_f, Z_f)$ be the germ in f of the continuous mapping Ψ.

We denote by I the unit interval with the trivial action of G, by $\tilde f\in C_G^\infty(M\times I, N)$ the mapping $\tilde f=f\circ\pi_M$ and by $Z_{\tilde f}=C_G^\infty(M\times I, N, \tilde f)$.

We obtain:

Lemma 1. *With the previous notations, there exists a germ $\Psi'_f\in C_f^0(Z_f, Z_f)$ such that*

$$\Psi'(g)(x, 1)=\Psi(g)(x)=g(x),\ \ \Psi'(g)(x, 0)=\Psi'(f)(x, t)=f(x),\ \ (\forall)x\in M,\ \ (\forall t)\in I.$$

Proof. First, we prove the local contractibility of the space Z_f i.e. for every neighborhood $\mathbf{N}_0$ of f in Z_f there exists a neighborhood $\mathbf{N}_1$ of f in Z_f, $\mathbf{N}_1 \subset \mathbf{N}_0$, and a mapping $\Phi \in C^{0,\infty}(N_1 \times I, \mathbf{N}_0)$ such that for every $g \in \mathbf{N}_1$,

$$(1) \qquad \Phi(g, 0) = f, \qquad (2) \qquad \Phi(g, 1) = g.$$

Indeed, using the equivariant embedding theorem of Palais and Mostow ([6], [5]), there exists a representation $G \subset O(n)$, which is interpreted as an action of G on R^n, and an equivariant embedding e of N in R^n. Moreover, we may choose e such that the range of $\mathbf{F}_N$ by e is an C^∞ — G-equivariant regular foliation $\mathbf{F}_{e(N)}$. Let J_e the G-bundle associated to $\mathbf{F}_{e(N)}$, vector subbundle of TR^n and $J_e^\perp$ the orthogonal subbundle of J_e related to a Riemannian G-invariant metric ou R^n. $J_e^\perp$ is a G-bundle and for every $x \in M$ the restriction $J_e^\perp | \gamma \mathbf{F}_{e(f(x))}$ to the leaf $\mathbf{F}_{e(f(x))}$ of $\mathbf{F}_{e(N)}$ at $e(f(x))$ is the normal bundle of $\mathbf{F}_{e(f(x))}$ in R^n related to the fixed previous metric. We have the mappings i, p:

$$e(N) \xrightarrow{\ i\ } J_e^\perp \xrightarrow{\ p\ } e(N)$$

which are G-equivariant. Let ξ a spray on $TR^n/_{e(N)}$ so that the associated exponential map is G-equivariant (the existence of a such spray is a classical fact). We denote by $\exp J_e^\perp$ the restriction of this map to $J_e^\perp$. Now as in the classical similar construction the mapping $\exp J_e^\perp$ defines a G-bundle $T(e(N), \varepsilon)$ in the disks of radius ε, $\varepsilon > 0$, such that the following diagram is commutative:

$$\begin{tikzcd}
T(e(N), \varepsilon) \arrow[r, "T"] \arrow[dr, shift left, "p"] & R^n \\
& e(N) \arrow[ul, shift left, "i"] \arrow[u, "e"']
\end{tikzcd}$$

with T a G-equivariant embedding. We remark that for every $x \in M$, the restriction $T(e(N), \varepsilon)\gamma | \mathbf{F}_{e(f(x))} = T_{e(f(x))}(e(N), \varepsilon)$ is a tubular neighborhood of the leaf $\mathbf{F}_{e(f(x))}$ (G-equivariant tubular neighborhood if this leaf is G-invariant).

Let $\mathbf{N}_1$ be a neighborhood of f in $\mathbf{N}_0$ such that for every $g \in \mathbf{N}_1$ and $x \in M$ the segment in R^n $[f(x), g(x)]$ is contained in $T_{e(f(x))}(e(N), \varepsilon)$. So, the mapping

$\Phi \in C^{0,\infty}(\mathbf{N}_1 \times I, \mathbf{N}_0)$, $\Phi(g, t)(x) = p((1-t)f(x) + tg(x))$ is well-defined and satisfies (1) and (2).

Now, we define $\Psi' \in C^0(\mathbf{N}_1, Z_{\bar{f}})$, $\Psi'(g)(x, t) = \Phi(g, t)(x)$, and lemma 1 is proved.

Let Ψ' defined in lemma 1 and let $\tilde{\Psi}' \in C^{0,\infty}(\mathbf{N}_1 \times M \times I, N)$ be the mapping associated to Ψ', $\tilde{\Psi}'(g, x, t) = \Psi'(g)(x, t)$. We denote by $\Gamma^{0,\infty}_f(\tilde{\Psi}'^* J)^G$

the $C^0_f(Z_f, C^\infty(M \times I)^G)$-modulus of germs in f of G-equivariant sections of the G-bundle $\tilde{\Psi}'^* J$ which are continuous on Z_f and of C^∞-classes on $M \times I$. Using the lemmas 3.1 and 3.2 from [8], we obtain:

Lemma 2. *$\Gamma^{0,\infty}_f(\tilde{\Psi}'^* J)^G$ is a $C^0_f(Z_f, C^\infty(M \times I)^G)$-modulus of finite type. Moreover, denoting by* $\mathrm{Ev}(f) : \Gamma^{0,\infty}_f(\tilde{\Psi}'^* J)^G \to \Gamma^\infty(\Psi'(f)^* J)^G = \Gamma^\infty(f^* J)^G$, $\mathrm{ev}_{M \times I}(f)$: $C^0_f(Z_f, C^\infty(M \times I)^G) \to C^\infty(M \times I)^G$ *the natural evaluation maps we have:*

$$\mathrm{Ker}\, Ev(f) = \mathrm{Ker}\, \mathrm{ev}_{M \times I}(f).\Gamma^{0,\infty}_f(\tilde{\Psi}'^* J)^G.$$

Also, denoting by $\partial \Psi'/\partial t$ the mapping of $\mathbf{N}_1 \times M \times I \to J$ which sends (g, x, t) into the tangent vector at $s=0$ to the curve $s \to \Psi'(g)(x, t+s)$, we have:

$$(3) \qquad \left(\frac{\partial \Psi'}{\partial t}\right)_f \in \Gamma^{0,\infty}_f(\tilde{\Psi}'^* J)^G.$$

For $g \in C^\infty(M \times I, N)$ let $t'_g : \Gamma^\infty(TM \times I) \to \Gamma^\infty(g^* TN)$,

$$\omega'_g : \Gamma^\infty(TN \times I) \to \Gamma^\infty(g^* TN),$$

$t'_g(\xi) = T'_g \circ \xi$, $\omega'_g(\eta) = \eta \circ \langle g, \pi_2 \rangle$, be the mappings defined in [4, II, §7].

Particularlly, if $g \in Z_{\bar{f}}$, by restriction we obtain:

$$t'^{G,\mathbf{F}}_g : \Gamma^\infty(L \times I)^G \to \Gamma^\infty(g^* J)^G, \quad \omega'^{G,\mathbf{F}}_g : \Gamma^\infty(J \times I)^G \to \Gamma^\infty(g^* J)^G,$$

where $t'^{G,\mathbf{F}}_g$ is an homeomorphism over $\langle g, \pi_2 \rangle^* : C^\infty(N \times I)^G \to C^\infty(M \times I)^G$.

Denoting by $\bar{\Psi} : Z_{\bar{f}} \to Z_{\bar{f}}$ the identity map, let $\bar{\Psi}_{\bar{f}}$ be the germ in $\bar{f}$ of $\bar{\Psi}$ and let

$$t^{G,\mathbf{F}}_{\bar{\Psi}} : C^0_{\bar{f}}(Z_{\bar{f}}, \Gamma^\infty(L \times I)^G) \to \Gamma^{0,\infty}_{\bar{f}}(\tilde{\Psi}'^* J)^G,$$

$$\omega^{G,\mathbf{F}}_{\bar{\Psi}} : C^0_f(Z_{\bar{f}}, \Gamma^\infty(J \times I)^G) \to \Gamma^{0,\infty}_{\bar{f}}(\tilde{\Psi}'^* J)^G,$$

be the mappings defined as follows:

Let $\xi \in C^0_{\bar{f}}(Z_{\bar{f}}, \Gamma^\infty(L \times I)^G)$, $\eta \in C^0_{\bar{f}}(Z_{\bar{f}}, \Gamma^\infty(J \times I)^G)$, and let $\tilde{\xi}$, $\tilde{\eta}$ be the representatives of ξ and η. Then $t^{G,\mathbf{F}}_{\bar{f}}(\tilde{\xi})$ is the germ at $\bar{f}$ of $g \to t'^{G,\mathbf{F}}_g(\tilde{\xi}(g))$ and $\omega^{G,\mathbf{F}}_{\bar{\Psi}}(\eta)$ is the germ at g of $g \to \omega'^{G,\mathbf{F}}_g(\tilde{\eta}(g))$.

We obtain:

Lemma 3. *Let $f \in C^\infty_{G,\mathbf{F}}(M, N)$ be the given IGF-stable mapping. Then $t^{G,\mathbf{F}}_{\bar{\Psi}} \oplus \omega^{G,\mathbf{F}}_{\bar{\Psi}}$ is a surjective mapping.*

Proof. $f = IG\mathbf{F}$-stable is equivalent with $t^{G,\mathbf{F}}_f \oplus \omega^{G,\mathbf{F}}_f$ a surjective map. It is easy to see that this last fact implies that $\tau = t^{G,\mathbf{F}}_{\bar{\Psi}}(f) + \omega^{G,\mathbf{F}}_{\bar{\Psi}}(f) : C^\infty(I, \Gamma^\infty(L)^G \oplus \Gamma^\infty(J)^G) \to C^\infty(I, \Gamma^\infty(f^* J)^G)$, where $\tau(h)(t) = t^{G,\mathbf{F}}_f(h(t)) + \omega^{G,\mathbf{F}}_f(h(t))$, is a surjective mapping. The surjectivity of τ is equivalent with the following equality:

$$\mathrm{Range}\, t^{G,\mathbf{F}}_{\bar{\Psi}} + \mathrm{Range}\, \omega^{G,\mathbf{F}}_{\bar{\Psi}} + \mathrm{Ker}\, \mathrm{Ev}(f) = \Gamma^{0,\infty}_f(\Psi'^* J)^G.$$

Denoting by $C=\Gamma_f^{0,\infty}(\tilde{\Psi}'^{*}J)^G/\mathrm{Range}\, t_{\Psi'}^{G,\mathrm{F}}$ a $C_f^0(Z_f, C^\infty(M\times I)^G)$-modulus of finite type, and by $\alpha=\pi_C\circ\omega_{\Psi'}^{G,\mathrm{F}}$, π_C being the canonical projection, the previons equality is equivalent with the following one $\alpha(C_{\bar{f}}^0(Z_{\bar{f}}),\Gamma^\infty(L\times\times I)^G+\mathrm{Ker}\,\mathrm{ev}_{M\times I}(f).C=C$.

Now, by the an equivariant version of Malgrange's preparation theorem (see [8]) there results: $\alpha(C_{\bar{f}}^0(Z_{\bar{f}}))=C$ and the lemma 3 is proved.

So, using lemmas 2,3, there exists

$$\xi\in C_{\bar{f}}^0(Z_{\bar{f}},\ \Gamma^\infty(L\times I)^G),\ \eta\in C_{\bar{f}}^0(Z_{\bar{f}}),\Gamma^\infty(J\times I)^G),$$

such that

$$t_{\Psi'}^{G,\mathrm{F}}(\xi)+\omega_{\Psi'}^{G,\mathrm{F}}(\eta)=\left(\frac{\partial\Psi''}{\partial t}\right)_f. \tag{4}$$

We can choose the representatives $\tilde{\xi}$ and $\tilde{\eta}$ of ξ and η respectivelly such that $\tilde{\xi}: Y\to\Gamma^\infty(L\times I)^G$, $\tilde{\eta}: Y\to\Gamma^\infty(J\times I)^G$ are continuous and such that we have

$$\left(\frac{\partial\Psi''}{\partial t}\right)(y)=t_y'^{G,\mathrm{F}}(\tilde{\xi}(y))+\omega_y'^{G,\mathrm{F}}(\tilde{\eta}(y)), \tag{5}$$

for all $y\in Y$, where Y is a suitable neighborhood of $\bar{f}$ in $Z_{\bar{f}}$. Clearly

$$\tilde{\xi}(\bar{f})=0,\ \tilde{\eta}(\bar{f})=0 \tag{6}$$

As in [4] II, we will obtain the diffeomorphisms of the theorem by integrating the vector fields $\tilde{\xi}(y)$ and $\tilde{\eta}(y)$ and evaluating at $t=1$.

For this fact we remark that the obvions modiffications in the proof of lemma 2 from [4] II §7, lead to the following:

Lemma 4. *There exists the neighborhoods O_M of o in $\Gamma^\infty(L\times I)^G$ and O_N of o in $\Gamma^\infty(J\times I)^G$ such that for each $\xi\in O_M$, $\eta\in O_N$ there exists $H_1(\xi)\in\in C^\infty(M\times I, M)$, $H_2(\eta)\in C^\infty(N\times I, N)$ such that $H_1(\xi)_t\in\mathrm{Diff}_{G,\mathrm{F}}^\infty(M)$, $H_2(\eta)_t\in\in\mathrm{Diff}_{G,\mathrm{F}}^\infty(N)$ for each $t\in I$, $H_1(\xi)_0=1_M$, $H_2(\eta)_0=1_N$ and:*

$$\frac{\partial H_1(\xi)_t}{\partial t}\circ H_1(\xi)_t^{-1}=\xi_t,\ \frac{\partial H_2(\eta)_t}{\partial t}\circ H_2(\eta)_t^{-1}=\eta_t,\ (\forall)t\in I. \tag{7}$$

Moreover, the mappings $H_1: O_M\to C^\infty(M\times I, M)$, $H_2: O_N\to C^\infty(N\times I, N)$ are continuous.

Now, we consider a neighborhood $\mathbf{N}_2\subset\mathbf{N}_1$ of f in Z_f such that $\Psi''(\mathbf{N}_2)Y\subseteq$ (the existence of $\mathbf{N}_2$ is given by the continuity of Ψ''). Then $\tilde{\xi}(\Psi''(g))$, $\tilde{\eta}(\Psi''(g))$ are defined, for all $g\in\mathbf{N}_2$. Since $\tilde{\xi}$, $\tilde{\eta}$ are continuous and $\tilde{\xi}(\bar{f})=0$, $\tilde{\eta}(f)=0$, we may choose a neighborhood $f\in\mathbf{N}_f^{G,\mathrm{F}}\subset\mathbf{N}_2$ so that $\tilde{\xi}(\Psi''(\mathbf{U}_f^{G,\mathrm{F}}))\subseteq O_M$, $\tilde{\eta}(\Psi''(\mathbf{U}_f^{G,\mathrm{F}}))\subseteq O_N$ (O_M, O_N as in lemma 4). Then $H_1(\tilde{\xi}(\Psi''(g))$ and $H_2(\tilde{\eta}(\Psi''(g)))$ are defined for all $g\in\mathbf{U}_f^{G,\mathrm{F}}$.

Let $H_1\in C^0(\mathbf{U}_f^{G,\mathrm{F}}, \mathrm{Diff}_{G,\mathrm{F}}^\infty(M)$, $H_2\in C^0(\mathbf{U}_f^{G,\mathrm{F}}, \mathrm{Diff}_{G,\mathrm{F}}^\infty(N)$,

$$H_1(g)=H_1(\tilde{\xi}(\Psi''(g)))_1,\ H_2(g)=H_2(\tilde{\eta}(\Psi''(g)))_1$$

From (7) it follows that $H_1(f)=1_M$, $H_2(f)=1_N$.

Finally, substituting $h=H_1(\tilde{\xi}(\Psi''(g)))$, $h'=H_2(\tilde{\eta}(\Psi'')g)))$, $y=\Psi''(g)$, $t=1$ in the conclusion of Lemma 1 from [4] II § 7, we obtain that $g=H_2(g)\circ f\circ\circ H_1(g)$ for all $g\in\mathbf{U}_f^G$. So, $\gamma_U=H_1\times H_2$ and the theorem is proved.

A remarkable example is the following. Let M, N be compact G-manifolds without boundary where the action of G on M and N are almost free i.e. every isotropy group is discrete. So, the orbits of G in M and N define the G-invariant foliations $\mathbf{F}_G(M)$ and $\mathbf{F}_G(N)$ and $C_{G,\mathrm{F}}^\infty(M,N)=C_G^\infty(M, N)$. The $G\mathbf{F}$-stability of $f\in C_G^\infty(M, N)$ is the stability of f in $C_G^\infty(M, N, f)==\{g\in C_G^\infty(M, N),\ \bar{g}=\bar{f}\}$ where $\bar{g}: M/G\to N/G$ is the mapping induced by g between the orbit spaces.

In particular if G gives actions which are simply transitive on the orbits, an example being the foliations on the total spaces of two principal G-bundles M, N by the orbits of the structural group G, we obtain the concept of $G\mathbf{F}$-stability of $f\in C_G^\infty(M, N)$ in $G_G^\infty(M, N, f)=C_f^\infty(M, N)=$ the space of C^∞-bundle morphisms between M, N, over the base morphism f.

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Received 21.II. 1985

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