ANALELE ŞTIINŢIFICE ALE UNIVERSITĂŢII „AL. I. CUZA" DIN IAŞI

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SUMAR

Analele ştiinţifice ale Universităţii „Al. I. Cuza" din Iaşi
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PROFESSOR RADU MIRON HIS 60-th ANNIVERSARY

Professor Radu Miron, Ph D, the present leader of the school of geometry in Iaşi, is one of the remarkable Romanian mathematicians who contributed by his prodigious scientific activity to the progress and affirmation of the Romanian mathematics in the whole world.

Following the ways most mathematicians walked, he knew how to get to new results amazing by their naturalness and by their beauty; he knew how to bring up-to-date some older theories, placing them on the orbit of the present development of mathematics, attaching them the necessary meaning and brilliance; he succeeded in solving difficult problems when other mathematicians' efforts were vain.

Prominent disciple of the geometers Al. Myller, O. Mayer and M. Haimovici, professor Radu Miron became well-known as a pioneer in the field of geometry, generously imparting his experience and his ideas to his students, to his candidates for a doctor's degree and to his collaborators within the country and from abroad.

He was born the 3rd of October in the village Codăeşti, the district of Vaslui, where he followed the elementary school. In 1948 he finished his secondary studies at the Industrial Lyceum in Bîrlad and in the same year, attracted by the mirage of mathematics and by the sense of accomplishing a destiny, he became a student of the Mathematics and Physics Unit of the University of Iaşi. Here, he distinguished himself from the very beginning through talent and assiduous work, he acquired the sympathy and the trust of his professors. Thus, starting with the third year as a student he became a „preparator" and from the fourth year, an assistant lecturer. In 1952 he successfully graduated from the University of Iaşi and in the following year he became a candidate for a doctor's degree at the Mathematics Institute of the Academy, the subsidiary of Iaşi, in the special subject of mechanics, under the scientific guidance of M. Haimovici, academician.

The climate of work, seriousness and research proper to the Seminary of Mathematics, the excellent library, the valuable tradition and the inventive tendency of its members had an overwhelming influence on young Radu Miron. Surrounded by such a peculiar atmosphere of study, he began his activity as a scholar in the well-known Seminary of Mathematics of Iaşi.

Impressed by the beauty of the subjects approached by M. Haimovici, Elie Cartan and Gh. Vrănceanu concerning the elaboration of a geometrical pattern of the movements of the nonholonomic mechanical systems, the young graduate R. Miron decided to work in this difficult field located between the differencial geometry and the analytic mechanics.

In 1957 he became a doctor in mathematical and physical sciences: in 1963 he was appointed chief of section in the Institute of Mathematics of the P.R.R. Academy - the subsidiary of Iași and a lecturer in the Mathematics-Mechanics Department, while in 1969 he became a professor at the same faculty where he is still on.

He was a competent and loyal leader of the faculty of Mathematics-Mechanics of the University „Al. I. Cuza" of Iași, between 1972 and 1976.

His thesis for a doctor's degree, The Problem of Geometrizing the Nonholonomic Mechanical Systems *that has in view how to apply the differential geometry to the domain of analytical mechanics, had a certain response in the world of mathematicians. Indeed, in this thesis the mathematician of Iași gave a solution to the famous problem of Hadamard-Cartan which refers to the creation of a geometric pattern that may fully describe the movements of the scleronomic nonholonomic mechanical systems. Over this subject, the great French geometer Elie Cartan had reckoned (Selecta, Paris, 1939, p. 81) that it is either impossible to solve it or that it should be treated differently from one case to another, which may diminish its theoretical interest.*

These ideas that consacrated him as a scientist and which, up to him, were an unsatisfied wish for many distinguished mathematicians, were taken over and enlarged upon by the author into a general and unitary theory of the distributions on the Riemannian manifolds.

We can state today that the theory of the nonholonomic manifolds of the Riemann spaces is a purely Romanian achievement, due, first of all, to Gh. Vrănceanu, M. Haimovici and R. Miron. Later on, the same line of research led him to the discovery of the Myller structures, to their equations and their fundamental theorems and to the elaboration of their geometry.

These researches were included in the monograph The Geometry of the Myller Structures *— published in 1966 at Editura Tehnică, București.*

The book comprises mainly the extension of Myller and Mayer's ideas concerning the most general notion of parallelism on the curve spaces.

It represents, by its own structure, a pious homage of the author to the founders of the Seminary of Mathematics in Iași.

The exceptional merits on the scientific fields brought him in 1969 the „Gh. Țițeica" Prize of the S.R.R. Academy. The lines of the research initiated by him and continued by his students and collaborators determined new investigations in the geometry of Riemann spaces, in the theory of G-structures, the submanifolds of affine connected spaces and so on.

The axiomatic method employed, professor Radu Miron succeeded in solving a half-a-century old problem, namely the substantiation on an axiomatic of Weyl type, by independent axioms, of the concept of n-dimensional affine pointwise space. The researches of this type were extended by the author all by himself or in collaboration with professor S. Golab from the University of Krakow.

Under the impulse of the former studies of M. Haimovici (1935) concerning Finsler's theory of subspaces, professor Radu Miron directed his scientific preoccupations towards the geometry of Finsler's spaces. The major importance of these spaces consists in their capacity of naturally modelling the physical fields.

Approaching, with courage and pertinence the large matter of the geometry of Finsler spaces, professor Radu Miron achieved, in 1975, together with the famous Japanese mathematician M. Matsumoto, the stable theory of these spaces.

His results were included by the Japanese specialist in his monograph Foundations of Finsler Geometry and Special Finsler Spaces *— of 1986, under the title* The Miron frame *and* the Fundamental Equations of the Miron frame.

This matter was further enlarged by him with the general theory of connections in Finsler spaces, the structure of the group of connection transformations, the submanifolds theory, the Finslerian pattern of relativity etc.

We may notice that this pattern completely solves the problem of gravity fields studied for almost 25 years by the Japanese school of Kyoto led by M. Matsumoto. Part of these researches were done in collaboration with M. Hashiguchi, H. Izumi, S. Watanabe and S. Ikeda from Japan and also with his students and collaborators from România. They constituted the foundation of the Romanian school of Finsler spaces created and directed by professor Radu Miron.

This school became well-known over the world once a Romanian-Japanese Symposium on Finsler geometry was organized by the University „Al. I. Cuza", in 1984, August, the 15-th up to the 25-th, when 16 of the best specialists in Japan had a participation. These researches led him to an obvious generalization of Finsler spaces, proposed by the analytical mechanics and made concrete through the concept of generalized Lagrange Spaces.

The creation and the development of the geometry of such spaces by him opened a new line of research materialized by very appreciated results all over the world.

The whole geometric edifice of these just-mentioned spaces, such as: Lagrange G-structures, subspaces, fundamental equations, the Lagrangean theory of relativity, and so on, constitutes a purely Romanian creation. The importance of these achievements was also testified by the invitation addressed to professor Radu Miron by the University of Sciences of Tokyo in vue of delivering some synthesis lectures destined to geometers and physicists. The method he elaborated in Finsler and Lagrange geometries allowed him to reach, for the first time in the speciality literature, to the theory of the total space of a vector bundle. His results were successfully taken over by the schools of geometry in Japan, Hungary, Italy and Romania and altogether included in an unitary manner in the monograph entitled „Vector Bundles. Lagrange Spaces. Applications to the Relativity Theory *— published this very year by Editura Academiei R.S.R., bearing the signature of professor Radu Miron and of his disciple Mihai Anastasiei. This book, based almost entirely on Romanian researches represents an original exposing of the theory of the vector bundles and of their applications to the relativity theory. His life-long scientific activity now reached a climax by the recent discovery of the Hamiltonian Spaces as an obvious conclusion of the Hamilton structures of the analytical mechanics.*

From them did he achieve by the modern means of the differential geometry, the theory of non-linear connections, Legendre transformations, metrical connections, Hamiltonean fundamental equations, the Kählerian pattern and applications in the theories of gravity and of the electromagnetism.

His results were published in well-known specialized reviews. For his remarkable successes obtained in differential geometry, the mathematician of

Iași was invited to deliver lectures at different imposing academic institutions in France, Great Britain, the Soviet Union, Hungary, Italy, Germany, Japan etc. Beginning with 1972 he was charged with Ph. D. programmes. Over six candidates passed under his guidance, having the thesis inspired by their master's researches.

Peerless teacher, having the gift of speech, his classes and his lectures are full of point and charm. His speeches are impressive by the exquisite exposing and rich information. Loved and esteemed by his students and by all his colleagues, professor Radu Miron gains the admiration for his enthusiasm for science, for the quality of his researches and for his major contribution to the progress of the old School of mathematics in Iași as a teacher and as a scientist.

He contributed to the ascent of the secondary and higher education by many conferences, lectures, lessons, textbooks and monographs. Each of them reflects his genuine spirit and his great care for the novelty and accessibility of the matters.

Together with some other distinguished professors of the department, in 1968–1969 he drafted the textbook comprising the courses of analytical and differential geometry in two volumes, destined to the first two-years students of the Mathematics Department.

Together with I. Pop he published, in 1974 at Editura Academiei R.S.R, the monograph entitled Algebraic Topology, *entirely written in the language of categories and in which the authors brought several original contributions out of which we mention the new concept of* Abstract Cech Cohomology associated to a projective system *which had the merit of considerably simplifying the study of Cech cohomology with coefficients in a presheaf. This paper destined to Ph D programmes doctor's degree and to the last-year students of the Mathematics Department was also a starting point for several candidates' theses.*

The celebration of Professor Radu Miron when having reached six decades of life and over 35 years of scientific activity is indeed a festive moment for the Seminary of Mathematics of Iași that had the major part in his training up and in his consecration in the field of Science and Teaching.

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NATURAL CONTINUITY OF A H-MAP

BY

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§0. **Introduction.** In a fundamental paper [4], H. Cartan discovered the so-called quasi-continuity property of a superharmonic function, namely that for each positive superharmonic function s on X, and $\varepsilon>0$, there exists an open set $\omega\subseteq X$, of capacity $<\varepsilon$, such that $s|_{X\setminus\omega}$ is continuous.

Fuglede [7] (see also Brelot [3]) proposes axiomatisations of this property. In [6] it is shown that, on harmonic spaces, the fact that every positive, superharmonic function is quasi-continuous, is equivalent with the axiom of domination.

In the setting of a H-cone, the notion of quasi-continuous element was introduced recently [1], and only as a technical tool.

One defines [5] the harmonic morphisms between harmonic spaces, as a *continuous* map, which preserves (super-)harmonicity. However, between the fine structures considered by Fuglede [5] or between H-cones, one is naturally led to abandone the condition of continuity (see [9], [2], [13]).

In this paper, we analyse the continuity property of the morphisms between H-cones of functions. The main result is contained in th. 1, which is an adaptation, to the frame of H-cones, of the results of Fuglede [9].

All the notions and notations about H-cones are as in [2]. If S is an H-cone of functions on the set X and $A\subseteq X$, then by : cl A, int A, ∂A we shall denote respectively : the *fine closure*, the *fine interior*, the *fine boundary* of the set A. If S and T are H-cones of functions on the sets X and Y, then the function $\varphi : Y\to X$ is called H-map if, for any bounded $s\in S$, $s\circ\varphi\in \tilde{T}$ (see [13]).

§1. **Quasi-continuous elements in a H-cone.** Let S be a standard H-cone of functions on the set X. For each $s\in S$ we denote :

$\mathcal{R}(s)=\{K\subseteq X \mid K$ *naturally compact*, $s|_K$ *naturally continuous*

$s\in S$ is called *quasi-continuous* if, for each $p\in S_0$ we have :

$$\bigwedge_{K\in\mathcal{R}(s)} B^{X\setminus K}p=0.$$

We denote by Q the set of all quasi-continuous elements of S. Then, the same proof as in [6] shows :

Proposition 1. a) *Q is a min-stable, convex subcone of S.*

b) *Q is a specific band,*

c) *Let $F\subseteq S$ be an increasing family, such that :*

$$\sup\{t(x) \mid t\in F\}=+\infty\,\forall,\ x\in X.$$

If $s\in S$ is such that $s\wedge t\in Q$, $\forall t\in F$, then $s\in Q$.