So

$$(u_\lambda + A'u_\lambda - u - A'u - B'u,\ J(u_\lambda - u)) \leq (2\lambda)^{-1}(\|u_\lambda - u - \lambda B'u\|^2 - \|u_\lambda - u\|^2),$$

giving

$$\|u_\lambda - u\|^2 \leq (B'u,\ J(u_\lambda - u)) + (2\lambda)^{-1}(\|u_\lambda - u - \lambda B'u\|^2 - \|u_\lambda - u\|^2) \to 0$$ as $\lambda \to 0^+$ since X is (UG) and u_λ is bounded. q.e.d.

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THE SEMI-INVARIANT SUBMANIFOLDS WITH SEMI-FLAT NORMAL CONNECTION OF A SASAKIAN SPACE FORM

BY

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1. Preliminaries. The semi-invariant submanifolds of a Sasakian manifold correspond to the CR-submanifolds of a Kaehler manifold and they have been introduced and studied by A. Bejancu and the present author in [2], [3], [4], [1], [9].

In [6], B. Y. Chen gives a classification theorem for CR-submanifolds with semi-flat normal connection in a complex space form $\tilde{M}(c)$, $c \neq 0$.

The main purpose of the present paper is to classify the semi-invariant submanifolds with semi-flat normal connection of a Sasakian space form $\tilde{M}(c)$. Our classification theorem for semi-invariant submanifolds with semi-flat normal connection of a Sasakian space form (Theorem 2.2) is simpler than the classification theorem of B. Y. Chen (Theorem 5.11 in [6]) because in our case, the f-structure defined on the normal bundle of a semi-invariant submanifold is parallel (Theorem 2.1). The existence of semi-invariant products with semi-flat normal connection is also studied (Theorem 2.3).

First, we recall the definitions and some fundamental results for later use.

Let $\tilde{M}$ be a Sasakian manifold of dimension $2n+1$ with the Sasakian structure (φ, ξ, η, g), where φ is a tensor field of type $(1, 1)$, ξ is a vector field, η is a 1-form and g is a Riemannian metric on $\tilde{M}$ which are related by:

$$\varphi^2 = -I + \eta \otimes \xi\ ;\ \varphi(\xi) = 0\ ;\ \eta(\xi) = 1\ ;\ \eta \circ \varphi = 0 \quad \text{and} \tag{1.1}$$
$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y)$$

for any vector fields X, Y tangent to $\tilde{M}$, where I denotes the identity tensor on $\tilde{M}$. We denote by $\tilde{\nabla}$ the Levi-Civita connection on $\tilde{M}$. Then, we also have ([5], p. 73)

(1.2) $(\tilde{\nabla}_X \varphi)Y = g(X, Y)\xi - \eta(Y)X\ ;\ \tilde{\nabla}_X \xi = -\varphi X$, for any vector fields X, Y tangent to $\tilde{M}$.

Now, let M be an m-dimensional Riemannian manifold isometrically immersed in $\tilde{M}$ such that the structure vector field ξ of $\tilde{M}$ is tangent to M. Denote by TM and $TM^\perp$ the tangent bundle of M and respectively the normal bundle to M and by $\{\xi\}$ the 1-dimensional distribution defined by ξ on M.

Definition. [2]. *The submanifold M of the Sasakian manifold $\tilde{M}$ is called a semi-invariant submanifold if it is endowed with the pair of distributions $(D, D^{\perp})$ satisfying conditions:*

(i) $TM = D \oplus D^{\perp} \oplus \{\xi\}$;

(ii) *the distribution D is invariant by φ, that is, $\varphi(D_x) = D_x$ for each $x \in M$;*

(iii) *the distribution $D^{\perp}$ is anti-invariant by φ, that is, $\varphi(D^{\perp}_x) \subset T_x M^{\perp}$ for each $x \in M$ and $\varphi^2(D^{\perp}_x) = D^{\perp}_x$ for each $x \in M$.*

Remark. By using (1.1) and the conditions (ii) and (iii) from the definition of a semi-invariant submanifold we obtain that the distribution $D, D^{\perp}$ and $\{\xi\}$ are mutually orthogonal to each other.

Throughout this paper we denote by $2p$ (resp. q) the dimension of D_x (resp. $D^{\perp}_x$). Then it is easily seen that when $p=0$ (resp. $q=0$), the semi-invariant submanifold M becomes an *anti-invariant submanifold*, [7], [10] (resp. an *invariant submanifold*, [8]). The semi-invariant submanifold M is called a *proper semi-invariant submanifold* if it is neither an invariant nor an anti-invariant submanifold, i.e. we have $p \neq 0, q \neq 0$. A *generic semi-invariant submanifold* is a semi-invariant submanifold characterized by the condition dim. $T_x M^{\perp} = q$, for each $x \in M$, [11], [2].

We denote by the same symbol g both Riemannian metrics on $\tilde{M}$ and M. The projection morphisms of TM onto D and $D^{\perp}$ are denoted respectively by P and Q. For every vector bundle H on M we denote by $\Gamma(H)$ the module of all differentiable sections of H.

The Gauss and Weingarten formulas for the submanifold M of $\tilde{M}$ are given respectively by

$$\text{(1.3)}\qquad \tilde{\nabla}_X Y = \nabla_X Y + h(X, Y), \qquad \tilde{\nabla}_X N = -A_N X + \nabla^{\perp}_X N;$$
$$X, Y \in \Gamma(TM), \qquad N \in \Gamma(TM^{\perp}),$$

where ∇ is the Levi-Civita connection on M, $\nabla^{\perp}$ is the linear connection induced by $\tilde{\nabla}$ on $TM^{\perp}$, h is the second fundamental form of M and A_N is the fundamental tensor of Weingarten with respect to the normal section N. The fundamental tensor A_N is related to h by

(1.4) $g(h(X, Y), N) = g(A_N X, Y)$ for any $X, Y \in \Gamma(TM)$ and $N \in \Gamma(TM^{\perp})$.

For all $X \in \Gamma(TM)$ we have

$$\text{(1.5)}\qquad X = PX + QX + \eta(X)\xi, \qquad \text{where } \eta(X) = g(X, \xi).$$

For every $N \in \Gamma(TM^{\perp})$ we put

(1.6) $\varphi N = tN + fN$, where tN and fN are the tangential and normal components of φN, respectively.

Now, suppose $\tilde{M}(c)$ be a Sasakian space form of constant φ-sectional curvature c. Then, the curvature tensor $\tilde{R}$ of $\tilde{M}(c)$ is given by [5, p. 97]:

$$\text{(1.7)}\qquad \begin{aligned} \tilde{R}(X, Y)Z = {} & [(c+3)/4]\{g(Y, Z)X - g(X, Z)Y\} + \\ & + [(c-1)/4]\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X, Z)\eta(Y)\xi - g(Y, Z)\eta(X)\xi + \\ & + g(Z, \varphi Y)\varphi X - g(Z, \varphi X)\varphi Y + 2g(X, \varphi Y)\varphi Z\}, \end{aligned}$$

for any vector fields X, Y, Z tangent to $\tilde{M}(c)$.

Recall that the equation of Ricci for the submanifold M is given by

(1.8) $R^{\perp}(X, Y; N_1, N_2) = \tilde{R}(X, Y; N_1, N_2) + g([A_{N_1}, A_{N_2}]X, Y)$; $X, Y \in \Gamma(TM)$, $N_1, N_2 \in \Gamma(TM^{\perp})$, where $\tilde{R}(X, Y; N_1, N_2) = g(\tilde{R}(X, Y)N_1, N_2)$, $R^{\perp}$ is the normal curvature tensor of M, and $[A_{N_1}, A_{N_2}] = A_{N_1}A_{N_2} - A_{N_2}A_{N_1}$.

For the semi-invariant submanifold M of the Sasakian manifold $\tilde{M}$, we denote by ν the orthogonal complementary vector bundle to $\varphi(D^{\perp})$ in $TM^{\perp}$. Thus we have the decomposition

$$\text{(1.9)}\qquad TM^{\perp} = \varphi D^{\perp} \oplus \nu \text{ and } g(\varphi D^{\perp}, \nu) = \{0\}.$$

In the sequel we denote by $2s = \dim \nu_x$, $x \in M$ and suppose $s \neq 0$. A subbundle μ of the normal bundle is said to be *parallel* if

$$\text{(1.10)}\qquad \nabla^{\perp}_X N \in \Gamma(\mu) \text{ for any } X \in \Gamma(TM) \text{ and } N \in \Gamma(\mu).$$

We need the following formulas obtained in [2].

Lemma 1.1. *Let M be a semi-invariant submanifold of a Sasakian manifold $\tilde{M}$. Then we have*

$$\text{(1.11)}\qquad h(X, \xi) = 0 \quad ; \quad \nabla_X \xi = -\varphi X, \text{ for any } X \in \Gamma(D),$$

$$\text{(1.12)}\qquad h(Y, \xi) = -\varphi Y; \quad \nabla_Y \xi = 0, \quad \text{for any } Y \in \Gamma(D^{\perp}),$$

$$\text{(1.13)}\qquad h(\xi, \xi) = 0 \quad ; \quad \nabla_\xi \xi = 0,$$

$$\text{(1.14)}\qquad A_{\varphi Y} Z = A_{\varphi Z} Y \text{ for any } Y, Z \in \Gamma(D^{\perp}),$$

$$\text{(1.15)}\qquad h(X, \varphi PY) + \nabla^{\perp}_X \varphi QY = fh(X, Y) + \varphi Q \nabla_X Y \text{ for any } X, Y \in \Gamma(TM),$$

$$\text{(1.16)}\qquad h(X, tN) + \nabla^{\perp}_X fN + \varphi Q A_N X = f\nabla^{\perp}_X N \text{ for any } X \in \Gamma(TM) \text{ and } N \in \Gamma(TM^{\perp}).$$

From (1.1), (1.6), (1.9) and the definition of a semi-invariant submanifold it follows that $f^3 + f = 0$, i.e., on the normal bundle of each semi-invariant submanifold f defines an *f-structure* [12]. The f-structure f is said to be *parallel* if we have $\nabla^{\perp}_X fN = f\nabla^{\perp}_X N$ for any $X \in \Gamma(TM)$ and $N \in \Gamma(TM^{\perp})$.

2. Semi-invariant submanifolds with semi-flat normal connection in $\tilde{M}(c)$. Definition. *The semi-invariant submanifold M of a Sasakian space form $\tilde{M}(c)$ is said to have semi-fllat normal connection if its normal curvature tensor $R^{\perp}$ is given by*

$$\text{(2.1)}\qquad \begin{aligned} & R^{\perp}(X, Y; N_1, N_2) = [(c-1)/2]g(X, \varphi PY)g(\varphi N_1, N_2), \\ & \text{for any } X, Y \in \Gamma(TM), \ N_1, N_2 \in \Gamma(TM^{\perp}). \end{aligned}$$

Remark. By using (1.7), (1.8) and (2.1) we obtain that a semi-invariant submanifold M of $\tilde{M}(c)$ has semi-flat normal connection if and only if

$$\text{(2.2)}\qquad \begin{aligned} g([A_{N_1}, A_{N_2}]X, Y) = [(c-1)/4] \{ & g(N_1, \varphi X)g(N_2, \varphi Y) - \\ & - g(N_1, \varphi Y)g(N_2, \varphi X)\} \end{aligned}$$

for any $X, Y \in \Gamma(TM)$ and $N_1, N_2 \in \Gamma(TM^\perp)$.

Thus, from (2.2) and the definition of a semi-invariant submanifold we get that if M is a semi-invariant submanifold with semi-flat normal connection of $\tilde{M}(c)$, then we have

$$g([A_{N_1}, A_{N_2}]X, U) = 0 \text{ and} \tag{2.3}$$

$$g([A_{N_1}, A_{N_2}]Z, W) = [(c-1)/4]\{g(N_1, \varphi Z)g(N_2, \varphi W) - g(N_1, \varphi W)g(N_2, \varphi Z)\} \tag{2.4}$$

for any $X \in \Gamma(D \oplus \{\xi\})$, $U \in \Gamma(TM)$, $Z, W \in \Gamma(D^\perp)$ and $N_1, N_2 \in \Gamma(TM^\perp)$.

Lemma 2.1. *Let M be a semi-invariant submanifold with semi-flat normal connection in $\tilde{M}(c)$. Then we have*

$$A_N X = 0 \textit{ and} \tag{2.5}$$

$$g(A_{\varphi Z}X, A_N W) = 0 \textit{ for any } X \in \Gamma(D \oplus \{\xi\}),\ Z, W \in \Gamma(D^\perp) \textit{ and } N \in \Gamma(\nu) \tag{2.6}$$

Proof. By using (1.1), (1.2), (1.3) and (1.4) we obtain

$g(A_N \varphi X, Y) = g(h(\varphi X, Y), N) = g(\tilde{\nabla}_Y \varphi X, N) = g(\varphi \tilde{\nabla}_Y X, N) = g(\varphi h(X, Y), N) = -g(A_{\varphi N}X, Y)$ whenever $X \in \Gamma(D \oplus \{\xi\})$, $Y \in \Gamma(TM)$, $N \in \Gamma(\nu)$. This shows that we have

$$A_N \varphi X = -A_{\varphi N}X \text{ for any } X \in \Gamma(D \oplus \{\xi\}) \text{ and } N \in \Gamma(\nu). \tag{2.7}$$

From (1.11), (1.12) and (1.13) we also have

$$A_N \xi = 0 \text{ whenever } N \in \Gamma(\nu). \tag{2.8}$$

Now, we take in (2.3), $X \in \Gamma(D \oplus \{\xi\})$, $U = \varphi X \in \Gamma(D)$, $N_1 = N \in \Gamma(\nu)$ and $N_2 = \varphi N \in \Gamma(\nu)$ and by using (2.7) and (2.8) obtain $0 = g([A_N, A_{\varphi N}]X, \varphi X) = -\|A_N \varphi X\|^2 - \|A_N X\|^2$ proving (2.5). Next, we take in (2.3), $X \in \Gamma(D \oplus \{\xi\})$, $U = W \in \Gamma(D^\perp)$, $N_1 = \varphi Z \in \Gamma(\varphi D^\perp)$, $N_2 = N \in \Gamma(\nu)$ and by using (2.5) we get $0 = g([A_{\varphi Z}, A_N]X, W) = -g(A_{\varphi Z}X, A_N W)$ proving (2.6).

Lemma 2.2. *Let M be a semi-invariant submanifold with semi-flat normal connection in $\tilde{M}(c)$. Then we have*

$$A_N Z = 0 \textit{ for any } Z \in \Gamma(D^\perp) \textit{ and } N \in \Gamma(\nu). \tag{2.9}$$

Proof. We take $X = \xi$ in (2.6) and obtain

$$g(A_{\varphi Z}\xi, A_N W) = 0 \text{ for any } Z, W \in \Gamma(D^\perp) \text{ and } N \in \Gamma(\nu). \tag{2.10}$$

On the other hand, by using (1.4), (1.11), (1.12) and (1.13) we get

$$A_{\varphi Z}\xi = -Z \text{ for any } Z \in \Gamma(D^\perp). \tag{2.11}$$

From (1.4), (2.10) and (2.11) we have

$$g(h(Z, W), N) = 0 \text{ for any } Z, W \in \Gamma(D^\perp) \text{ and } N \in \Gamma(\nu). \tag{2.12}$$

Then (1.4), (2.5) and (2.12) imply $g(A_N Z, U) = 0$ for any $Z \in \Gamma(D^\perp)$, $U \in \Gamma(TM)$ and $N \in \Gamma(\nu)$ proving (2.9).

Theorem 2.1. *Let M be a semi-invariant submanifold with semi-flat normal connection in $\tilde{M}(c)$. Then the f-structure f is parallel.*

Proof. By using (2.5), (2.8) and (2.9) in (1.16) we obtain

$$\nabla^\perp_X fN = f\nabla^\perp_X N \text{ for any } X \in \Gamma(TM) \text{ and } N \in \Gamma(\nu). \tag{2.13}$$

On the other hand, by taking $N = \varphi Z \in \Gamma(\varphi D^\perp)$, we have $tN = \varphi N = -Z \in \Gamma(D^\perp)$. $fN = 0$ and (1.16) implies

$$f\nabla^\perp_X \varphi Z = -h(X, Z) + \varphi Q A_{\varphi Z}X \text{ for any } X \in \Gamma(TM). \tag{2.14}$$

Since $f\nabla^\perp_X \varphi Z \in \Gamma(\nu)$, $\varphi Q A_{\varphi Z}X \in \Gamma(\varphi D^\perp)$ and $h(X, Z) \in \Gamma(\varphi D^\perp)$ (by means of (2.5) and (2.9)), from (2.14) we get

$$f\nabla^\perp_X \varphi Z = \nabla^\perp_X f\varphi Z = 0 \text{ for any } X \in \Gamma(TM) \text{ and } Z \in \Gamma(D^\perp). \tag{2.15}$$

Thus, (2.13) and (2.15) imply that the f-structure f is parallel and the proof is complete.

We recall that a submanifold M of $\tilde{M}$ is said to be *flat* (respectively to have *flat normal connection*) if its curvature tensor $R \equiv 0$ (respectively, its normal curvature tensor $R^\perp \equiv 0$).

New we state and prove the following classification theorem.

Theorem 2.2. *Let M be a semi-invariant submanifold in a Sasakian space form $\tilde{M}^{2n+1}(c)$. Then M has semi-flat normal connection in $\tilde{M}^{2n+1}(c)$ if and only if M is one of the following:*

(a) *a totally geodesic invariant submanifold $\tilde{M}^{2n+1}(c)$,*

(b) *a flat anti-invariant submanifold tangent to ξ of a totally geodesic invariant submanifold $\tilde{M}^{2q+1}(c)$ of $\tilde{M}^{2n+1}(c)$,*

(c) *a generic proper semi-invariant submanifold with flat normal connection in a totally geodesic invariant submanifold $\tilde{M}^{2p+2q+1}(c)$ of $\tilde{M}^{2n+1}(c)$.*

Proof. Suppose M be a semi-invariant submanifold with semi-flat normal connection in $\tilde{M}^{2n+1}(c)$.

If $q = 0$, then M is an invariant submanifold of $\tilde{M}^{2n+1}(c)$ and by (2.5) we have that M is totally geodesic immersed in $\tilde{M}^{2n+1}(c)$. Thus, M is itself a Sasakian space form $\tilde{M}^{2p+1}(c)$.

Assume that M is not an invariant submanifold of $\tilde{M}^{2n+1}(c)$, i.e., $q > 0$.

If $p = 0$, then M is an anti-invariant submanifold and (2.9) and (1.12) imply $h(X, Y) \in \Gamma(\varphi D^\perp)$ for any $X, Y \in \Gamma(TM)$. From this and (1.15) we obtain

$$\nabla^\perp_X \varphi Y = \varphi \nabla_X Y \text{ for any } X, Y \in \Gamma(TM). \tag{2.16}$$

Therefore by using (1.2), (1.3) and (2.16), considering the normal component of $\tilde{\nabla}_X \nabla^\perp_Y \varphi Z$ we obtain

$$\nabla^\perp_X \nabla^\perp_Y \varphi Z = \varphi \nabla_X \nabla_Y Z \text{ for any } X, Y, Z \in \Gamma(TM). \tag{2.17}$$

Thus the normal curvature tensor $R^\perp$ is given by

$$R^\perp(X, Y)\varphi Z = \varphi(R(X, Y)Z)\ ;\ X, Y, Z \in \Gamma(TM). \tag{2.18}$$

From (2.1) and (2.18) we have

(2.19) $$Q(R(X, Y)Z)=0 \text{ for any } X, Y, Z\in\Gamma(TM).$$

On the other hand, by using the equation of Gauss, (1.12), (1.13) and (1.14) we get

(2.20) $$\eta(R(X, Y)Z)=0.$$

From (1.12), (2.9), (2.16), (2.19) and (2.20) we conclude that M is a flat anti-invariant submanifold tangent to ξ of a totally geodesic invariant submanifold $\tilde{M}^{2q+1}(c)$ of $\tilde{M}^{2n+1}(c)$. If M is neither invariant nor anti-invariant submanifold of $\tilde{M}^{2n+1}(c)$, then (2.5) and (2.9) imply

(2.21) $$h(X, Y)\in\Gamma(\varphi D^{\perp}) \text{ for any } X, Y\in\Gamma(TM).$$

By using (1.15) and (2.21) we get $\nabla^{\perp}_X\varphi Z=\varphi Q\nabla_X Z\in\Gamma(\varphi D^{\perp})$ for any $X\in\Gamma(TM)$ and $Z\in\Gamma(D^{\perp})$, i.e., the normal subbundle $\varphi D^{\perp}$ is parallel. Then, the fundamental theorem of submanifolds shows that M lies in a totally geodesic invariant submanifold $\tilde{M}^{2p+2q+1}(c)$ of $\tilde{M}^{2n+1}(c)$ and (2.1) implies that M has flat normal connection in $\tilde{M}^{2p+2q+1}(c)$.

The converse of this is trivial.

Remarks. (i). In the particular case where $\tilde{M}^{2n+1}(c)$ is $S^{2n+1}(1)$, the $(2n+1)$-dimensional unit sphere endowed with the natural Sasakian structure of constant φ-sectional curvature $c=1$, then Theorem 2.2 gives the classification of semi-invariant submanifolds with flat normal connection of $S^{2n+1}(1)$. By this theorem we have that the study of proper semi-invariant submanifolds with flat normal connection in $S^{2n+1}(1)$ is equivalent to the study of generic proper semi-invariant submanifolds with flat normal connection in $S^{2p+2q+1}(1)$.

(ii). If $\tilde{M}^{2n+1}(c)$ is $R^{2n+1}(-3)$, the $(2n+1)$-dimensional Euclidean space with standard Sasakian structure of constant φ-sectional curvature $c=-3$, then Theorem 2.2 shows that the study of proper semi-invariant submanifolds with semi-flat normal connection in $R^{2n+1}(-3)$ is equivalent to the study of generic proper semi-invariant submanifolds with flat normal connection in $R^{2p+2q+1}(-3)$.

Now, we recall that a semi-invariant submanifold M of a Sasakian manifold $\tilde{M}$ is called a *semi-invariant product* if M is locally a Riemannian product $M_1\times M_2$, where M_1 is an invariant submanifold of $\tilde{M}$ and M_2 is an anti-invariant submanifold of $\tilde{M}$ such that the structure vector field ξ of $\tilde{M}$ is normal to M_2, [9]. We also recall that a semi-invariant submanifold M is said to be *$(D, D^{\perp})$-geodesic* (respectively *D-geodesic*) if $h(X, Z)=0$ for any $X\in\Gamma(D)$ and $Z\in\Gamma(D^{\perp})$ (resp. if $h(X, Y)=0$ for all $X, Y\in\Gamma(D)$).

By Corollary 1 of [1] we have that a semi-invariant submanifold M is a semi-invariant product if and only if $A_{\varphi Z}X=0$ for any $Z\in\Gamma(D^{\perp})$ and $X\in\Gamma(D)$. From this and (2.5) we obtain

Lemma 2.3. *Let M be a semi-invariant product with semi-flat normal connection in $\tilde{M}(c)$. Then M is $(D, D^{\perp})$-geodesic and D-geodesic.*

From Theorem 3.2, Corollaries 3.1 and 3.2 in [9], Theorem 2.2 and Lemma 2.3 we have the following theorem.

Theorem 2.3. (i). *There exist no proper semi-invariant products with semi-flat normal connection in Sasakian space forms $\tilde{M}(c)$ with $c\neq-3$.*

(ii). *If M is a proper semi-invariant product with semi-flat normal connection in $R^{2n+1}(-3)$, then M is a generic, $(D, D^{\perp})$-geodesic and D-geodesic semi-invariant product with flat normal connection in $R^{2p+2q+1}(-3)$.*

Remark. From the assertion (i) of Theorem 2.3 it follows that there exist no proper semi-invariant products with flat normal connection in $S^{2n+1}(1)$.

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