$$\int_{t_0}^{t} \beta(t, s)\alpha(s)\tilde{\omega}(s, 2\rho\psi(s))ds < (\varepsilon/2C) \text{ and } \int_{t}^{\infty} \alpha(s)\tilde{\omega}(s, 2\rho\psi(s))ds < (\varepsilon/2C).$$

Hence, for $t \geq t_1$, $|x(t) - y(t)| < \varepsilon$ and since $\varepsilon > 0$ is arbitrary, it follows that $\lim_{t\to\infty} |x(t) - y(t)| = 0$. The proof of the converse is exactly the same as for Theorem 3.1.

Remark 3.5. In particular case of differential inclusions (see Remark 3.3) Theorem 3.4 reduces to Theorem 3.3 [13].

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ON PSEUDO-KAEHLERIAN FINSLER SPACES

BY

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Osculating Riemann metrics associated to Finsler metrics were considered previously by O. V a r g a [12] and A. M o o r [11]. Also T. M a e b a s h i [8] has investigated subspaces of Finsler spaces admitting a preferential direction ; yet his results are true for the case $R^i_{jk} = \delta N^i_j/\delta x^k - \delta N^i_k/\delta x^j = 0$ only. Moreover, G. A s a n o v [3] has shown that given a 4-dimensional manifold with full parallelism structure $\{S_i\}_{1\leq i\leq 4}$ (which stands for space-time), the osculating Riemann metric (at the preferential direction $S = \sum_{i=1}^{4} S_i$) associated to Berwald-Moore metric of the space represents an approximation of the Finsler metric (up to terms of second order in the standard deviations of the supporting elements from the preferential direction S) and thus the basic structure of the real space-time is Finslerian. Clearly the cases considered in [8], [11], [12] as well as in [2], [3] are subject to topological restrictions (such as the vanishing of Euler-Poincaré characteristic) even if not mentioned.

Our purpose is to examine osculating objects at a preferential direction of a differentiable manifold associated to several finslerian geometrical objects such as finslerian almost complex structures, finslerian almost hermitian structures, etc. Attention is devoted to the integrable case, i.e. $R^i_{jk} = 0$, indicated by M. M a t s u m o t o - L. T a m a s s y [10] to be of particular interest; the main result is that a pseudo-Kaehlerian Finsler space (M, E, J) induces locally osculating Kaehlerian structures on M.

A result of T. M a e b a s h i [8] is reconsidered in §1.

1. The notations and conventions throughout the present paper are a combined version of those from B. T. M. H a s s a n [7] and M. M a t s u m o t o [9].

Let M be a n-dimensional real C^∞-differentiable manifold and let j be the natural imbedding of M into the total space $T(M)$ of the tangent bundle over M. Let $\tilde{M} = T(M) \setminus j(M)$ and $\pi : \tilde{M} \to M$ be the natural projection ; moreover we denote by $\pi^{-1}TM$ the bundle induced by $T(M)$ and π. A finslerian almost complex structure on M is a bundle morphism $J : \pi^{-1}TM \to \pi^{-1}TM$ such that $J^2 = -I$, where I denotes the identity transformation on $\pi^{-1}TM$.

Let (U, x^i) be a local coordinate system on M and $(\tilde{U}, x^i, y^i)$ the naturally induced coordinates on $\tilde{M}$, where $\tilde{U} = \pi^{-1}(U)$. Let $\tilde{\partial}_i$ be the natural lift of the locally defined tangent vector fields $\partial/\partial x^i$, i.e. $\tilde{\partial}_i(\tilde{x}) = (\tilde{x}, \partial/\partial x^i|_{\pi\tilde{x}})$, $\tilde{x} \in \tilde{M}$. We put $J\tilde{\partial}_i = F^j_i(x, y)\tilde{\partial}_j$: then $F^j_i F^k_j = -\delta^k_i$. It is easily seen that a differentiable manifold carrying a finslerian almost complex structure should be even-dimensional. Note that any finslerian almost complex structure on a

$2m$-dimensional manifold M is equivalent to a $GL(m, C)$-principal subbundle of the Finsler principal bundle $\pi^{-1}L(M)\to\tilde{M}$, such as constructed in M. Matsumoto [9].

Any almost complex structure J on M admits a natural lift $\bar{J}:\pi^{-1}TM\to\pi^{-1}TM$ defined by $\bar{J}_{\tilde{x}}\bar{X}=(\tilde{x}, J_{\pi\tilde{x}}(\hat{\pi}\bar{X}))$, $\bar{X}\in\pi_{\tilde{x}}^{-1}TM$, $\tilde{x}\in\tilde{M}$, where $\hat{\pi}:\tilde{M}\times T(M)\to T(M)$ denotes the projection on the second factor of $\tilde{M}\times T(M)$.

Let (M, E) be a Finsler space with the Finsler energy $E: T(M)\to\mathbf{R}_+$ and the associated metric tensor g with local components $g_{ij}(x, y)=(1/2)\partial^2E/\partial y^i\partial y^j$. Suppose M carries a finslerian almost complex structure J: then (M, E, J) is said to be an *almost hermitian Finsler space* if $g(J\bar{X}, J\bar{Y})=g(\bar{X}, \bar{Y})$ for any finslerian vector fields $\bar{X}, \bar{Y}$ on M. Moreover let ∇ denote the unique regular Cartan connection of (M, E). Then (M, E, J) is said to be a *pseudo-Kaehlerian Finsler space* if the Cartan connection is almost complex, i.e. $\nabla J=0$.

Now suppose the manifold M has vanishing Euler-Poincaré characteristic: then there exists a tangent vector field S on M such that $S(x)\neq 0$ for each $x\in M$. Such a vector field is refered to as a choise of *preferential direction* on M and fixed for this section.

Given a finslerian almost complex structure J on M the *osculating almost complex structure* associated to J is defined by

$$(1.1)\qquad (J_SX)_x=\hat{\pi}J_{S(x)}(S(x), X_x).$$

for any tangent vector field X on M, $x\in M$. If $J_j^i(x)$ stand for the local components of J_S, then (1.1) is locally equivalent to

$$(1.2)\qquad J_j^i(x)=F_j^i(x, S(x)).$$

Note that if J is an almost complex structure on M, then J coincides with the osculating almost complex structure associated to its natural lift. Conversely, let J_S be the osculating structure associated to a finslerian almost complex structure J on M. Then the natural lift of J_S coincides with J along $S(M)$ only.

Given a Finsler space (M, E) the *osculating Riemann metric* associated to g is defined by

$$(1.3)\qquad g_S(X, Y)_x=g(\bar{X}, \bar{Y})_{S(x)}$$

for any tangent vector fields X, Y on M and $x\in M$. Here $\bar{X}, \bar{Y}$ denote the natural lifts of X, Y, e.g. $\bar{X}(\tilde{x})=(\tilde{x}, X(\pi\tilde{x}))$, for any $\tilde{x}\in\tilde{M}$. Let $h_{ij}(x)$ be the local components of g_S. Note that (1.3) is locally equivalent to

$$(1.4)\qquad h_{ij}(x)=g_{ij}(x, S(x)).$$

According to (1.1), (1.3) if *(M, E, J) is an almost hermitian Finsler space, then (M, g_S, J_S) is an almost hermitian manifold*. However, the consequences of the pseudo-Kaehlerian condition of (M, E, J) are not obvious for the osculating Riemann space at the preferential direction S on M.

Now let ∇ be a regular connection in $\pi^{-1}TM$ and $\beta:\pi^{-1}TM\to N$ its horizontal lift, where N denotes the non-linear connection associated to ∇. Let

$\gamma:\pi^{-1}TM\to\mathrm{Ker}\,(d\pi)$ be the vertical lift and consider the vector bundle morphism $L: T(\tilde{M})\to\pi^{-1}TM$ defined by $L(\tilde{X})=(\tilde{x}, (d\pi)\tilde{X})$ for any $\tilde{X}\in T_{\tilde{x}}(\tilde{M})$, $\tilde{x}\in\tilde{M}$. Let J be a finslerian almost complex structure on M and denote by N_J^h the finslerian tensor field defined as follows

$$N_J^h(\bar{X}, \bar{Y})=[J\bar{X}, J\bar{Y}]^h-J[J\bar{X}, \bar{Y}]^h-J[\bar{X}, J\bar{Y}]^h-[\bar{X}, \bar{Y}]^h,$$

for any finslerian vector fields $\bar{X}, \bar{Y}$ on M, where $[\bar{X}, \bar{Y}]^h=L[\beta\bar{X}, \beta\bar{Y}]$ and so on. It is well known ([1]) that the vanishing of N_J^h is equivalent to the existence of a canonical complex structure on each integral manifold of the distribution N, provided that $R^1=0$. Here

$$R^1(\bar{X}, \bar{Y})=-\gamma^{-1}([\beta\bar{X}, \beta\bar{Y}]_v),$$

while $[\beta\bar{X}, \beta\bar{Y}]_v$ denotes the vertical part of $[\beta\bar{X}, \beta\bar{Y}]$ with respect to the direct sum decomposition

$$(1.5)\qquad T(\tilde{M})=N\oplus\ker\,(d\pi).$$

Finally we get a formula which will be used later. Let F_i^k, A_{ij}^k and N_{ij}^k be respectively the local components of J, N_J^h and of the Nijenhuis torsion tensor field of the osculating almost complex structure J_s associated to J. Then, using (1.2), we have

$$(1.6)\qquad \begin{aligned}N_{ij}^k={}&A_{ij}^k+\left(F_i^m\frac{\partial F_j^k}{\partial y^t}-F_j^m\frac{\partial F_i^k}{\partial y^t}\right)\left(\frac{\partial S^t}{\partial x^m}+N_m^t\right)+F_m^k\frac{\partial F_i^m}{\partial Y^t}\left(\frac{\partial S^t}{\partial x^j}+N_j^t\right)-\\&-F_m^k\frac{\partial F_j^m}{\partial y^t}\left(\frac{\partial S^t}{\partial x^i}+N_i^t\right).\end{aligned}$$

where N_i^j and S_i^j stand for the local components of the non-linear connection N and of the preferential direction S, respectively.

2. Recall that given a non-linear connection N on $\tilde{M}$ its local coefficients are subject of the transformation law

$$N'^k_m(x', y')=N_i^j(x, y)\frac{\partial x'^k}{\partial x^j}\frac{\partial x^i}{\partial x'^m}-\frac{\partial^2x'^k}{\partial x^i\partial x^j}\frac{\partial x^i}{\partial x'^m}y^j,$$

under any transformation

$$(2.1)\qquad x'^i=x'^i(x^1,\dots,x^n),\quad y'^i=\frac{\partial x'^i}{\partial x^j}y^j,\quad \det\left|\frac{\partial x'^i}{\partial x^j}\right|=0$$

of the local coordinates on $\tilde{M}$. On the other hand, if there is a choice of preferential direction S on M, we have

$$\frac{\partial S'^j}{\partial x'^i}=\frac{\partial S^m}{\partial x^k}\frac{\partial x^k}{\partial'^i}\frac{\partial x'^j}{\partial x^m}+\frac{\partial^2x'^j}{\partial x^k\partial x^m}\frac{\partial x^k}{\partial x'^i}S^m$$

under (2.1). Consequently the vectors $\left(\frac{\partial}{\partial x^i}+\frac{\partial S^j}{\partial x^i}\frac{\partial}{\partial y^j}\right)_{\tilde{x}}$, $i=1,\dots,n$, generate a subspace of $T_{\tilde{x}}(\tilde{M})$, denoted by $D_{\tilde{x}}$, at any $\tilde{x}\in S(M)$. Then the assignement

$D: \tilde{x} \to D_{\tilde{x}}$ is a n-dimensional distribution on $S(M)$, which is said to be the *non-linear connection associated to* S.

Now suppose that the non-linear connection N has a vanishing R^1 torsion tensor field. Since $[\beta X, \beta Y]_v = -\gamma R^1(X, Y)$, the distribution N is involutive and hence integrable by the classical theorem of Frobenius. Thus, for each $\tilde{x} \in \tilde{M}$ there is a maximal integral manifold $V_{\tilde{x}}$ of N passing through $\tilde{x}$, that is $\tilde{x} \in V_{\tilde{x}}$, $T_{\tilde{y}}(V_{\tilde{x}}) = N_{\tilde{y}}$ for any $\tilde{y} \in \tilde{V}_x$. Since $N_{\tilde{x}}$ is a direct sum complement of $\operatorname{Ker}(d_{\tilde{x}}\pi)$ in $T_{\tilde{x}}(\tilde{M})$, it follows that $d_{\tilde{x}}\pi: N_{\tilde{x}} \to T_x(M)$, $x = \pi(\tilde{x})$, is an isomorphism. Thus $\pi: V_{\tilde{x}} \to M$ is a local diffeomorphism, by the classical inverse function theorem. Consequently there exist the open neighbourhoods $U_{\tilde{x}}$ of $\tilde{x}$, respectively U_x of $x = \pi(\tilde{x})$, such that $U_{\tilde{x}} \subset V_{\tilde{x}}$, $U_x \subset M$ and $\pi: U_{\tilde{x}} \to U_x$ is a diffeomorphism. Let $S: U_x \to T(M)$ be the local tangent vector field defined by $S = (\pi | U_{\tilde{x}})^{-1}$. Since for each $y \in U_x$

$$\delta/\delta x^i|_{S(y)} \in N_{S(y)} = T_{S(y)}(V_{\tilde{x}}), \quad i = 1, \ldots, n,$$

then there is a differentiable curve $c_i: (-\varepsilon', \varepsilon') \to V_{\tilde{x}}$ with initial data $(S(y), \partial/\partial x^i|_{S(y)})$. If c_i is written locally $c_i(t) = (a_i^j(t), b_i^j(t))$, $|t| < \varepsilon'$, then

$$da_i^j/dt(0) = \delta_i^j, \tag{2.2}$$

$$db_i^j/dt(0) = -N_i^j(y, S(y)). \tag{2.3}$$

Since $U_{\tilde{x}}$ is open in $V_{\tilde{x}}$ and c_i is continuous, there is $\varepsilon > 0$ such that $c_i(t) \in U_{\tilde{x}}$, $|t| < \varepsilon$. Let us put by definition $x_i(t) = \pi(c_i(t))$, $|t| < \varepsilon$. Then clearly

$$c_i(t) = S(x_i(t)), \quad i = 1, \ldots, n. \tag{2.4}$$

Using (2.2) – (2.4) it follows

$$db_i^j/dt(0) = \partial S^j/\partial x^m(x_i(0))\, da_i^m/dt(0) = \partial S^j/\partial x^i(y).$$

Then, because of (2.3) we have

$$N_i^j(y, S(y)) = -\partial S^j/\partial x^i(y), \tag{2.5}$$

for any $y \in U_x$. Thus we conclude

Proposition 2.1. *Let N be a non-linear connection on M with a vanishing R^1 torsion tensor field. Then for each $x \in M$, there exist an open neighbourhood U_x of x and a local vector field $S: U_x \to T(M)$ such that $S(y) \neq 0$ for any $y \in U_x$ and the restriction of N to $S(U_x)$ is exactly the non-linear connection associated to S.*

Now let (M, E, J) be an almost hermitian Finsler space. Suppose that the non-linear connection N of the Cartan connection has a vanishing R^1 torsion tensor field. Let $S: U_x \to T(M)$ be the local vector field defined on the open neighbourhood U_x of $x \in M$ with the properties of the proposition 2.1. Let g be the Finsler metric tensor associated to the energy E. Denote by g_S (respect. J_S) the osculating Riemann metric (respect. the osculating almost complex structure) associated to the restriction of g (respect. of J) to $S(U_x)$. The following result (strictly local in nature) occurs.

Theorem 2.2. *With the notations above, for each $x \in M$ (U_x, g_S, J_S) is a hermitian manifold, provided that $N_j^h = 0$.*

Proof. Since (2.5) holds for any $y \in U_x$, then from (1.6) we have $N_{ki}^{k}(y) = A_{ki}^k(y, S(y))$. Thus $N_j^h = 0$ implies that J_S is integrable. Consequently U_x admits a complex manifold structure which induces J_S. Finally g_S is compatible with J_S due to the similar property of (g, J).

Note that the natural lift of J_S in the theorem above and $J|_{S(U_x)}$ actually coincide.

3. Let ∇ be a regular connection in $\pi^{-1}TM$ and let S be a choice of preferential direction on M. The *osculating linear connection associated to* ∇ is defined by

$$(\overset{S}{\nabla}_X Y)_x = \hat{\pi}(\nabla_{\beta\bar{X}} \bar{Y})_{S(x)} \tag{3.1}$$

for any tangent vector fields X, Y on M and $x \in M$. Here $\bar{X}$, $\bar{Y}$ denote the natural lifts of X, Y respectively. Note that the restriction to $S(M)$ of the natural lift of $\overset{S}{\nabla}_X Y$ and $\nabla_{\beta\bar{X}} \bar{Y}$ actually coincide. Moreover we have

$$[\beta X, \beta \bar{Y}]_h = \beta \overline{[X, Y]}, \tag{3.2}$$

where the left hand side denotes the horizontal part of $[\beta \bar{X}, \beta \bar{Y}]$ with respect to the direct sum decomposition (1.5). Let T_S denote the torsion tensor field of $\overset{S}{\nabla}$. Then using (3.2) and $\operatorname{Ker} L = \operatorname{Ker}(d\pi)$, we obtain

$$(T_S(X, Y))_x = \pi(T(\bar{X}, \bar{Y}))_{S(x)}, \tag{3.3}$$

here

$$T(\bar{X}, \bar{Y}) = \tilde{T}(\beta\bar{X}, \beta\bar{Y}), \quad \tilde{T}(\tilde{X}, \tilde{Y}) = \nabla_{\tilde{X}} L\tilde{Y} - \nabla_{\tilde{Y}} L\tilde{X} - L[\tilde{X}, \tilde{Y}],$$

where $\tilde{X}$, $\tilde{Y}$ are tangent vector fields on $\tilde{M}$. Let J_S be the osculating almost complex structure associated with the finslerian almost complex structure J on M. Note that for any tangent vector field X on M we have

$$\overline{J_S X}|_{S(M)} = \bar{J}_S \bar{X}|_{S(M)}, \tag{3.4}$$

where $\bar{X}$, $\overline{J_S X}$, $\bar{J}_S$ are the natural lifts of X, $J_S X$, J_S respectively. Using (3.4) we derive

$$((\overset{S}{\nabla}_x J_S) Y)_x = \hat{\pi}(\nabla_{\beta\bar{X}} J) \bar{Y})_{S(x)}. \tag{3.5}$$

With the notations in section 1, if (M, E) is a Finsler space, from (1.4) we obtain

$$\partial h_{ij}/\partial x^k = \partial g_{ij}/\partial x^k + \partial g_{ij}/\partial y^m \cdot \partial S^m/\partial x^k.$$

Then

$$X(g_S(Y, Z)) = X^i Y^j Z^k \left(\frac{\partial g_{jk}}{\partial x^i} + \frac{\partial g_{jk}}{\partial y^m} \frac{\partial S^m}{\partial x^i} \right) + X^i g_{jk} \frac{\partial (Y^j Z^k)}{\partial x^i}. \tag{3.6}$$

Suppose ∇ is a regular connection in $\pi^{-1}TM$ whose non-linear connection restricted to $S(M)$ is the non-linear connection associated to S. As Y^j, Z^k depend solely on positional arguments, (3.6) leads to

$$(X(g_S(Y, Z))_x = ((\beta\bar{X})(g(\bar{Y}, \bar{Z})))_{S(x)},$$

and thus

$$(\overset{S}{\nabla}_X g_S)(Y, Z)_x = (\nabla_{\beta\bar{X}} g)(\bar{Y}, \bar{Z})_{S(x)}. \tag{3.7}$$

The formulae (3.3), (3.5), (3.7) lead to the following

Theorem 3.1. *Let (M, E, J) be a pseudo-kaehlerian Finsler space and let ∇ and g be the Cartan connection and the metric tensor associated to E, respectively. Let S be a choice of a preferential direction on M and denote by g_S and J_S the osculating structures associated to g and J, respectively. If the restriction to $S(M)$ of the non-linear connection of ∇ coincides with the non-linear connection associated to S, then the osculating connection associated to ∇ is the Riemannian connection defined by g_S. Moreover (M, g_S, J_S) is a kaehlerian manifold.*

Finally let (M, E, J) be a pseudo-kaehlerian Finsler space such that the non-linear connection of the Cartan connection has a vanishing R^1 torsion tensor field. If $x \in M$, let S be the preferential direction of an open neighbourhood U_x of x as defined in proposition 2.1. Let J_S and g_S be the osculating structures associated respectively to $J|_{S(U_x)}$ and to the restriction to $S(U_x)$ of the metric tensor defined by E. Then combining proposition 2.1 and theorem 3.1 we have

Theorem 3.2. *With the notations above, for each $x \in M$ (U_x, g_S, J_S) is a kaehlerian manifold.*

4. We reconsider now some results of T. Maebashi [8].
Let (M, E) be a Finsler space with metric tensor g_{ij}. Let $\tilde{x}_0 \in \tilde{M}$ be a fixed tangent direction on M. According to lemma 1, p. 63, [8], the vector field Φ defined by

$$\Phi(x) = (y_0^i + (C^i_{kl}(x_0, y_0)\gamma^l_{00}(x_0, y_0) - \gamma^i_{0k}(x_0, y_0))(x^k - x_0^k))\partial/\partial x^i|_x \tag{4.1}$$

with respect to the local coordinates $(\pi^{-1}(U), x^i, y^i)$, $\tilde{x}_0 \in \pi^{-1}(U)$, $x_0 = \tilde{\pi}(x_0)$, is a solution of

$$\Phi(x_0) = \tilde{x}_0, \quad \partial\Phi^i/\partial x^j + \Gamma^i_{jk}(M_\Phi)\Phi^j = 0. \tag{4.2}$$

Here we have used the notations

$$C^i_{jk} = g^{il}C_{jlk}, \quad \gamma^i_{jk} = g^{it}\gamma_{jtk}, \quad \gamma^i_{00} = \gamma^i_{jk}y^j y^k, \quad \gamma^i_{0j} = \gamma^i_{kj}y^k,$$

$$C_{ijk} = \frac{1}{4}\frac{\partial^3 E}{\partial y^i \partial y^j \partial y^k}, \qquad \gamma_{ijk} = \frac{1}{2}\left(\frac{\partial g_{ji}}{\partial x^i} + \frac{\partial g_{ij}}{\partial x^k} - \frac{\partial g_{jk}}{\partial x^i}\right).$$

Also $\Gamma^i_{jk}(M_\varphi)$ stand for the components of the Levi-Civita connection of the Riemann space $M_\Phi = (M, g_\Phi)$, where g_Φ is the metric tensor of M with local components $g_{ij}(x, \Phi(x))$. Let $N^i_j(M_\Phi = \Gamma^i_{jk}(M_\Phi)y^k$ be the non-linear connection associated with $\Gamma^i_{jk}(M_\Phi)$. By (4.2) the restriction of $N^i_j(M_\Phi)$ to $\Phi(M)$ is given by $N^i_j(M_\Phi) = -\partial\Phi^i/\partial x^j$ and therefore for the torsion tensor field $R^1(M_\Phi)$ we have

$$R^i_{jk}(M) =_\Phi \frac{\delta N^i_j(M_\Phi)}{\delta x^k} - \frac{\delta N^i_k(M_\Phi)}{\delta x^j} = 0.$$

On the other hand, if $R^i_{jkm}(M_\Phi)$ denote the components of the curvature tensor of $\Gamma^i_{jk}(M_\Phi)$, we have $R^i_{jk}(M_\Phi) = y^t R^i_{tjk}(M_\Phi)$, which shows that generally $R^i_{jk}(M_\Phi)$ is not vanishing, a contradiction. The explanation follows as this. Indeed (4.1) is subject to (4.2); yet (4.1) does not define globally a vector field on M, i.e. the components of Φ from (4.1) do not change as the components of a vector field under a transformation of the local coordinates.

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