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# GAUGE THEORY ON A VECTOR BUNDLE (II)

## Local gauge invariance and Lagrangians for gauge fields

BY

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**1. Introduction.** This is the second paper in a series we devote to the gauge theory on a vector bundle. We recall that the first paper was concerned with relativistic Finsler geometry and the global gauge invariance of Lagrangians on a vector bundle, see [2].

It is the purpose of the present paper to study the local gauge invariance and Lagrangians for gauge fields. More precisely, in §1 we introduce two kinds of gauge fields and by (1.18) and (1.19) define the horizontal and vertical gauge covariant derivatives. Then we prove that the local gauge invariant Lagrangian $\mathcal{L}_0'(x, v)$ is obtained from the global gauge invariant Lagrangian $\mathcal{L}_0(x, v)$ by replacing $\dfrac{\delta Q^A}{\delta x^\alpha}$ and $\dfrac{\partial Q^A}{\partial v^i}$ from $\mathcal{L}_0(x, v)$ by $\overset{(h)}{D}_\alpha Q^A$ and respectively $\overset{(v)}{D}_i Q^A$. Next in §2 we define by (2.38), (2.39) and (2.40) the horizontal, mixed and respectively vertical strength fields and obtain the corresponding Lagrangians (2.58), (2.59) and respectively (2.60). It is noteworthy that all strength fields are Finsler tensor fields on a vector bundle and Lagrangians for gauge fields are invariant to both the relativistic transformations on the vector bundle and the local gauge action of the Lie group involved in the theory.

We have to note that the terminology and notations from [2] are used in the present paper, sometimes without any notice. However, we recall that we have the following range for indices

$$\alpha, \beta, \gamma, \ldots = 0, 1, 2, 3 \,;\; i, j, k, \ldots = 1, \ldots, m \,;\; A, B, C, \ldots = 1, \ldots p \,;$$
$$a, b, c, \ldots = 1, \ldots, n.$$

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**1. Local gauge invariance of Lagrangians on a vector bundle.** Let $M$ be the Minkowski 4-dimensional space and $E$ be the vector bundle over $M$ with $m$-dimensional fibers. We recall from [2] that relativistic transformations on $E$ are given by

$$\text{(1.1)} \qquad \begin{cases} \tilde{x}^\alpha = L^\alpha_\beta x^\beta + A^\alpha \\ \tilde{v}^i = A^i_j(x) v^j \end{cases}$$

where in the first line we put the inhomogeneous Lorentz transformations on $M$.

Next, we consider $p$ scalar fields $Q^A(x, v)$ on $E$ and an $n$-dimensional Lie group $G$ which has a $p$-dimensional representation. Suppose the action of $G$ on physical fields $Q^A(x, v)$ is locally, that is, the constants $\varepsilon^a$ from [2] are now replaced by differentiable functions $\varepsilon^a(x, v)$. More precisely, the local action of $G$ on $Q^A(x, v)$ is given by

$$Q'^A(x, v)=Q^A(x, v)+\varepsilon^a(x, v)[X_a]^A_B\cdot Q^B(x, v). \tag{1.2}$$

where $\{X_a\}$ is a basis for the Lie algebra of $G$. We assume on $E$ there exists a non-linear connection $HE=(N^i_\alpha(x, v))$ and by using (2.2) from [2] and (1.2) from the present paper obtain

$$\frac{\delta Q'^A}{\delta x^\alpha}=\frac{\delta Q^A}{\delta x^\alpha}+\frac{\delta\varepsilon^a}{\delta x^\alpha}[X_a]^A_B Q^B+\varepsilon^a[X_a]^A_B\frac{\delta Q^B}{\delta x^\alpha}, \text{ and} \tag{1.3}$$

$$\frac{\partial Q'^A}{\partial v^i}=\frac{\partial Q^A}{\partial v^i}+\frac{\partial\varepsilon^a}{\partial v^i}[X_a]^A_B Q^B+\varepsilon^a[X_a]^A_B\frac{\partial Q^B}{\partial v^i}. \tag{1.4}$$

We recall that in [2] we have studied the Lagrangian

$$\mathcal{L}_0(x, v)=\mathcal{L}\left(Q^A(x, v), \frac{\delta Q^A}{\delta x^\alpha}, \frac{\partial Q^A}{\partial v^i}, \gamma_{\alpha\beta}(x, v), g_{ij}(x, v)\right). \tag{1.5}$$

This Lagrangian is supposed to be invariant with respect to the global action of $G$ on $Q^A(x, v)$. But, in general, $\mathcal{L}_0(x, v)$ is not invariant with respect to the transformations (1.3) and (1.4). In order to get a local gauge invariant Lagrangian we consider the *gauge fields* $H^a_\alpha(x, v)$ and $V^a_i(x, v)$ and suppose the new Lagrangian $\mathcal{L}'_0(x, v)$ is a scalar field on $E$ given by

$$\mathcal{L}'_0(x, v)=\mathcal{L}'\left(Q^A(x,v), \frac{\delta Q^A}{\delta x^\alpha}, \frac{\partial Q^A}{\partial v^i}, H^a_\alpha(x, v), V^a_i(x, v), \gamma_{\alpha\beta}(x, v), g_{ij}(x, v)\right) \tag{1.6}$$

where $\mathcal{L}'$ is a differentiable function.
Moreover, we suppose $H^a_\alpha(x, v)$ and $V^a_i(x, v)$ are the components of $n$-horizontal and respectively $n$ vertical Finsler differential 1 — forms on $E$, *i.e.*, we have

$$H^a_\alpha(x, v)=L^\beta_\alpha\tilde{H}^a_\beta(\tilde{x}, \tilde{v}), \text{ and respectively} \tag{1.7}$$

$$V^a_i(x, v)=A^j_i(x)\tilde{V}^a_j(\tilde{x}, \tilde{v}). \tag{1.8}$$

We now suppose the local action of $G$ on gauge fields is given by the following inhomogeneous transformations

$$\delta'(H^a_\alpha(x, v))=\varepsilon^b(x, v)[L_b]^a_c H^c_\alpha(x, v)+\frac{\delta\varepsilon^a(x, v)}{\delta x^\alpha}, \text{ and} \tag{1.9}$$

$$\delta'(V^a_i(x, v))=\varepsilon^b(x, v)[D_b]^a_c V^c_i(x, v)+\frac{\partial\varepsilon^a(x, v)}{\partial v^i}, \tag{1.10}$$

where $[L_b]^a_c$ and $[D_b]^a_c$ are stillunknown coefficients.



The local gauge invariance of the Lagrangian (1.6) is expressed as follows:

$$\begin{aligned}\frac{\partial\mathcal{L}'}{\partial Q^A}\delta'(Q^A)&+\frac{\partial\mathcal{L}'}{\partial\left(\frac{\partial Q^A}{\partial v^i}\right)}\Bigg|\frac{\delta Q^A}{\delta x^\alpha}=\text{const.}\quad\delta'\left(\frac{\partial Q^A}{\partial v^i}\right)+\\&+\frac{\partial\mathcal{L}'}{\partial\left(\frac{\delta Q^A}{\delta x_\alpha}\right)}\delta'\left(\frac{\delta Q^A}{\delta x^\alpha}\right)+\frac{\partial\mathcal{L}'}{\partial H^a_\alpha}\delta'(H^a_\alpha)+\frac{\partial\mathcal{L}'}{\partial V^a_i}\delta'(V^a_i)=0,\end{aligned} \tag{1.11}$$

where we have put

$$\delta'(Q^A)=\varepsilon^a(x, v)[X_a]^A_B Q^B(x, v), \tag{1.12}$$

$$\delta'\left(\frac{\delta Q^A}{\delta x_\alpha}\right)=\varepsilon^a(x, v)[X_a]^A_B\frac{\delta Q^B}{\delta x^\alpha}+\frac{\delta\varepsilon^a}{\delta x^\alpha}[X_a]^A_B Q^B, \text{ and} \tag{1.13}$$

$$\delta'\left(\frac{\partial Q^A}{\partial v^i}\right)=\varepsilon^a(x, v)[X_a]^A_B\frac{\partial Q^B}{\partial v^i}+\frac{\partial\varepsilon^a}{\partial v^i}[X_a]^A_B Q^B. \tag{1.14}$$

By using (1.19), (1.10) and (1.12) — (1.14) in (1.11) and taking account of arbitrariness of $\varepsilon^a(x, v)$ we obtain that (1.11) is equivalent with the following equations

$$\begin{aligned}\frac{\partial\mathcal{L}'}{\partial Q^A}[X_a]^A_B Q^B&+\frac{\partial\mathcal{L}'}{\partial\left(\frac{\partial Q^A}{\partial v^i}\right)}\Bigg|\frac{\delta Q^A}{\delta x^\alpha}=\text{const.}\quad[X_a]^A_B\frac{\partial Q^B}{\partial v^i}+\\&+\frac{\partial\mathcal{L}'}{\partial\left(\frac{\delta Q^A}{\delta x^\alpha}\right)}[X_a]^A_B\frac{\delta Q^B}{\delta x^\alpha}+\frac{\partial\mathcal{L}'}{\partial H^b_\alpha}[L_a]^b_c H^c_\alpha+\frac{\partial\mathcal{L}'}{\partial V^b_i}[D_a]^b_c V^c_i=0,\end{aligned} \tag{1.15}$$

$$\frac{\partial\mathcal{L}'}{\partial\left(\frac{\delta Q^A}{\delta x^\alpha}\right)}[X_a]^A_B\cdot Q^B+\frac{\partial\mathcal{L}'}{\partial H^a_\alpha}=0, \text{ and} \tag{1.16}$$

$$\frac{\partial\mathcal{L}'}{\partial\left(\frac{\partial Q^A}{\partial v^i}\right)}[X_a]^A_B Q^B+\frac{\partial\mathcal{L}'}{\partial V^a_i}=0. \tag{1.17}$$

From (1.16) and (1.17) it follows that $\frac{\delta Q^A}{\delta x^\alpha}$, $\frac{\partial Q^A}{\partial v^i}$ and the gauge fields $H^a_\alpha$ and $V^a_i$ enter into the new Lagrangian only by means of

$$\overset{(h)}{D}_\alpha Q^A(x, v)=\frac{\delta Q^A}{\delta x^\alpha}(x, v)-H^a_\alpha(x, v)[X_a]^A_B Q^B(x, v), \text{ and} \tag{1.18}$$

$$\overset{(v)}{D}_i Q^A(x, v)=\frac{\partial Q^A}{\partial v^i}(x, v)-V^a_i(x, v)[X_a]^A_B Q^B(x, v). \tag{1.19}$$

We call $\overset{(h)}{D}_\alpha Q^A(x, v)$ and $\overset{(v)}{D}_i Q^A(x, v)$ the *horizontal* and respectively *vertical gauge covariant derivatives* of the physical fields $Q^A(x, v)$. By the above study we can express the Lagrangian we look for as follows

$$\mathcal{L}_0'(x, v) = \mathcal{L}''(Q^A(x, v), \overset{(h)}{D}_\alpha Q^A(x, v), \overset{(v)}{D}_i Q^A(x, v), \gamma_{\alpha\beta}(x, v), g_{ij}(x, v)), \tag{1.20}$$

where $\mathcal{L}''$ is a differentiable function.

We are now studying (1.15) in order to get differentiable function $\mathcal{L}''$. First, by direct calculations we obtain

$$\frac{\partial \mathcal{L}}{\partial Q^A} = \frac{\partial \mathcal{L}''}{\partial Q^A}\Bigg|_{\substack{\overset{(v)}{D}_i Q^A = \mathrm{const.} \\ \overset{(h)}{D}_\alpha Q^A = \mathrm{const.}}} - \tag{1.21}$$

$$- \frac{\partial \mathcal{L}}{\partial(\overset{(h)}{D}_\alpha Q^B)} [X_a]^B_A H^a_\alpha - \frac{\partial \mathcal{L}''}{\partial(\mathcal{L}'' Q^B)} [X_a]^B_A V^a_i,$$

$$\frac{\partial \mathcal{L}'}{\partial\left(\dfrac{\delta Q^A}{\delta x^\alpha}\right)} = \frac{\partial \mathcal{L}''}{\partial(\overset{(h)}{D}_\alpha Q^A)}, \tag{1.22}$$

$$\frac{\partial \mathcal{L}'}{\partial\left(\dfrac{\partial Q^A}{\partial v^i}\right)}\Bigg|_{\frac{\delta Q^A}{\delta x^\alpha} = \mathrm{const.}} = \frac{\partial \mathcal{L}''}{\partial(\overset{(v)}{D}_i Q^A)}, \tag{1.23}$$

$$\frac{\partial \mathcal{L}'}{\partial H^a_\alpha} = - \frac{\partial \mathcal{L}''}{\partial(\overset{(h)}{D}_\alpha Q^A)} [X_a]^A_B Q^B, \text{ and} \tag{1.24}$$

$$\frac{\partial \mathcal{L}'}{\partial V^a_i} = - \frac{\partial \mathcal{L}''}{\partial(\overset{(v)}{D}_i Q^A)} [X_a]^A_B Q^B. \tag{1.25}$$

Then by using (1.18), (1.19) and (1.21) — (1.25) in (1.15) we get

$$\frac{\partial \mathcal{L}''}{\partial Q^A} [X_a]^A_B Q^B + \frac{\partial \mathcal{L}''}{\partial(\overset{(h)}{D}_\alpha Q^A)} [X_a]^A_B \overset{(h)}{D}_\alpha Q^B + \tag{1.26}$$

$$+ \frac{\partial \mathcal{L}''}{\partial(\overset{(v)}{D}_i Q^A)} [X_a]^A_B \overset{(v)}{D}_i Q^B + \frac{\partial \mathcal{L}''}{\partial(\overset{(h)}{D}_\alpha Q^A)} \{[X_a, X_b]^A_C - [L_a]^c_b [X_c]^A_C\} Q^C H^b_\alpha +$$

$$+ \frac{\partial \mathcal{L}''}{\partial(\overset{(v)}{D}_i Q^A)} \{[X_a, X_b]^A_C - [D_a]^c_b [X_c]^A_C\} Q^C V^b_i = 0,$$

where $[X_a, X_b]$ is the Lie bracket from the Lie algebra of $G$.

We further denote by $C_a{}^c{}_b$ the structure constants of the Lie group $G$, i. e., we have



$$[X_a, X_b] = C_a{}^c{}_b X_c. \tag{1.27}$$

In order to simplify (1.26) we take

$$[L_a]^c_b = [D_a]^c_b = C_a{}^c{}_b, \text{ and by using (1.27) obtain} \tag{1.28}$$

$$\left\{\frac{\partial \mathcal{L}''}{\partial Q^A} Q^B + \frac{\partial \mathcal{L}''}{\partial(\overset{(h)}{D}_\alpha Q^A)} \overset{(h)}{D}_\alpha Q^B + \frac{\partial \mathcal{L}''}{\partial(\overset{(v)}{D}_i Q^A)} \overset{(v)}{D}_i Q^B\right\} [X_a]^A_B = 0. \tag{1.29}$$

Now, it is important to note that by (1.28) the local gauge transformations of gauge fields $H^a_\alpha(x, v)$ and $V^a_i(x, v)$ become

$$\delta^*(H^a_\alpha) = \varepsilon^b(x, v) C_b{}^a{}_c H^c_\alpha(x, v) + \frac{\delta \varepsilon^a}{\delta x^\alpha}(x, v), \tag{1.30}$$

and respectively,

$$\delta^*(V^a_i) = \varepsilon^b(x, v) C_b{}^a{}_c V^c_i(x, v) + \frac{\partial \varepsilon^a}{\partial v_i}(x, v). \tag{1.31}$$

On the other hand, by using (1.12) — (1.14), (1.18), (1.19), (1.30) and (1.31) we obtain for gauge covariant derivatives $\overset{(h)}{D}_\alpha Q^A$, $\overset{(v)}{D}_i Q^A$ the same gauge transformations as for $Q^A$. More precisely, we get

$$\delta^*(\overset{(h)}{D}_\alpha Q^A) = \varepsilon^a(x, v) [X_a]^A_B \overset{(h)}{D}_\alpha Q^B, \text{ and} \tag{1.32}$$

$$\delta^*(\overset{(v)}{D}_i Q^A) = \varepsilon^a(x, v) [X_a]^A_B \overset{(v)}{D}_i Q^B. \tag{1.33}$$

Since the Lagrangian (1.5) is supposed to be invariant with respect to the global action of $G$ on $Q^A(x, v)$ we conclude that the Lagrangian

$$\mathcal{L}_0'(x, v) = \mathcal{L}(Q^A(x, v), \overset{(h)}{D}_\alpha Q^A(x, v), \overset{(v)}{D}_i Q^A(x, v)), \tag{1.34}$$

where $\mathcal{L}$ is the function in (1.5) is invariant with respect to all local gauge transformations (1.2),(1.3) and (1.4). Hence we proved in this section that in order to get a local gauge invariant Lagrangian $\mathcal{L}_0'(x, v)$ from a global gauge invariant Lagrangian $\mathcal{L}_0(x, v)$ we just replace $\dfrac{\delta Q^A}{\delta x^\alpha}$ and $\dfrac{\partial Q^A}{\partial v^i}\left(\text{which do not enter into } \dfrac{\delta Q^A}{\delta x^\alpha}\right)$ by $\overset{(h)}{D}_\alpha Q^A$ and respectively $\overset{(v)}{D}_i Q^A$.

**2. The Lagrangians for gauge fields on a vector bundle.** In the present section we look for Lagrangians of gauge fields which are invariant with respect to the local gauge action of the Lie group $G$. First, by using (1.30) and (1.31) we derive

$$\delta^*\left(\frac{\delta H^a_\alpha}{\delta x^\beta}\right) = \frac{\delta \varepsilon^b}{\delta x^\beta} C_b{}^a{}_c H^c_\alpha + \varepsilon_b C_b{}^a{}_c \frac{\delta H^c_\alpha}{\delta x_\beta} + \frac{\delta}{\delta x^\beta}\left(\frac{\delta \varepsilon^a}{\delta x^\alpha}\right), \tag{2.1}$$

$$\delta^*\left(\frac{\delta V^a_i}{\delta x^\alpha}\right) = \frac{\delta \varepsilon^b}{\delta x^\alpha} C_b{}^a{}_c V^c_i + \varepsilon^b C_b{}^a{}_c \frac{\delta V^c_i}{\delta x^\alpha} + \frac{\delta}{\delta x^\alpha}\left(\frac{\partial \varepsilon^a}{\partial v^i}\right), \tag{2.2}$$

$$\delta^*\left(\frac{\partial H^a_\alpha}{\partial v^i}\right)=\frac{\partial \varepsilon^b}{\partial v^i}C_b{}^a{}_c H^c_\alpha+\varepsilon^b C_b{}^a{}_c\frac{\partial H^c_\alpha}{\partial v^i}+\frac{\partial}{\partial v^i}\left(\frac{\delta \varepsilon^a}{\delta x^\alpha}\right). \tag{2.3}$$

$$\delta^*\left(\frac{\partial V^a_i}{\partial v^j}\right)=\frac{\partial \varepsilon^b}{\partial v^j}C_b{}^a{}_c V^c_i+\varepsilon^b C_b{}^a{}_c\frac{\partial V^c_i}{\partial v^j}+\frac{\partial}{\partial v^j}\left(\frac{\partial \varepsilon^a}{\partial v^i}\right). \tag{2.4}$$

Since we have two kinds of gauge fields we look for Lagrangians of the following form

$$L_0=L\left(H^a_\alpha,\ V^a_i,\ \frac{\delta H^a_\alpha}{\delta x^\beta},\ \frac{\delta V^a_i}{\delta x^\alpha},\ \frac{\partial H^a_\alpha}{\partial v^i},\ \frac{\partial V^a_i}{\partial v^j},\ g_{\alpha\beta},\ g_{ij}\right) \tag{2.5}$$

Thus the condition of local gauge invariance of $L_0$ is given by

$$\delta^*(L_0)=\frac{\partial L}{\partial H^a_\alpha}\delta^*(H^a_\alpha)+\frac{\partial L}{\partial V^a_i}\delta^*(V^a_i)+\frac{\partial L}{\partial\left(\frac{\delta H^a_\alpha}{\delta x^\beta}\right)}\delta^*\left(\frac{\delta H^a_\alpha}{\delta x^\beta}\right)+ \tag{2.6}$$

$$+\frac{\partial L}{\partial\left(\frac{\delta V^a_i}{\delta x^\alpha}\right)}\delta^*\left(\frac{\delta V^a_i}{\delta x^\alpha}\right)+\frac{\partial L}{\partial\left(\frac{\partial H^a_\alpha}{\partial v^i}\right)}\delta^*\left(\frac{\partial H^a_\alpha}{\partial v^i}\right)+\frac{\partial L}{\partial\left(\frac{\partial V^a_i}{\partial v^j}\right)}\delta^*\left(\frac{\partial V^a_i}{\partial v^j}\right)=0.$$

We have to remark that (2.6) expresses the local gauge invariance of $L_0$ on the domain of a coordinates neighborhood of $E$. Hence, before we go further we should prove the invariance of (2.6) with respect to the relativistic transformations (1.1). To this end, by using (1.7) and (1.8) we first obtain

$$\frac{\delta H^a_\alpha}{\delta x^\gamma}=L^\beta_\alpha L^\mu_\gamma\frac{\delta \tilde H^a_\beta}{\delta \tilde x^\mu}, \tag{2.7}$$

$$\frac{\delta V^a_i}{\delta x^\alpha}=\left(\frac{\partial A^j_i}{\partial \tilde x^\gamma}\tilde V^a_j+A^j_i(x)\frac{\delta \tilde V^a_j}{\delta \tilde x^\gamma}\right)L^\gamma_\alpha, \tag{2.8}$$

$$\frac{\partial H^a_\alpha}{\partial v^i}=L^\gamma_\alpha A^j_i(x)\frac{\partial \tilde H^a_\gamma}{\partial \tilde v^j}, \tag{2.9}$$

$$\frac{\partial V^a_i}{\partial v^j}=A^k_i(x)A^h_j(x)\frac{\partial \tilde V^a_k}{\partial \tilde v^h}. \tag{2.10}$$

Then by using (2.7) — (2.10), (1.7) and (1.8) we easy infer

$$\frac{\partial L}{\partial \tilde H^a_\gamma}=\frac{\partial L}{\partial H^a_\alpha}L^\gamma_\alpha, \tag{2.11}$$

$$\frac{\partial L}{\partial \tilde V^a_j}=\frac{\partial L}{\partial V^a_i}A^j_i(x)+\frac{\partial L}{\partial\left(\frac{\delta V^a_i}{\partial x^\alpha}\right)}\frac{\partial A^j_i}{\partial \tilde x^\beta}L^\beta_\alpha, \tag{2.12}$$

$$\frac{\partial L}{\partial\left(\frac{\delta \tilde H^a_\beta}{\delta \tilde x^\mu}\right)}=\frac{\partial L}{\partial\left(\frac{\delta H^a_\alpha}{\delta x^\gamma}\right)}L^\beta_\alpha L^\mu_\gamma, \tag{2.13}$$



$$\frac{\partial L}{\partial\left(\frac{\delta \tilde V^a_j}{\delta \tilde x^\beta}\right)}=\frac{\partial L}{\partial\left(\frac{\delta V^a_i}{\delta x^\alpha}\right)}A^j_i(x)L^\beta_\alpha, \tag{2.14}$$

$$\frac{\partial L}{\partial\left(\frac{\partial \tilde H^a_\gamma}{\partial \tilde v^j}\right)}=\frac{\partial L}{\partial\left(\frac{\partial H^a_\alpha}{\partial v^i}\right)}L^\gamma_\alpha A^j_i(x),\ \text{and} \tag{2.15}$$

$$\frac{\partial L}{\partial\left(\frac{\partial \tilde V^a_k}{\partial \tilde v^h}\right)}=\frac{\partial L}{\partial\left(\frac{\partial V^a_i}{\partial v^j}\right)}A^k_i(x)A^h_j(x). \tag{2.16}$$

On the other hand, by using again (1.7), (1.8) and (2.7) — (2.10) we get

$$\delta^*\left(\frac{\delta H^a_\alpha}{\delta x^\gamma}\right)=\delta^*\left(\frac{\delta \tilde H^a_\beta}{\delta \tilde x^\mu}\right)L^\beta_\alpha L^\mu_\gamma, \tag{2.17}$$

$$\delta^*\left(\frac{\delta V^a_i}{\delta x^\alpha}\right)=\left(\frac{\partial A^j_i}{\partial \tilde x^\beta}\delta^*(\tilde V^a_j)+A^j_i(x)\delta^*\left(\frac{\delta \tilde V^a_j}{\delta \tilde x^\beta}\right)\right)L^\beta_\alpha, \tag{2.18}$$

$$\delta^*\left(\frac{\partial H^a_\alpha}{\partial v^i}\right)=L^\beta_\alpha A^j_i(x)\delta^*\left(\frac{\partial \tilde H^a_\beta}{\partial \tilde v^j}\right), \tag{2.19}$$

$$\delta^*\left(\frac{\partial V^a_i}{\partial v^j}\right)=A^k_i(x)A^h_j(x)\,\delta^*\left(\frac{\partial \tilde V^a_k}{\partial \tilde v^h}\right), \tag{2.20}$$

$$\delta^*(H^a_\alpha)=L^\beta_\alpha\delta^*(\tilde H^a_\beta),\ \text{and} \tag{2.21}$$

$$\delta^*(V^a_i)=A^j_i(x)\delta^*(\tilde V^a_j). \tag{2.22}$$

Further, taking account of (2.7) — (2.22) we obtain

$$\frac{\partial L}{\partial H^a_\alpha}\delta^*(H^a_\alpha)+\frac{\partial L}{\partial\left(\frac{\delta H^a_\alpha}{\delta x^\beta}\right)}\delta^*\left(\frac{\delta H^a_\alpha}{\delta x^\beta}\right)=\frac{\partial L}{\partial \tilde H^a_\gamma}\delta^*(\tilde H^a_\gamma)+\frac{\partial L}{\partial\left(\frac{\delta \tilde H^a_\gamma}{\delta \tilde x^\mu}\right)}\delta^*\left(\frac{\delta \tilde H^a_\gamma}{\delta \tilde x^\mu}\right), \tag{2.23}$$

$$\frac{\partial L}{\partial V^a_i}\delta^*(V^a_i)+\frac{\partial L}{\partial\left(\frac{\delta V^a_i}{\delta x^\alpha}\right)}\delta^*\left(\frac{\delta V^a_i}{\delta x^\alpha}\right)=\frac{\partial L}{\partial \tilde V^a_j}\delta^*(\tilde V^a_j)+\frac{\partial L}{\partial\left(\frac{\delta \tilde V^a_j}{\delta \tilde x^\gamma}\right)}\delta^*\left(\frac{\delta \tilde V^a_j}{\delta \tilde x^\gamma}\right), \tag{2.24}$$

$$\frac{\partial L}{\partial\left(\frac{\partial H^a_\alpha}{\partial v^i}\right)}\delta^*\left(\frac{\partial H^a_\alpha}{\partial v^i}\right)=\frac{\partial L}{\partial\left(\frac{\partial \tilde H^a_\gamma}{\partial \tilde v^j}\right)}\delta^*\left(\frac{\partial \tilde H^a_\gamma}{\partial \tilde v^j}\right), \tag{2.25}$$

$$\frac{\partial L}{\partial\left(\frac{\partial V^a_i}{\partial v^j}\right)}\delta^*\left(\frac{\partial V^a_i}{\partial v^j}\right)=\frac{\partial L}{\partial\left(\frac{\partial \tilde V^a_k}{\partial \tilde v^h}\right)}\delta^*\left(\frac{\partial \tilde V^a_k}{\partial \tilde v^h}\right). \tag{2.26}$$

Thus due to (2.23) — (2.26) we have the invariance of the condition (2.6) with respect to the relativistic transformations (1.1) on E.

Then by using (1.40) and (1.41) of [2] we obtain

$$
\begin{aligned}
\frac{\partial L}{\partial\left(\frac{\delta H^a_\alpha}{\delta x^\beta}\right)}\frac{\delta}{\delta x^\beta}\left(\frac{\delta \varepsilon^a}{\delta x^\alpha}\right) &= \frac{1}{2}R^i_{\beta\alpha}\frac{\partial\varepsilon^a}{\partial v^i}\frac{\partial L}{\partial\left(\frac{\delta H^a_\alpha}{\delta x^\beta}\right)}+\\
&+\frac{1}{2}\left\{\frac{\partial L^a}{\partial\left(\frac{\delta H^a_\alpha}{\delta x^\beta}\right)}+\frac{\partial L}{\partial\left(\frac{\delta H^a_\beta}{\delta x^\alpha}\right)}\right\}\frac{\delta}{\delta x^\beta}\left(\frac{\delta\varepsilon^a}{\delta x^\alpha}\right), \text{ and}
\end{aligned}
\tag{2.27}
$$

$$
\begin{aligned}
&\frac{\partial L}{\partial\left(\frac{\partial H^a_\alpha}{\partial v_i}\right)}\frac{\partial}{\partial v^i}\left(\frac{\delta\varepsilon^a}{\delta x^\alpha}\right)+\frac{\partial L}{\partial\left(\frac{\delta V^a_i}{\delta x^\alpha}\right)}\frac{\delta}{\delta x^\alpha}\left(\frac{\partial\varepsilon^a}{\partial v^i}\right)=\\
&=\left\{\frac{\partial L}{\partial\left(\frac{\partial H^a_\alpha}{\partial v^i}\right)}+\frac{\partial L}{\partial\left(\frac{\delta V^a_i}{\delta x^\alpha}\right)}\right\}\frac{\partial}{\partial v^i}\left(\frac{\delta\varepsilon^a}{\delta x^\alpha}\right)+\frac{\partial N^j_\alpha}{\partial v^i}\frac{\partial\varepsilon^a}{\partial v^j}\frac{\partial L}{\partial\left(\frac{\delta V^a_i}{\delta x^\alpha}\right)}.
\end{aligned}
\tag{2.28}
$$

We now replace $\delta^*(H^a_\alpha)$, $\delta^*(V^a_i)$, $\delta^*\left(\frac{\delta H^a_\alpha}{\delta x^\beta}\right)$, $\delta^*\left(\frac{\delta V^a_i}{\delta x^\alpha}\right)$, $\delta^*\left(\frac{\partial H^a_\alpha}{\partial v^i}\right)$ $\delta^*\left(\frac{\partial V^a_i}{\partial v^j}\right)$ from (1.30), (1.31), (2.1) — (2.4) and then by using (2.27), (2.28) and the arbitrariness of functions $\varepsilon^a(x, v)$ we obtain that $L_0$ is invariant to the local gauge action of $G$ if and only if the function $L$ from (2.5) is a solution of equations

$$
\begin{aligned}
&C_b{}^a{}_c\left\{\frac{\partial L}{\partial H^a_\alpha}H^c_\alpha+\frac{\partial L}{\partial V^a_i}V^c_i+\frac{\partial L}{\partial\left(\frac{\delta H^a_\alpha}{\delta x^\beta}\right)}\frac{\delta H^c_\alpha}{\delta x^\beta}+\right.\\
&\left.+\frac{\partial L}{\partial\left(\frac{\delta V^a_i}{\delta x^\alpha}\right)}\frac{\delta V^c_i}{\delta x^\alpha}+\frac{\partial L}{\partial\left(\frac{\partial H^a_\alpha}{\partial v^i}\right)}\frac{\partial H^c_\alpha}{\partial v^i}+\frac{\partial L}{\partial\left(\frac{\partial V^a_i}{\partial v^j}\right)}\frac{\partial V^c_i}{\partial v^j}\right\}=0,
\end{aligned}
\tag{2.29}
$$

$$
\frac{\partial L}{\partial H^a_\alpha}+\frac{\partial L}{\partial\left(\frac{\delta H^b_\gamma}{\delta x^\alpha}\right)}C_a{}^b{}_c H^c_\gamma+\frac{\partial L}{\partial\left(\frac{\delta V^b_i}{\delta x^\alpha}\right)}C_a{}^b{}_c V^c_i=0,
\tag{2.30}
$$

$$
\begin{aligned}
&\frac{\partial L}{\partial V^a_i}+\frac{\partial L}{\partial\left(\frac{\partial H^b_\alpha}{\partial v^i}\right)}C_a{}^b{}_c H^c_\alpha+\frac{\partial L}{\partial\left(\frac{\partial V^b_j}{\partial v^i}\right)}C_a{}^b{}_c V^c_j+\\
&+\frac{1}{2}\frac{\partial L}{\partial\left(\frac{\delta H^a_\alpha}{\delta x^\beta}\right)}R^i_{\beta\alpha}+\frac{\partial L}{\partial\left(\frac{\delta V^a_j}{\delta x^\alpha}\right)}\frac{\partial N^i_\alpha}{\partial v^j}=0,
\end{aligned}
\tag{2.31}
$$



$$
\frac{\partial L}{\partial\left(\frac{\delta H^a_\alpha}{\delta x^\beta}\right)}+\frac{\partial L}{\partial\left(\frac{\delta H^a_\beta}{\delta x^\alpha}\right)}=0,
\tag{2.32}
$$

$$
\frac{\partial L}{\partial\left(\frac{\partial H^a_\alpha}{\partial v^i}\right)}+\frac{\partial L}{\partial\left(\frac{\delta V^a_i}{\delta x^\alpha}\right)}=0, \text{ and}
\tag{2.33}
$$

$$
\frac{\partial L}{\partial\left(\frac{\partial V^a_i}{\partial v^j}\right)}+\frac{\partial L}{\partial\left(\frac{\partial V^a_j}{\partial v^i}\right)}=0.
\tag{2.34}
$$

From (2.32) — (2.34) it follows that the four derivatives $\frac{\delta H^a_\alpha}{\delta x^\beta}$, $\frac{\partial H^a_\alpha}{\partial v^i}$, $\frac{\delta V^a_i}{\delta x^\alpha}$ and $\frac{\partial V^a_i}{\partial v^j}$ enter into the Lagrangian we look for, only by means of $A^a_{\alpha\beta}$, $B^a_{\alpha i}$ and $K^a_{ij}$ given by

$$
A^a_{\alpha\beta}=\frac{\delta H^a_\alpha}{\delta x^\beta}-\frac{\delta H^a_\beta}{\delta x^\alpha},
\tag{2.35}
$$

$$
B^a_{\alpha i}=\frac{\partial H^a_\alpha}{\partial v^i}-\frac{\delta V^a_i}{\delta x^\alpha}, \text{ and respectively}
\tag{2.36}
$$

$$
K^a_{ij}=\frac{\partial V^a_i}{\partial v^j}-\frac{\partial V^a_j}{\partial v^i}.
\tag{2.37}
$$

Now, by using the above functions, the structure constants of $G$, the gauge fields and the non — linear connection $HE=(N^i_\alpha(x, v))$ on $E$ we define the following differentiable functions

$$
R^a_{\alpha\beta}=A^a_{\alpha\beta}-C_b{}^a{}_c H^c_\alpha H^b_\beta+R^i_{\alpha\beta}V^a_i,
\tag{2.38}
$$

$$
P^a_{\alpha i}=B^a_{\alpha i}-C_b{}^a{}_c H^c_\alpha V^b_i+\frac{\partial N^j_\alpha}{\partial v^i}V^a_j, \text{ and}
\tag{2.39}
$$

$$
S^a_{ij}=K^a_{ij}-C_b{}^a{}_c V^c_i V^b_j.
\tag{2.40}
$$

In order to get the behaviour of $R^a_{\alpha\beta}$, $P^a_{\alpha i}$, $S^a_{ij}$ with respect to the relativistic transformations (1.1) we first note that (2.7) — (2.10) imply

$$
A^a_{\alpha\beta}=L^\mu_\alpha L^\nu_\beta \tilde{A}^a_{\mu\nu},
\tag{2.41}
$$

$$
B^a_{\alpha i}=L^\beta_\alpha A^j_i(x)\tilde{B}^a_{\beta j}-\frac{\partial A^j_i}{\partial x^\alpha}V^a_j, \text{ and}
\tag{2.42}
$$

$$
K^a_{ij}=A^k_i(x)A^h_j(x)\tilde{K}^a_{kh}.
\tag{2.43}
$$

Then taking account of (1.16) and (1.24) from [2] and of (1.7), (1.8) and (2.41) — (2.43) from the present paper we obtain that $R^a_{\alpha\beta}$, $P^a_{\alpha i}$ and $S^a_{ij}$ are Finsler tensor fields of type $\binom{0\ 0}{2\ 0}$, $\binom{0\ 0}{1\ 1}$ and respectively $\binom{0\ 0}{0\ 2}$ with respect to the lower indices. Moreover, $R^a_{\alpha\beta}$ and $S^a_{ij}$ are anti-symmetric Finsler tensor fields. With respect to the upper index „$a$" we shall prove in the next paper that all these Finsler tensor fields satisfy some homogeneous gauge transformations given by the adjoint representation of the Lie group $G$.

We next consider the Lagrangian $L_0$ as a differentiable function

$$L_0 = L'(R^a_{\alpha\beta},\ P^a_{\alpha i},\ S^a_{ij},\ \gamma_{\alpha\beta},\ g_{ij}). \tag{2.44}$$

It is easy to verify that $L_0$ given by (2.44) is satisfying equations (2.32), (2.33) and (2.34). Moreover, this Lagrangian satisfies (2.30) and (2.31). In order to justify the last assertion by means of (2.35) — (2.40) we derive

$$\frac{\partial L'}{\partial\left(\dfrac{\delta H^a_\alpha}{\delta x^\beta}\right)} = 2\frac{\partial L'}{\partial(R^a_{\alpha\beta})}, \tag{2.45}$$

$$\frac{\partial L'}{\partial\left(\dfrac{\delta V^a_i}{\delta x^\alpha}\right)} = -\frac{\partial L'}{\partial(P^a_{\alpha i})}, \tag{2.46}$$

$$\frac{\partial L'}{\partial\left(\dfrac{\partial H^a_\alpha}{\partial v^i}\right)} = \frac{\partial L'}{\partial(P^a_{\alpha i})}, \tag{2.47}$$

$$\frac{\partial L'}{\partial\left(\dfrac{\partial V^a_i}{\partial v^j}\right)} = 2\frac{\partial L'}{\partial(S^a_{ij})}, \tag{2.48}$$

$$\frac{\partial L'}{\partial H^a_\alpha} = C_a{}^b{}_c\left\{2\frac{\partial L'}{\partial(R^b_{\alpha\beta})}H^c_\beta + \frac{\partial L'}{\partial(P^b_{\alpha i})}V^c_i\right\},\quad \text{and} \tag{2.49}$$

$$\frac{\partial L'}{\partial V^a_i} = \frac{\partial L'}{\partial(R^a_{\alpha\beta})}R^i_{\alpha\beta} + \frac{\partial L'}{\partial(P^b_{\alpha i})}C_c{}^b{}_a H^c_\alpha + \frac{\partial L'}{\partial(P^a_{\alpha j})}\frac{\partial N^i_\alpha}{\partial v^j} + 2\frac{\partial L'}{\partial(S^b_{ij})}C_a{}^b{}_c V^c_j. \tag{2.50}$$

Taking account of (2.45) — (2.50) we easy obtain that $L_0$ given by (2.44) satisfies (2.30) and (2.31). Thus all Lagrangians given by (2.44) are satisfying (2.30) — (2.34).

We are now concerned with equation (2.29). First, by using (2.45) — (2.50) in (2.29) we obtain

$$\begin{aligned}&\left\{\frac{\partial L'}{\partial(R^e_{\alpha\beta})}\left(2C_a{}^e{}_d C_b{}^a{}_c H^c_\alpha H^d_\beta + C_b{}^e{}_c V^c_i R^i_{\alpha\beta} + 2C_b{}^e{}_c\frac{\delta H^c_\alpha}{\delta x^\beta}\right)\right\} + \\ &+\left\{\frac{\partial L'}{\delta(P^e_{\alpha i})}\left(C_b{}^a{}_c C_a{}^e{}_d V^d_i H^c_\alpha + C_b{}^a{}_c C_d{}^e{}_a H^d_\alpha V^c_i + C_b{}^e{}_c\frac{\partial N^j_\alpha}{\partial v^i}V^c_j - C_b{}^e{}_c\frac{\delta V^c_i}{\delta x^\alpha} + \right.\right.\\ &\left.\left.+ C_b{}^e{}_c\frac{\partial H^c_\alpha}{\partial v^i}\right)\right\} + \left\{2\frac{\partial L'}{\partial(S^e_{ij})}\left(C_b{}^a{}_c C_a{}^e{}_d V^d_j V^c_i + C_b{}^e{}_c\frac{\partial V^c_i}{\partial v^j}\right)\right\} = 0.\end{aligned} \tag{2.51}$$

Denote by $\Sigma_1$ the expression in (2.51) which lies inside the first two brackets $\{,\}$. Then by using the Jacoby identity for structure constants

$$C_a{}^d{}_b C_c{}^b{}_e + C_c{}^d{}_b C_e{}^b{}_a + C_e{}^d{}_b C_a{}^b{}_c = 0,\quad \text{we get} \tag{2.52}$$



$$\begin{aligned}\Sigma_1 &= \frac{\partial L'}{\partial(R^e_{\alpha\beta})}\left(C_b{}^e{}_c A^c_{\alpha\beta} + (C_c{}^e{}_a C_b{}^a{}_d + C_d{}^e{}_a C_c{}^a{}_b)H^c_\alpha H^d_\beta + C_b{}^e{}_c R^i_{\alpha\beta}V^c_i\right) = \\ &= C_b{}^e{}_c\frac{\partial L'}{\partial(R^e_{\alpha\beta})}R^c_{\alpha\beta}.\end{aligned} \tag{2.53}$$

In a similar way, the expression $\Sigma_3$ which lies between the last two brackets in (2.51) becomes

$$\Sigma_3 = C_b{}^e{}_c\frac{\partial L'}{\partial(S^e_{ij})}\cdot S^c_{ij}. \tag{2.54}$$

Finally, by using (2.36), (2.39) and (2.52) the remaining expression $\Sigma_2$ in (2.51) becomes

$$\begin{aligned}\Sigma_2 &= \frac{\partial L'}{\partial(P^e_{\alpha i})}\left(C_b{}^e{}_c B^c_{\alpha i} + (C_d{}^e{}_c C_a{}^c{}_b + C_a{}^e{}_c C_b{}^c{}_d)H^a_\alpha V^d_i + C_b{}^e{}_c\frac{\partial N^j_\alpha}{\partial v^i}V^c_j\right) = \\ &= C_b{}^e{}_c\frac{\partial L'}{\partial(P^e_{\alpha i})}P^c_{\alpha i}.\end{aligned} \tag{2.55}$$

Therefore, due to (2.53) — (2.55) we obtain for (2.51) the following nice form

$$C_b{}^e{}_c\left\{\frac{\partial L'}{\partial(R^e_{\alpha\beta})}R^c_{\alpha\beta} + \frac{\partial L'}{\partial(P^e_{\alpha i})}P^c_{\alpha i} + \frac{\partial L'_i}{\partial(S^e_{ij})}S^c_{ij}\right\} = 0. \tag{2.56}$$

We now suppose $G$ is a semi-simple Lie group. Thux the Killing form $h = (h_{ab})$ given by

$$h_{ab} = C_a{}^d{}_c C_b{}^c{}_d. \tag{2.57}$$

is non-degenerate on the Lie algebra of $G$. We have to remark that the class of semi-simple Lie groups is of special interest in physics.

Next, we consider the pseudo-Riemannian metrics $\gamma_{\alpha\beta}(x, v)$ and $g_{ij}(x, v)$ on the vector bundles $HE$ and respectively $VE$. Then we define the following differentiable functions :

$$L'_H = -\frac{1}{4}h_{ab}\gamma^{\alpha\beta}(x, v)\gamma^{\varepsilon\gamma}(x, v)R^a_{\alpha\gamma}(x, v)R^b_{\beta\varepsilon}(x, v), \tag{2.58}$$

$$L'_{HV} = -\frac{1}{2}h_{ab}\gamma^{\alpha\beta}(x, v)g^{ij}(x, v)P^a_{\alpha i}(x, v)P^b_{\beta j}(x, v),\quad \text{and} \tag{2.59}$$

$$L'_V = -\frac{1}{4}h_{ab}g^{hk}(x, v)g^{ij}(x, v)S^a_{hi}(x, v)S^b_{kj}(x, v). \tag{2.60}$$

First we easy can see that $L'_H$, $L'_{HV}$ and $L'_V$ are in fact invariant with respect to the relativistic transformations (1.1) Thus they define globally three scalar fields on $E$. Then we claim that $L'_H$, $L'_{HV}$ and $L'_V$ satisfy the condition (2.6). In fact, they are satisfying (2.30) —(2.34) since the derivatives of gauge fields appear only by means of $R^a_{\alpha\beta}$, $P^a_{\alpha i}$ and $S^a_{ij}$. Therefore, the only condition we have to verify is (2.29). We do this only for $L'_H$. The calculations for $L'_{HV}$ and $L'_V$ are analogous to those ones which follow.

First, by (2.58) we have

$$\frac{\partial L'_H}{\partial(R^e_{\alpha\beta})} = -h_{ea}\gamma^{\alpha\gamma}(x, v)\gamma^{\beta\mu}(x, v)R^a_{\gamma\mu}(x, v), \text{ and} \tag{2.61}$$

$$\frac{\partial L'_H}{\partial(P^e_{\alpha i})} = 0; \quad \frac{\partial L'_H}{\partial(S^e_{ij})} = 0. \tag{2,62}$$

Then taking account of (2.52), (2.57) and (2.61) we obtain

$$C_b{}^e{}_c \frac{\partial L'_H}{\partial(R^e_{\alpha\beta})} R^c_{\alpha\beta} = -C^f_{ed}C_a{}^d{}_f C_b{}^e{}_c \gamma^{\alpha\gamma}\gamma^{\beta\mu} R^a_{\gamma\mu} R^c_{\alpha\beta}. \tag{2.63}$$

We have to remark that in case of semi-simple Lie groups the structure constants are totally anti-symmetric, i.e., we have

$$C_a{}^c{}_b = -C_b{}^c{}_a = -C_a{}^b{}_c. \tag{2.64}$$

Then by using (2.52) and (2.64) we get

$$C_b{}^e{}_c \frac{\partial L'_H}{\partial(R^e_{\alpha\beta})} R^c_{\alpha\beta} = 0. \tag{2.65}$$

Hence due to (2.62) and (2.65) we see that $L'_H$ satisfies (2.56) which is equivalent with (2.29). Therefore, $L'_H$ is a Lagrangian for gauge fields $H^a_\alpha$ and $V^a_i$ which is local gauge invariant with respect to the action of $G$. The same assertion holds true for $L'_{HV}$ and $L'_V$.

In case of an abelian Lie group ($C_b{}^a{}_c = 0$) we see that (2.56) is identically satisfied by any Lagrangian for gauge fields. Thus any Lagrangian given by (2.44) is a local gauge invariant Lagrangian. However, in case of Lie group $U(1)$ which is involved in the electromagnetic field theory we define some Lagrangians as in (2.58) — (2.60).

First, since $U(1)$ is 1 — dimensional we denote by $H_\alpha(x, v)$ and $V_i(x, v)$ the horizontal and respectively vertical gauge fields. Then (2.38), (2.39) and (2.40) become

$$R_{\alpha\beta} = \frac{\delta H_\alpha}{\delta x^\beta} - \frac{\delta H_\beta}{\delta x^\alpha} + R^i_{\alpha\beta} V_i, \tag{2.66}$$

$$P_{\alpha i} = \frac{\partial H_\alpha}{\partial v^i} - \frac{\delta V_i}{\partial x^\alpha} + \frac{\partial N^j_\alpha}{\partial v^i} V_j, \text{ and respectively} \tag{2.67}$$

$$S_{ij} = \frac{\partial V_i}{\partial v^j} - \frac{\partial V_j}{\partial v^i}. \tag{2.68}$$

We now consider the following three Lagrangians for gauge fields:

$$L'_H = -\frac{1}{4}\gamma^{\alpha\beta}(x, v)\gamma^{\gamma\varepsilon}(x, v)R_{\alpha\gamma}(x, v)R_{\beta\varepsilon}(x, v), \tag{2.69}$$



$$L'_{HV} = -\frac{1}{2}\gamma^{\alpha\beta}(x, v)g^{ij}(x, v)P_{\alpha i}(x, v)P_{\beta j}(x, v), \text{ and} \tag{2.70}$$

$$L'_V = -\frac{1}{4}g^{hk}(x, v)g^{ij}(x, v)S_{hi}(x, v)S_{kj}(x, v). \tag{2.71}$$

Clearly, all these Lagrangians are local gauge invariant Lagrangians.

As in the classical gauge theory, the Finsler tensor fields given by (2.38) — (2.40) are called the *strength fields* with respect to the gauge fields $H^a_\alpha(x, v)$ and $v^a_i(x, v)$ More precisely, $R^a_{\alpha\beta}$, $P^a_{\alpha i}$ and $S^a_{ij}$ are called the *horizontal, mixed* and respectively *vertical strength fields*. The local action of $G$ on strength fields in going to be studied in the third paper of this series.

The full Lagrangian for the physical fields $Q^A(x, v)$ is the sum of a Lagrangian for gauge fields and a local gauge invariant Lagrangian. Therefore, summing up the results we obtained in the present paper we conclude that we have the following three full Lagrangians:

$$\mathscr{L}_H(x, v) = \mathscr{L}'_0(x, v) + L'_H(x, v), \tag{2.72}$$

$$\mathscr{L}_{HV}(x, v) = \mathscr{L}'_0(x, v) + L'_{HV}(x, v) \text{ and} \tag{2.73}$$

$$\mathscr{L}_V(x, v) = \mathscr{L}'_0(x, v) + L'_V(x, v), \tag{2.74}$$

where $\mathscr{L}'_0(x, v)$ is given by (1.34) and $L'_H$, $L'_{HV}$ and $L'_V$ are given by (2.58) — (2.60) or by (2.69) — (2.71). Moreover, according to the elementary theory of P.D.E. we see that any Lagrangian given by

$$\mathscr{L}(x, v) = \mathscr{L}'_0(x, v) + L'_H(x, v) + L'_{HV}(x, v) + L'_V(x, v), \text{ is a full} \tag{2.75}$$

Lagrangian for matter fields $Q^A(x, v)$.

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