Corollary 4.2. *Let L be a weak super SMP-space with the Egoroff property, $f \in L^+$, and $g_k \downarrow 0$ in L. Then there exists an increasing sequence P_m of band projections on L such that $P_m f \uparrow f$ and $P_m g_k \downarrow_k 0$ f-uniformly for every $m \in \mathbf{N}$.*

The space c of all real convergent sequences provides an example of a Riesz space without the principal projection property and satisfying the requirements of Corollary 4.2.

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ON THE THERMOELASTICITY OF MATERIALS WITH VOIDS

BY

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1. Introduction. Recently [1], Batra has proposed a principle of virtual work for the nonlinear theory of thermoelasticity. An account of the research on formulations of principle of virtual work for continuum mechanics can be found in [2–4].

The purpose of the present paper is to extend the result established by Batra in [1] to the case of nonlinear theory of thermoelastic materials with voids. In this theory, the bulk density is written as the product of two fields, the matrix material density field and the volume fraction field (cf. [5.6]). This representation introduces an additional degree of kinematic freedom. The intended applications of the theory of elastic materials with voids are to geological materials like rock and soils and to manufactured porous materials (ceramics and pressed powders). The nonlinear theory of thermoelastic materials with voids has been studied in various paper (see [5], [7–9]).

In section 2 we give a resume of the basic equations of the nonlinear theory of thermoelastic materials with voids and some preparatory results. The next section is devoted to a formulation of the principle of virtual work. From it are derived the equations of motion, the balance of equilibrated force, the constitutive equations for the temperature and the boundary conditions for stress and equilibrated stress.

2. Basic equations. Preliminaries. We consider a body $\mathcal{B}$ which occupies a closed, bounded, regular region of Euclidean three-dimensional space in a fixed reference configuration B_R. The motion of $\mathcal{B}$ is given by a smooth, differentiable map $\mathcal{X}$ in $B_R \times (0, t_0)$,

$$\mathbf{x} = \mathcal{X}(\mathbf{X}, t). \tag{2.1}$$

We denote by B_t the current configuration of the body $\mathcal{B}$.

We suppose that the map $\mathcal{X}$ is invertible with a smooth, differentiable inverse on $B_R \times (0, t_0)$. Let $\mathbf{F} = \nabla_X \mathcal{X}$ be the deformation gradient. Then

$$\det \mathbf{F} > 0. \tag{2.2}$$

The principle of balance of mass can be expressed in the form

(2.3) $$\rho_R = \rho \det\mathbf{F}.$$

where ρ_R is the mass density in the configuration B_R and ρ is the mass density at the present time t.

The concept of a distributed body asserts that the mass density ρ has the decomposition (cf.[5–6])

(2.4) $$\rho = \nu\gamma,$$

where γ is the density of the matrix material and ν is the volume fraction field. In the reference configuration B_R we have

(2.5) $$\rho_R = \nu_R\gamma_R.$$

We define the gradient of the volume fraction field by

(2.6) $$\mathbf{G} = \nabla_{\mathbf{X}}\nu.$$

We denote by Θ the absolute temperature and assume that it is always positive.

The equations of motion of the continua can be written in the form

(2.7) $$\mathrm{div}\mathbf{T} + \rho\mathbf{b} = \rho\ddot{\mathbf{x}},$$

where $\mathbf{T}$ is the Cauchy stress, $\mathbf{b}$ is the body force per unit mass and a superposed dot denotes the material time derivative.

The local form of the balance of the equilibrated forces is (cf. [5])

(2.8) $$\mathrm{div}\mathbf{H} + g + \rho\ell = \rho k\ddot{\nu},$$

where $\mathbf{H}$ is the Cauchy equilibrated stress, g is the intrinsic equilibrated body force, ℓ is the extrinsic equilibrated body force, and k is the equilibrated inertia.

A thermoelastic material with voids is characterized by the following constitutive equations which hold at each point $\mathbf{X}$ and for all time

(2.9) $$\begin{gathered} e = e(\mathbf{F},\eta,\nu,\mathbf{G}), \quad \mathbf{T}^T = \rho\mathbf{F}(\partial_{\mathbf{F}}e)^T, \quad \Theta = \partial_\eta e, \quad \mathbf{H} = \rho(\partial_{\mathbf{g}}e)\mathbf{F}^T \\ g = -\rho\partial_\nu e, \mathbf{q} = \mathbf{q}(\mathbf{F},\eta,\nu,\mathbf{G},\mathbf{g}) \end{gathered}$$

where e is the internal energy per unit mass, η is the entropy per unit mass, $\mathbf{q}$ is the heat-flux vector and $\mathbf{g} = \Delta_{\mathbf{X}}\Theta$.

The local form of the energy balance for thermoelastic materials with voids reduces to

(2.10) $$\rho\Theta\dot{\eta} = -\ \mathrm{div}\ \mathbf{q} + \rho r,$$

where r is the specific heat supply.

Thus, the basic equations of the theory are: the equation of motion (2.7), the balance of equilibrated force (2.8), the energy equations (2.10) and the constitutive equation (2.9). To these equations we must adjoin the boundary conditions and the initial conditions.

We assume that the boundary conditions have the form

(2.11) $$\mathbf{x} = \hat{\mathbf{x}} \quad \text{on } S_1 \times (0,t_0); \qquad \mathbf{t} = \hat{\mathbf{t}} \quad \text{on } S_2 \times (0,t_0),$$

(2.12) $$\nu = \hat{\nu} \quad \text{on } S_3 \times (0,t_0); \qquad h = \hat{h} \quad \text{on } S_4 \times (0,t_0),$$

(2.13) $$\vartheta = \hat{\vartheta} \quad \text{on } S_5 \times (0,t_0); \qquad \mathbf{q} = \hat{\mathbf{q}} \quad \text{on } S_6 \times (0,t_0),$$

where $\hat{\mathbf{x}}$, $\hat{\mathbf{t}}$, $\hat{\nu}$, $\hat{\mathbf{h}}$, $\hat{\vartheta}$ and $\hat{\mathbf{q}}$ are prescribed functions and $S_k (k = 1,2,\ldots,6)$ are subsets of ∂B_t so that

$$\overline{S}_1 \cup S_2 = \overline{S}_3 \cup S_4 = \overline{S}_5 \cup S_6 = \partial\mathbf{B}_t, \qquad S_1 \cap S_2 = S_3 \cap S_4 = S_5 \cap S_6 = \phi.$$

It follows from (2.7) and (2.9) that the equations of motion become

$$\mathrm{div}\left[\rho(\partial_{\mathbf{F}}e)\mathbf{F}^T\right] + \rho\mathbf{b} = \rho\ddot{\mathbf{x}}.$$

Similarly, the equation (2.8) can be written in the form

$$\mathrm{div}\left[\rho(\partial_{\mathbf{G}}e)\mathbf{F}^T\right] - \rho\partial_\nu e + \rho\ell = \rho k\ddot{\nu}.$$

The boundary conditions $(2.11)_2$, $(2.12)_2$ reduces to

(2.16) $$\rho(\partial_{\mathbf{F}}e)\mathbf{F}^T\mathbf{n} = \hat{\mathbf{t}} \quad \text{on } S_2 \times (0,t_0),$$

and

$$\rho(\partial_{\mathbf{G}} e)\mathbf{F}^T \cdot \mathbf{n} = \widehat{h} \quad \text{on } S_4 \times (0, t_0), \tag{2.17}$$

respectively.

A comparison motion for $\mathcal{B}$ is defined (cf.[1])

$$\mathbf{x}^* = \chi(\mathbf{X}, t) + \varepsilon \overline{\mathbf{x}}(\mathbf{X}, t) = \chi^*(X, t), \qquad (\mathbf{X}, t) \in \mathbf{B}_R \times (0, t_0). \tag{2.18}$$

At each time and for each value of the scalar parameter ε, χ^* maps B_R onto a configuration $B_t^* = B_t^*(\varepsilon)$. Cleary, the deformation gradieent of the comparison motion is

$$\mathbf{F}^* = \nabla_{\mathbf{X}} \chi^* = \mathbf{F} + \varepsilon \nabla_{\mathbf{X}} \overline{\mathbf{x}}. \tag{2.19}$$

The comparison fields for the specific entropy, volume distribution fraction and temperature are defined on $B_t \times (0, t_0) \times R$, respectvely, by

$$\eta^* = \eta + \varepsilon \overline{\eta}, \quad \nu^* = \nu + \varepsilon \overline{\nu}, \quad \vartheta^* = \vartheta + \varepsilon \overline{\vartheta}, \tag{2.20}$$

where $\overline{\eta}$, $\overline{\nu}$ and $\overline{\vartheta}$ are arbitrary smooth fields over $B_t \times (0, t_0)$. The gradient of the comparison volume fraction is defined by

$$\mathbf{G}^* = \nabla_{\mathbf{X}} \nu^* = \mathbf{G} + \varepsilon \overline{\mathbf{G}}$$

where $\overline{\mathbf{G}} = \nabla_{\mathbf{X}} \overline{\nu}$.

3. The principle of virtual work. In this section we extend the principle of virtual work proposed by Batra [1] for the classical thermoelasticity to the nonlinear theory of thermoelastic materials with voids.

The process $(\mathbf{x}, \nu, \eta)$ is said to be admissible if the fields $\mathbf{x}, \nu$ and η are of class C^2 on $B_t \times (0, t_0)$ and of class C^1 on $B_R \times (0, t_0)$.

An admissible process $(\mathbf{x}, \nu, \eta)$ is said to be kinematically admissible if $\mathbf{x}, \nu$ and the corresponding comparison fields $\mathbf{x}^*, \nu^*$ satisfy the boundary conditions $(2.11)_1$ and $(2.12)_1$.

We introduce the notations

$$V(B_t) = \int_{B_t} \rho e \, dv \tag{3.1}$$

and

(3.2)

$$\overline{\delta} W(B_t) = \int_{B_t} \rho \left[(\mathbf{b} - \ddot{\mathbf{x}}) \cdot \overline{\mathbf{x}} + \vartheta \overline{\nu} \right] dv + \int_{B_t} \rho \ell - \mathbf{k} \ddot{\nu}) \overline{\nu} dv + \int_{S_2} \widehat{\mathbf{t}} \cdot \overline{\mathbf{x}} da + \int_{S_4} \widehat{h} \overline{\nu} da.$$

The principle of virtual work for themoelastic materials with voids is expressed in the form : *the deformation of the thermoelastic materials with voids is such as to satisfy the condition*

$$\delta V(B_t) = \overline{\delta} W(B_t), \tag{3.3}$$

for all kinematicall admissible process.

We use the notation

$$\delta V(B_t) = \lim_{\varepsilon \to 0} \frac{d}{d\varepsilon} V(B_t^*). \tag{3.4}$$

Let us study the implication of the above principle. By (3.1), as in [1], we obtain

(3.5)

$$\delta V(B_t) = \int_{B_t} \{ \rho(\partial_\nu e) \overline{\eta} - \overline{\mathbf{x}} \cdot \operatorname{div} \left[\rho(\partial_{\mathbf{F}} e) \mathbf{F}^T \right] + \rho(\partial_\nu e) \overline{\nu} -$$
$$- \operatorname{div} \left[\rho(\partial_{\mathbf{G}} e) \mathbf{F}^T \right] \overline{\nu} \} \, dv + \int_{S_2} \overline{\mathbf{x}} \cdot \left[\rho(\partial_{\mathbf{F}} e) \mathbf{F}^T \right] \mathbf{n} da + \int_{S_4} \overline{\nu} \left[\rho(\partial_{\mathbf{G}} e) \mathbf{F}^T \right] \cdot \mathbf{n} da.$$

It follows from (3.2) ,(3.3) and (3.5) that

(3.6)

$$\int_{B_t} \{ \overline{\mathbf{x}} \left[-\rho \ddot{\mathbf{x}} + \rho \mathbf{b} + \operatorname{div} \left(\rho(\partial_{\mathbf{F}} e) \mathbf{F}^T \right) \right] - \overline{\eta} (\rho \vartheta - \rho \delta_\eta e) +$$
$$\left[\rho \ell - \rho k \ddot{\nu} + \operatorname{div} \left(\rho(\partial_{\mathbf{G}} e) \mathbf{F}^T \right) - \rho \partial_\nu e \right] \overline{\nu} \} \, dv + \int_{S_2} \left[\widehat{\mathbf{t}} - \rho(\partial_{\mathbf{F}} e) \mathbf{F}^T \mathbf{n} \right] \cdot \overline{\mathbf{x}} da +$$
$$+ \int_{S_4} \left[\widehat{h} - \rho(\partial_{\mathbf{G}} e) \mathbf{F}^T \cdot \mathbf{n} \right] \overline{\nu} da = 0,$$

for any $\overline{\mathbf{x}}$, and $\overline{\nu}$ that vanish on S_1 and S_3, respectively, and for any $\overline{\nu}$. Thus, from (3.6), we obtain

$$\rho \ddot{\mathbf{x}} = \rho \mathbf{b} + \operatorname{div} \left(\rho(\partial_{\mathbf{F}} e) \mathbf{F}^T \right), \tag{3.7}$$

$$\partial k \ddot{\nu} = \rho \ell + \operatorname{div} \left(\rho(\partial_{\mathbf{G}} e) \mathbf{F}^T - \rho \partial_\nu e, \right) \tag{3.8}$$

$$\vartheta = \partial_\nu e, \tag{3.9}$$

$$\rho(\partial_{\mathbf{F}} e) \mathbf{F}^T \mathbf{n} = \widehat{\mathbf{t}} \quad \text{on } S_2 \times (0, t_0), \tag{3.10}$$

$$(3.11) \qquad \rho(\partial_{\mathbf{G}} e)\mathbf{F}^t \cdot \mathbf{n} = \widehat{h} \quad \text{on } S_4 \times (0, t_0).$$

We conclude that the principle of virtual work (3.3) is equivalent to the equations of motion, the balance of equilibrated force and the constitutive equations for the temperature.

Remark 1. If we postulate a form of the principle of virtual work that holds over an arbitrary part of the body B_t, then the principle of virtual work is equivalent to the equations of motions, the balance of equilibrated force, the constitutive equation for the temperature and the Cauchy principle for the stress and equilibrated stress(see Batra [1]).

Remark 2. We can also use the principle of virtual work in the case where ϑ is considered independent constitutive variable. Thus assume that $e = e(\mathbf{F}, \vartheta, \nu, \mathbf{G})$ and introduce the specific free-energy Ψ by $\Psi = e - \vartheta\eta$.

We have

$$(3.12) \qquad \overline{\eta} = (\partial_{\mathbf{F}}\eta) \cdot \overline{\mathbf{F}} + (\partial_\vartheta \eta)\overline{\vartheta} + (\partial_\nu \eta)\overline{\nu} + (\partial_{\mathbf{G}}\eta) \cdot \overline{\mathbf{G}}.$$

Repeating the previous argument, we obtain

$$(3.13) \quad \delta V(B_t) = \int_{B_t} -\overline{\mathbf{x}} \cdot \operatorname{div}\left[\rho(\partial_{\mathbf{F}}\Psi + \vartheta\partial_{\mathbf{F}}\eta)\mathbf{F}^T\right] dv + \int_{B_t} \rho(\partial_\vartheta \Psi + \eta + \vartheta\partial_\vartheta \eta)\overline{\vartheta} dv +$$

$$+ \int_{B_t} \rho(\partial_\nu \Psi + \vartheta\partial_\nu \eta)\overline{\nu} dv - \int_{B_t} \operatorname{div}\left[\rho(\partial_{\mathbf{G}}\Psi + \vartheta\partial_{\mathbf{G}}\eta)\mathbf{F}^T\right]\overline{\nu} dv +$$

$$+ \int_{S_2} \overline{\mathbf{x}} \cdot \rho(\partial_{\mathbf{F}}\Psi + \vartheta\partial_{\mathbf{F}}\nu)\mathbf{F}^T \mathbf{n} da + \int_{S_4} \rho(\partial_{\mathbf{G}}\Psi + \vartheta\partial_{\mathbf{G}}\eta)\mathbf{F}^T \cdot \mathbf{n}\overline{\nu} da$$

and

$$(3.14) \quad \overline{\delta} W(B_t) = \int_{B_t} \rho(\mathbf{b} - \ddot{\mathbf{x}}) \cdot \overline{\mathbf{x}} dv + \int_{B_t} \rho(\ell - k\ddot{\nu})\overline{\nu} dv + \int_{B_t} \rho\vartheta(\partial_\vartheta \eta)\overline{\vartheta} dv -$$

$$- \int_{B_t} \overline{\mathbf{x}} \cdot \operatorname{div}\left[\rho\vartheta(\partial_{\mathbf{F}}\eta)\mathbf{F}^T\right] dv + \int_{B_t} \rho\vartheta(\partial_\nu \eta)\overline{\nu} dv - \int_{B_t} \overline{\nu} \operatorname{div}\left[\rho\vartheta(\partial \mathbf{G}\eta)\mathbf{F}^T\right] dv +$$

$$+ \int_{S_2} \overline{\mathbf{x}} \cdot \rho\vartheta(\partial_{\mathbf{F}}\eta)\mathbf{F}^T \mathbf{n} da + \int_{S_2} \widehat{\mathbf{t}} \cdot \overline{\mathbf{x}} da + \int_{S_4} \widehat{h}\overline{\nu} da + \int_{S_4} \rho\vartheta(\partial_{\mathbf{G}}\eta)\mathbf{F}^T \cdot \mathbf{n}\overline{\nu} da.$$

From the statement of the principle of virtual work given above, we obtain the equation of motion, the balance of equilibrated, force, the boundary conditions for the stress and the equilibrated stress and the constitutive equation

$$\eta = -\partial_\vartheta \Psi.$$

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