Conversely, we have

Theorem 5.4. *If M is a C^ω-manifold and M^{*n} has a positive definite fundamental tensor $g^{ij}(x,p)$ and H^{2n} has an N-almost Hermitian connection ∇ without curvature, then in every point $x \in M^{*n}$ we can find an open set $U_x \subset M^{*n}, x \in U_x$ and there exist n orthonormal d-covector fields on $M^{*n}|_{U_x}$.*

Proof. Let us consider the system of partial differential equations $\nabla_X Z = 0$. This system is completely integrable by virtue of the fact that the curvature R of ∇ is zero. That means, there exists an open set U_x in every point $x \in M^{*n}$ and exist $2n$ linearly independent vector fields $Z_1, \ldots, Z_{2n}$ on $\pi^{*-1}(U_x) \subset T^*M$. We can decompose the vectors $Z_1, \ldots, Z_{2n}$ to the horizontal and vertical parts in the form: $Z_1 = Z_1^H + Z_V^1, \ldots, Z_{2n} = Z_{2n}^H + Z_V^{2n}$. We can take on $\pi^{*-1}(U_x)$ n linearly independent vertical vector fields from the vertical vector fields $\{Z_V^1, \ldots, Z_V^{2n}\}$. If we orthonormalize the above n vertical vectors by means of the fundamental tensor g^{ij}, then they determine n orthonormal d-covector fields on $M^{*n}|_{U_x}$.

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Department of Mathematics
National Defense Academy
Yokosuka 239, Japan

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A VARIATIONAL CHARACTERIZATION OF THE EUCLIDEAN n-SPHERE

BY

GIOVANNI ROTONDARO

Let M be a closed orientable C^∞ n-manifold and $f : M \to E^{n+1}$ be a C^∞ immersion into Euclidean $(n+1)$-space, such that the origin O of coordinates does not lie in $f(M)$. Set

$$\tau_M(f) = \int_M |f|^{-n} dV,$$

where $|\ |$ is the Euclidean norm, and dV is the volume element of the induced Riemannian metric on M. If $I(M, E^{n+1})$ is the set of all such immersions, then τ_M is a real functional on this set.

Denote by ν the Gauss normal field of f, and consider the normal deformation

$$\overline{f} = f + t\varphi\nu,$$

where $\varphi : M \to R$ is a C^∞ real function, and the real parameter t varies in a small neighbourhood of zero. Following the conventions of [2], w denote by δ the operator $(\partial/\partial t)_{t=0}$, and call f a critical point of τ_M if

$$\delta\tau_M(f) = 0$$

for every choise of φ.

Theorem. *The functional τ_M has a critical point if and only if M is diffeomorphic to an Euclidean sphere. In fact, every such critical immersion is an embedding of M as an Euclidean sphere with centre 0.*

Proof. We have $\delta\tau_M(f) = \int_M (\delta |f|^{-n} dV + |f|^{-n} \delta dV)$. Setting $p = -f \cdot \nu$, since $|\overline{f}|^2 + |f|^2 + \varphi^2 t^2 - 2pt$, we obtain $\delta |f|^{-n} = np\varphi |f|^{-n-2}$. By a calculation very similar to those carried out in [2, p.127], we have $\delta dV = -n\varphi H dV$, where H is the first mean curvature, i.e. the arithmetic mean of the principal curvatures. Then

$$\delta\tau_M(f) = n \int_M \varphi |f|^{-n-2}(p - H|f|^2) dV.$$

By a standard principle of calculus of variations, $\delta\tau_M(f) = 0$ for all φ if and only if $p - H|f|^2 = 0$. We claim that this implies $|f| =$ constant. If we accept for the moment this fact, then the proof is achieved. In fact, if $|f|$ is a constant, then M is an immersed ovaloid. Therefore, by Hadamard's theorem on ovaloids [1, p.121], f must be a diffeomorphism onto the n-sphere of E^{n+1} with centre O and radius $|f|$.

To prove the claim, denote by Δ and Δ_1 respectively the Laplacian and the first Beltrami differntial parameter on M. If $f = (f^1, f^2, \dots, f^n)$, then $\Delta(f^i)^2 = = 2f^i\Delta f^i + 2\Delta_1(f^i)$. Exploiting the well-known identity $\Delta f = nh$ [1, p.198] it follows, by a standard computation, that $\Delta\,|\,f\,|^2 = 2n(1 - Hp)$. Now, if $p - H\,|\,f\,|^2 = 0$, then $Hp \leq 1$, and hence $\Delta\,|\,f\,|^2 \geq 0$. Therefore, by Bochner's lemma [2, p.195], $|\,f\,|$ must be constant.

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Dipartamento di Matematica e Applicationi "R. Caccioppoli"
Università degli Studi di Napoli
Via Mezzocannone, 8
80134 Napoli — Italia

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HOMOLOGY OF $P(w_0, w_1, w_2)$

BY

LOUIS FUNAR

Let $w = (w_0, \dots, w_n)$ be a set of positive integers and denote by S the polynomial ring $\mathbf{C}[x_0, \dots, x_n]$ graded by $\deg x_i = w_i \quad i = \overline{0, n}$. Then the projective variety $\operatorname{Proj}(S) = P(w)$ is called the weighted projective space of specified weights. For general w, $P(w)$ is a singular space. Its singularities are quotient ones, whose germs are isomorphic to $(\mathbf{C}^n/H, 0)$ where $H \subset \mathrm{GL}(n, C)$ is a small abelian group, and whose corresponding links are generalized lens spaces [2, 3, 4]. It is known that the rational cohomology groups of $P(w)$ and $\mathbf{C}P^n$ agree [6] as graded abelian groups.

Our main result is that $H_\cdot(P(w))$ is torsion free for $n = 2$. This seems to have some significance when we to folow the lines developed in[1] in studying the quasi not-homogeneous singularities. We shall give a cell decomposition for $P(w)$ in same manner as Dold [5] has done for lens spaces. Then an elementary combinatorial computation will answer our question.

We restrict our attention to the case $n = 2$ and remember the an alternative way to describe $P(w)$ is as a quotient of CP^2 under the action of $G = Z/W_0Z \times \dots \times Z/W_2Z$

$$(k_0, k_1, k_2)(z_0, z_1, z_2) = (\xi_{w_0}^{k_0}, \xi_{w_1}^{k_1}, \xi_{w_2}^{k_2}),$$

where ξ_j is a primitive root of unity of order j. Now consider the Hopf fibration

$$\begin{tikzcd} S^1 \arrow[r] & S^5 \arrow[d] \arrow[dr, "\pi"] & \\ & CP^2 \arrow[r] & P(W) \end{tikzcd}$$

and set π for the composition of the canonical projections. Our aim is to give a G - equivariant cell decomposition of $\mathbf{C}P^2$. We shall define a decomposition of S^5 satisfying

i) the decomposition is G-equivariant;
ii) any set of the decomposition is stable with respect to the action of S^1 coming from the Hopf fibration;