17. S i n g a l, M.K. and M a t h u r, A s h a – *On nearly-compact spaces*, Boll. Un. Mat. Ital. 4 (1969), 702-710.

18. T h o m p s o n, T. – *S-closed spaces*, Proc. Amer. Math. Soc. 60 (1976), 335-338.

19. V e l i č k o, N. – *H-closed topological spaces*, Amer. Math. Soc. Transl. 78 (1968), 103-118.

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NEW RESULTS ON THE OPERATOR DIFFERENTIAL EQUATIONS

BY

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1.Some notions and notations. The class C of continuous complex valued functions of a non-negative real variable forms a commutative algebra without zero divisors where the product is defined as the finite convolution $fg = \int_0^t f(t-\tau)g(\tau)d\tau$, the sum and scalar product are defined in the usual way. The quotient field of this algebra is the operator field M of M i k u s i n s k i [1]. The elements of M are $\frac{f}{g}$ *the convolution quotient*, where f and g are in C, $g \neq 0$. Some special elements from M : s; the differential operator, $\ell = \{1\} = \frac{1}{s}$; the integral operator and 1; the unit element. In M we have an equivalence relation: $\frac{f}{g} = \frac{h}{k} \iff fk = gh$ (in C), so an element $\frac{f}{g}$ in M is a representative of an equivalence class. We denote by f of $\{f(t)\}$ the representation of $f(t) \in C_{[0,\infty)}$ while $x(\lambda) = q\{x(\lambda,t)\}$ stands for operation function, where $q \in M$ and $\{x(\lambda,t)\}$ is a parametric function. $M(\lambda)$ is the set of all mappings of $0 \leq \lambda < \infty$ into M. Finally $D(T) : 0 \leq \lambda \leq \Lambda < \infty$, $0 \leq t \leq T < \infty$ is the domain R^2. After this introduction we define some subspaces which are needed for later developments.

2.Subspaces $J(D), C_s(\lambda), \bar{C}_s(\lambda), A_q, A_s$ and A_ε.

$J(D)$: is defined as the set of all complex-valued functions $f(\lambda,t)$, such that (i) for every $T < \infty$, $f(\lambda,t)$ is defined, bounded and measurable over $D(T)$, (ii) for every $\lambda \in [0,\Lambda]$, $f(\lambda,t) \in C$. $J(\lambda)$ is the algebra which lies in $M(\lambda)$, its elements are of the form $f(\lambda) = q\{f(\lambda,t)\}$, $q \in M$ and $f(\lambda,t) \in J(D)$.

$C_s(\lambda)$: is a commutative algebra which lies in $M(\lambda)$, its elements are of the form $f(\lambda) = s^\beta\{f(\lambda,t)\}$, $\beta > 0$ and $f(\lambda,t) \in J(D)$ [5].

$\bar{C}_s(\lambda)$: is a vector space that contains $C_s(\lambda)$ [5].

$K(D)$: is a subset of $J(D)$ and its elements are of the form of the finite sum $\Sigma_\nu A_\nu(t)e^{i\sigma_\nu(t)\lambda}$ where $\sigma_\nu(t)$ is +ve real function and $A_\nu(t), \sigma_\nu(t)$ are in C.

K: is the class of all numerical bounded functions of the form of the finite sum $\Sigma_\nu A_\nu(t)e^{i\nu\lambda}$ where A_ν is a numerical constant and λ in $[0, \Lambda]$. Now we can define our spaces. By A_q we denote the set of all members of the form $b + q\, g(\lambda)$, where b is a numerical constant, $q \in M$ and $g(\lambda) \in K$. For $q = s^{-\beta} = \left\{\frac{t^{\beta-1}}{\Gamma(\beta)}\right\}$, $\beta > 0$ and t in $[0, T]$, we denote the set $A_q(\lambda)$ by $A_s(\lambda)$. It is easy to see that $A_s \subset C_s$. By $A_\varepsilon(\lambda)$ we denote the set of all elements of the form $a + \varepsilon\, f(\lambda)$ where a is a constant operator, $f(\lambda) = p\{f(\lambda, t)\}$, $p \in M$, $f(\lambda, t) \in K(D)$ and ε is sufficiently small, consequently $A_\varepsilon \subset J(\lambda)$.

In the following section we introduce some operations in $J(\lambda)$.

3.Limit, derivative and integral in $J(\lambda)$.

Definition 1. *The function $f(\lambda)$ has a limit $g \in M$ at the point $\lambda = \lambda_0$ if and only if $f(\lambda)$ can be written in the form $f(\lambda) = q\{f(\lambda, t)\}$ where $q \in M$, $\{f(\lambda, t)\} \in J(D)$ and $g = q\{\lim_{\lambda \to \lambda_0} f(\lambda, t)\}$ such that $\{\lim_{\lambda \to \lambda_0} f(\lambda, t)\}$ is in C (in the usual sense).*

Definition 2. *An element $f(\lambda) \in J(\lambda)$ has its derivative in $J(\lambda)$ if and only if $f(\lambda)$ can be written in the form $q\{f(\lambda, t)\}$ where $\{f(\lambda, t)\}$ has $\{f'_\lambda(\lambda, t)\}$ belonging to $J(D)$ and $\frac{df}{d\lambda} = q\{f'_\lambda(\lambda, t)\}$.*

Definition 3. *For every $f(\lambda) \in J(\lambda)$, $\int_0^\lambda f(\lambda)d\lambda = q\{\int_0^\lambda f(\lambda, t)d\lambda\}$, where $q \in M$ and $f(\lambda, t) \in J(D)$.*

Proposition 1. *The defined limit (derivative and integral) does not depend on the representative choosen from the class $f(\lambda)$ i.e. $f(\lambda)$ can be represented in infinitely many ways but the definition of limit (derivative and integral) is unique.*

Proof. The proof of the unicity of the derivative and integral of the function $f(\lambda)$ in $J(\lambda)$ is formally the same as in [1], one has only to take care of the changed class of functions. We shall investigate only the unicity of the defined limit. Let a_i and b be functions of class C such that $q_i = \frac{a_i}{b}$, $f(\lambda) = q_i\{f_i(\lambda, t)\} = q_j\{f_j(\lambda, t)\}$, $i \neq j$, $\{f_i(\lambda, t)\} \in J(D)$ and $\{\lim_{\lambda \to \lambda_0} f_i(\lambda, t)\} = \{h_i(t)\}$, where $\{h_i(t)\} \in C$ and $i = 1, 2, \ldots$, then

$$\{a_i(t)\}\{f_i(\lambda, t)\} = \{a_j(t)\}\{f_j(\lambda, t)\}$$

i.e. $\{\int_0^t a_i(t-\tau)f_i(\lambda, \tau)d\tau\} = \{\int_0^t a_j(t-\tau)f_j(\lambda, \tau)d\tau\}$ in $J(D)$. Thus, for $\lambda \to \lambda_0$ we have $\{\int_0^t a_i(t-\tau)h_i(\tau)d\tau\} = \{\int_0^t a_j(t-\tau)h_j(\tau)d\tau\}$ in $J(D)$, i.e.

$$a_i h_i = a_j h_j.$$

Dividing this equality by b, we have $q_i h_i = q_j h_j$, which proves that there can be only one limit.

4.Binary relations and operator of absolute value in $J(D)$. In $J(D)$ we shall define $f \leq g$ if and only if f and g are real and $f(\lambda, t) \leq g(\lambda, t)$ for every (λ, t) in $D(T)$.

Definition 4. *By $|f|$ we denote the mapping $J(D) : f \to |f| = \{|f(\lambda, t)|\}$ which is called the operator of absolute value.*

The properties of this operator are given in [2].

Remarks.

(i) The equality $|fg| = |f||g|$ generally does not hold. For if that were the case then we should have for $f = \ell |\int_0^t g(\lambda, \tau)d\tau| = \int_0^t |g(\lambda, t)|d\tau$, which need not be true at all.

(ii) It will be observed that if $g \in C$, then the inverse $\frac{1}{g}$ cannot be a function $\in C$ [1], but it is an operator in M and so if f and g are in C such that $\frac{f}{g} \in M$ and $\frac{f}{g} \notin C$ then the expression $|\frac{f}{g}|$, does not make sense in M.

Definition 5. *In $J(\lambda)$, $f(\lambda)$ approximates $g(\lambda)$ with factor $q \in M$ and measure ε_T, if $f - g = q\{u(\lambda, t)\}$, where $\{u(\lambda, t)\} \in J(D)$ and $\{|u(\lambda, t)|\} \leq \varepsilon_T \ell$.*

After these definitions we are in a position to consider some aspects of the differential equations in the Mikusinski field.

5.Linear differential equations with variable coefficients in $M(\lambda)$. We shall consider the differential equations

$$\frac{d^n}{d\lambda^n}x(\lambda) + a_{n-1}(\lambda)\frac{d^{n-1}}{d\lambda^{n-1}}x(\lambda) + \ldots + a_1\frac{d}{d\lambda}x(\lambda) + a_0(\lambda)x(\lambda) = 0 \tag{1}$$

where $x^{(k)}(\lambda)$ are in $M(\lambda)$, $k = 0, 1, \ldots, n$.

Proposition 2. *If $X \subset M(\lambda)$ is the set of all solutions of equation (1) then X is a linear space over M.*

The proof is formally the same as by M i k u s i n s k i [1], one has only to take care of the changed class of functions.

Given the constant operators $a_k^0 \in M$ and $f_k \in M(\lambda)$, with $a_k(\lambda) = a_k^0 + f_k(\lambda)$, $k = 0, 1, \ldots, n-1$ we can interpret equation (1) in the form of nonhomogeneous differential equation

$$\left(\frac{d^n}{d\lambda^n} + a_{n-1}^0\frac{d^{n-1}}{d\lambda^{n-1}} + \ldots + a_1^0\frac{d}{d\lambda} + a_0^0\right)x(\lambda) = -\sum_{k=0}^{n-1} f_k(\lambda)x^{(k)}(\lambda) \tag{2}$$

or

$$F\left(\frac{d}{d\lambda}\right)x(\lambda)=f(\lambda) \tag{3}$$

where $F\left(\frac{d}{d\lambda}\right)=\left(\frac{d^n}{d\lambda^n}+a_{n-1}^0\frac{d}{d\lambda^{n-1}}+\ldots+a_1^0\frac{d}{d\lambda}+a_0^0\right)$ and

$$f(\lambda)=-\sum_{k=0}^{n-1}f_k(\lambda)x^{(k)}(\lambda). \tag{4}$$

First we shall deal with the homogeneous equation

$$F\left(\frac{d}{d\lambda}\right)x(\lambda)=\left(\frac{d}{d\lambda^n}+a_{n-1}^0\frac{d}{d\lambda^{n-1}}+\ldots+a_1^0\frac{d}{d\lambda}+a_0^0\right)x(\lambda)=0 \tag{5}$$

for which the corresponding characteristic equation

$$F(r)=r^n+a_{n-1}^0r^{n-1}+\ldots+a_1^0r+a_0^0=0. \tag{6}$$

Definition 6. *The polynomial $F(r)$, $r\in M$ is said irreducible if it cannot be decomposed into the product of two polynomials of degree ≥ 1.*

For instance, the equation $q^2=\{t\sin\ln t\}$, $q\in M$ has no solution in M [1], this example shows that M is not algebraically closed.

Definition 7. *Equation (5) is said to be logarithmic if it has n linearly independent solutions (logarithmic solutions).*

Definition 8. *Equation (5) is said to be miscellaneous (Mixed) if the number of logarithmic solutions is less than n.*

Definition 9. *Equation (5) is called pure if it has no solution.*

6.Stability of the solution of equation (1) with coefficients in $A_\varepsilon(\lambda)$. When $\lambda\to\infty$. Here we are interested in the case when equation (5) has n logarithmic solutions of the form

$$x_k=q_k\{\exp z_k(t)\},\ k=1,2,\ldots,n \tag{7}$$

where $q_k\in M$, $z_k(t)=-\alpha_k(t)+i\beta_k(t)$, $\alpha_k(t)>0$, $\beta_k(t)$ and $\alpha_k(t)$ are bounded real functions in C. Thus by means of notations $A_\varepsilon(\lambda)$ and (7), we can verify that the general solution of equation (3) is given by

$$x(\lambda)=q\{x(\lambda,t)\}=\sum_{k=1}^{n}c_kq_k\left\{e^{z_k(t)\lambda}\right\}+$$
$$+\frac{\varepsilon pq}{v}\int_0^\lambda\{v_k(t)\}\left\{e^{z_k(t)(\lambda-u)}\right\}\{f(u,t)\}du$$

where $v=v(z_1,z_2,\ldots,z_n)\neq 0$ is Vandermonde determinant of order n ($v\in C\Rightarrow\frac{1}{v}\notin C$ but $\frac{1}{v}\in M$ [1]), $v_k(t)$ is v after the k^{th} column is replaced by $(0,0,\ldots,1)$, $x(\lambda)=q\{x(\lambda,t)\}$, $\{x(\lambda,t)\}\in J(\lambda)$, c_k $(k=1,2,\ldots,n)$ are arbitrary constant operators, p,q are in M and $\{f(\lambda,t)\}=-\sum_{k=1}^{n}\{f_k(\lambda,t)\}\{x^{(k)}(\lambda,t)\}$ where $f_k\in K(D)$.

Remark. According to the Definitions 1, 4 and Remark (ii), we cannot apply the theory of stability for the expression (8) directly by using the ideas of the well-known methods of classical analysis which depend on the absolute value and the limits. Thus we shall modify the solution given in (8) to a form which is suitable to the applications of stability theory according to our given definitions and notations in the sense of the space of operators.

Let us remark that all the products of the functions of class C which are given in our problems are in the sense of the defined convolutions. Since c_kq_k, $(k=1,2,\ldots,n)$ and $\frac{pq}{v}$ are in M then we can write $c_kq_k=\frac{h_k}{h}$ and $\frac{pq}{v}=\frac{g_1}{g}$ where h_k,h,g_1 and $g(t)$ are all in C. Let $\frac{1}{gh}=q_1\in M$, $h_kg=\{\gamma_k(t)\}$ and $g_1hv_k=\{\delta_k(t)\}$. By means of these notations we can modify equation (8) to the form

$$\begin{aligned}q\{x(\lambda,t)\}&=q_1\sum_{k=1}^{n}[\{\gamma_k(t)\}\left\{e^{z_k(t)\lambda}\right\}+\\&+\int_0^\lambda\{\delta_k(t)\}\left\{e^{z_k(t)(\lambda-u)}\right\}\{f(u,t)\}du],\\\{x(\lambda,t)\}&=\frac{q_1}{q}\sum_{k=1}^{n}[\{\gamma_k(t)\}\left\{e^{z_k(t)\lambda}\right\}+\\&+\int_0^\lambda\{\delta_k(t)\}\left\{e^{z_k(t)(\lambda-u)}\right\}\{f(u,t)\}du].\end{aligned} \tag{9}$$

Since $x(\lambda)$ as in (8) is a solution of the homogeneous equation (1) with coefficients in $A_\varepsilon(\lambda)$, then according to Propositions 1 and 2, its product by an arbitrary constant operator (w.r.t.λ) is again a solution of the same equation, thus if $\frac{q_1}{q}\in C$ we can see that the parametric function as in (9) is the suitable form of a solution of equation (1) which agrees with notions and notations of our space. Thus we can employ to it the theory of stability when $\lambda\to\infty$ in the sense of the field M.

But when $\frac{q_1}{q}\notin C$ the following theorem does not applicable, and the problem needs further investigation.

Theorem 1. *The solution (9) of equation (1) with coefficients in $A_\varepsilon(\lambda)$ tends operationally to zero when $\lambda \to \infty$, if $q/q_1 \in C$ provided that equation (5) has n linearly independent solutions of type (7).*

In fact the proof in the case when (5) is a logarithmic equation and its n linearly independent solutions of type (7) is given in [2].

Remark. The property of existence of the solution of equation (1) with coefficients in $A_\varepsilon(\lambda)$ in the case when (5) is pure or mixed and the behaviour of the solution when $\lambda \to \infty$ remain to be investigated.

7.Form of the formal solution of equation (1) with coefficients in $A_q(\lambda)$ and in $A_s(\lambda)$. We first investigated the linear differential equation in M. Among many results let us cite the differential equation with coefficients in $A_q(\lambda)$. According to our consideration, equation (1) can be rewritten in the form of an inhomogeneous equation as in (2) where the coefficients of l.h.s. are numerical constants, and the characteristic equation (6) which corresponds to equation (5) has n roots; denote these roots by r_k, $k = 1, 2, \ldots, n$.

Theorem 2. *The formal solution of equation (1) with coefficients in $A_q(\lambda)$, has the form:*

$$x_k(\lambda) = \left(1 + \sum_{j=1}^{\infty} q^j y_{kj}\right) \exp\left(r_k + \sum_{j=1}^{\infty} q^j \rho_{kj}\right) \tag{10}$$

where r_k is one of the numerical roots of equation (6), ρ_{kj}, $j = 1, 2, \ldots$, are complex number and $y_{jk} \in K$.

Proof. The possibility of the formation of the function y_{kj} and numbers ρ_{kj} can be established formally according to the well-known methods of classical analysis. By substituting $x_k(\lambda)$ in (2) and equating the coefficients of the powers of q with some modifications. We can construct y_{kj} in the form of the finite sum $\sum_{\nu=0} \alpha_\nu \dfrac{d^{i\nu\lambda}}{F'(r_k + i\nu)} \in K$ and ρ_{kj} in the form $\alpha_0/F'(r_k)$, where α_0, α_ν are numerical constants and $F'(r_k), F'(r_k + i\nu)$ are the value of the derivative of the characteristic equation (6) at r_k and $r_k + i\nu$ respectively. We omit the details of the proof which are easy to reconstruct as in the earlier paper [3], one replaces only $s^{-\beta}$ by q.

The question arises whether the given method of the formal solution of equation (1) can be applied when $a_k = a_k^0 + q_k f_k(\lambda)$, $(k = 0, 1, \ldots, n)$, where q_k, a_k^0 are constant operators in M and $f_k(\lambda)$ $(k = 0, 1, \ldots, n)$ are in K or $K(D)$. Let us note that:

1-the given method of the formal solution does not depend on the representative choice of q from the class $K_q(\lambda)$.

2-according to the roots r_k $(k = 0, 1, \ldots, n)$ of equation (5) we have n or less expressions as in (10). Let the expressions (10) with their derivatives exist in $M(\lambda)$ and satisfy equation (1).

The following question arises: *are these expressions linearly independent?* which is outside the scope of the analysis here, and we hope to discuss it later.

8.On the existence of the formal solution. Among many forms of solutions let us cite the form (10) with $q = s^\alpha$ where α is a real number.

i.**Case** $\alpha < 0$.

We have investigated this case the roots of equation (6) are distinct [3] or has any form [5]. Moreover we gave an answer for the open question on the convergence of the formal solution (10) in the operational sense [3].

ii.**Case** $\alpha > 0$.

The expression (10) does not exists whenever the coefficients ρ_{kj} and the functions y_{kj} are sufficiently small because the expressions $\exp\left(s^{j\alpha}\rho_{kj}\right)$, $j = 1, 2, \ldots$ has no meaning in the field M of operators [1].

9.Approximate solutions of equation (1). The equation (1) with coefficients in $A_s(\lambda)$ is considered. With this consideration the coefficients of equation (5) will be numerical constants we investigated the approximate solution x_m of equation (1) which tends operationally to an accurate solution $x(\lambda)$ as in (10), where $q = s^{-\beta}$, $\beta > 0$ and $x_m(\lambda)$ has the same form of $x(\lambda)$ with summation up to m. Taking in our consideration that the ordinary power series $\sum_{j=0}^{\infty} |r_{jk}| z^j$ and $\sum_{j=0}^{\infty} |y_{kj}^{(i)}| z^j$, $(i = 0, 1, \ldots, i \leq n)$ have +ve radius of convergence [3].

Our investigation based on the modification of the solutions such that we can use the ideas of classical analysis to obtain new results in the field of Mikusinski opertors according to the given definitions and notations. We shall give the following propositions which are important for the problem which we had been discussing in [3]. In the same time the situations are illustrated in a way to show the distinction between the problems in the classical case and the problem in the space of operators.

Let $\sum_{j=0}^{\infty} |r_{kj}| z^j$ and $\sum_{j=0}^{\infty} |y_{kj}^{(i)}| z^j$ be as considered above, then each convegent series $\sum_{j=0}^{\infty} r_{kj} s^{-\beta j}$ and $\sum_{j=0}^{\infty} y_{kj}^{(i)} s^{-\beta j}$ can be represented by $r_{k0} + f$ and $y_{k0} + g(\lambda)$ respectively where $f = \{f(t)\} \in C$ and is a parametric function which for a fixed λ is a function in C. It should be observed that in this case the exponential $\exp(f\lambda)$ does exist and has the form $\exp(f\lambda) = 1 + h(\lambda)$

where $h(\lambda) = \{h(\lambda, t)\}$ is a parametric function, and all its derivatives are parametric in $[0, \Lambda]$ [1].

Proposition 4. *Consider $p \in C$, $g(\lambda)$ and f as defined above then the expressions: $px(\lambda) = P[(r_{k0} + f)\exp(y_{k0} + g(\lambda))]$ is parametric functions in D.*

Proof. Since the formulae for the derivative of product (convolution) of the parametric functions $g(\lambda) = \{g(\lambda, t)\}$ and $e^{f\lambda} = 1 + \{h(\lambda, t)\}$ are valid formally as in the usual form and the product (sum) of the parametric functions is again a parametric function [1]; thus with our suppositions we can show that $px(\lambda)$ is a parametric function in D.

We can easily verify the same results of the last proposition to the expression: $px^{(i)}(\lambda)$, $i = 0, 1, 2, \ldots, i \leq n$.

Proposition 5. *Let the numerical function $u(\lambda)$ be the general solution of the ordinary differential equation (5) and $v(\lambda - \eta)$ be the solution of the same equation such that $v^{(i)}(0) = 0$, $i = 0, 1, 2, \ldots, n-2$ and $v^{(n-1)}(0) = 1$, then*

(i) The general solution $x(\lambda)$ of the operational equation (1) as we considered and its derivatives can be represented as

$$x^{(i)}(\lambda) = q\left\{x^{(i)}(\lambda, t)\right\} = u^{(i)}(\lambda) + s^{-\beta} q \int_0^\lambda v^{(i)}(\lambda - \eta)\{f(\eta, t)\}d\eta,$$

$$i = 0, 1, 2, \ldots, n-2,$$

where $q \in M$ and $\left\{x_\lambda^{(i)}(\lambda, t)\right\}$ are parametric functions.

(ii) if $q \in M$ such that (i) holds, then $q^{-1} \in C$.

Proof. We omit the details of the proof of part (i) which are easy to reconstruct as Proposition 3 one has only to take care of the different class of functions. It remains to prove (ii). if (i) holds then

$$\left\{x^{(i)}(\lambda, t) - \int_0^\lambda v^{(i)}(\lambda, \eta) \int_0^t f(\eta, t) \frac{(t-\tau)^{\beta-1}}{\Gamma(\beta)}\right\} = q^{-1} u(\lambda).$$

Since l.h.s. is a parametric function, thus $q^{-1}u(\lambda)$ is also a parametric function, but $u(\lambda)$ is a numerical function of the variable λ only. Hence $q^{-1} \in C$.

Propositions 4 and 5 seem to be easy but they are very important for the field M of operators as we show. According to Definition 4 the absolute value of operator is connected to the function which for fixed λ belongs to the class C, this must be the reason that the relation of expression

$|x^{(i)}(\lambda) - x_m^{(i)}(\lambda)|$ is considered, see relation (8) in [4] and also $|q^{-1}u^{(i)}(\lambda)|$ does not hold if $q^{-1} \notin C$ and all given inequalities in [4] do not make sense because $|q^{-1}|$ has no meaning in M.

REFERENCES

1. M i k u s i n s k i, J. – *Operational Calculus*, Pergamon, 1959.
2. S h a r k a w i, I.E. – *On a solution of the operator differential equation with variable coefficients*, Pure Math. Manuscript, Calcutta, vol.6 (1987), 125-132.
3. S h a r k a w i, I.E. – *The formal solution of Mikusinski operational differential equations with variable coefficients*, An.Şt. Univ. "Al.I.Cuza" Iaşi, Matematica, 36 (1990), 105-110.
4. S h a r k a w i, I.E. – *Approximate solutions of the operator L.D.E. with variable coefficients*, Jour.Inst., Com.Sci. (Math.Ser.) Calcutta, vol.1, No.2, (1988), 111-119.
3. S t a n k o v i c, B. – *The existence and the unicity of solution of a system of operator differential equations*, Publ.de L'Inst. Math. Nouvelle série, Beograd, Tome I (23), (1969), 85-92. Univ. "Al.I.Cuza" Iaşi, Matematica, 36 (1990), 105-110.

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