CORSINI PIERGIULIO - *PROLEGOMENA OF HYPERGROUP THEORY* (Supplement to Rivista di Matematica Pura ed Applicata, Aviani Editore (1993), 215 pages)

The first hyperstructure was that of hypergroup (multigroup), a natural generalzation of groups, introduced by F.Marty in 1934 and independently by H.S.Hall in 1937. Since then the investigation into hypergroup theory knew no respite and it has developped in large sitting of hyperstructures (multialgebras). Despite this developpement of research on the topic there has been no introducting text on it. Corsini's book is intended to fill this gap. It is a detailed survey of the most important results on hypergroup theory and on some other reprsentative hyperstructures. The book is, in fact, a new enlarged edition of the Italian version dated 1986.

The first section (*Introduction*) contains the fundamental definitions and results on hypergroups. The second section (*Homomorphisms*) deals with homomorphisms of various types and correlated equivalences. The results of the third section (*Complete Parts*) are presented in the context of general theory of hypergroups and also in connection of the heart's structure (the heart of a hypergroup H is the intersection of all subhypergroups of H which are complete parts). Sections fourth (*Subhypergroups*) and fifth (*Regular Hypergroups*) deal with some special types of subhypergroups (closed, normal and invertible, conjugable). The hypergroups of type U and those of type C are also briefly presented. In other four sections, namely *Join Spaces, Canonical Hypergroups and their Generalizations, m-complete Hypergroups, Cogroups, Cyclic Hypergroups*, the most important results on some remarkable classes of hypergroups as m-complete hypergroups, 1–hypergroups, cogroups, cyclic hypergroups, are given. The section *Associativity Semihypergroups and Hypergroups* presents associativity semihypergroups and hypergroups as well as (partial) τ-groupoids and constructions of partial τ-groupoids. In the next section constructions on hypergroups and semihypergroups are given. In connection with the hypergroup theory the notions of hyperring, hypermodule, vectorial hyperspace and the principal results concerning them are presented in section *Hyperrings, Hypermodules and Vectorial Hyperspaces*.

The book (which closes with *Index of Symbols, Analytical Index* and with an exhaustive bibliography of 384 titles) is self–contained and the organization of the material aims successfully at making proofs as clear as possible. Each section begins by brief hystorical comments and references and is illustrated with meaningful examples.

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A LINEAR H_∞ -CONTROL PROBLEM WITH IMPULSIVE DISTURBANCES

BY

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1. Introduction and problem formulation. Consider the following input-output system with impulsive disturbances

$$(1.1)\qquad \begin{cases} y'(t) = Ay(t) + B_2u(t) + \sum_{i=0}^{\infty} B_1 w_i \otimes \delta(iT), & t \in \mathbb{R}^+ = [0,+\infty) \\ y(0) = y_0 \end{cases}$$

$$(1.2)\qquad z(t) = Cy(t) + Du(t), \quad t \in \mathbb{R}^+,$$

where A is the infinitesimal generator of a C_0-semigroup e^{At} on $Y, B_1 \in$ $\in L(W,Y)$, $B_2 \in L(U, D(A^*)')$, $C \in L(Y,Z)$ and $D \in L(U,Z)$. Here Y, U, W, Z are real Hilbert spaces; we denote by $(.,.), (.,.)_U, (.,.)_W, (.,.)_Z$ the scalar products and by $|.|, |.|_U, |.|_W, |.|_Z$ the corresponding norms.

We have denoted by A^* the adjoint of A and by $D(A^*)'$ the dual of $D(A^*)$. The pairing between $D(A^*)$ and $D(A^*)'$ will also be denoted by $(.,.)$.

In system (1.1)-(1.2), $T > 0$ is a given positive number, $\delta(iT), i \in \mathbb{N}$ is the Dirac measure supported in iT; $y(t) \in Y$ is the state, $u(t) \in U$ is the control, $w_i \in W, i \in \mathbb{N}$ are the disturbances and $z(t) \in Z$ is the control output.

Hypotheses :

(i) For every $\tilde{T} > 0$,the map $B_2^* e^{A^*t}$ can be extended as a continuous map from Y to $L^2(0,\tilde{T};U)$, i.e.,there exists a positive constant $C_{\tilde{T}}$ such that

$$\int_0^{\tilde{T}} |B_2^* e^{A^*t} y|_U^2 dt \le C_{\tilde{T}} |y|^2, \quad \forall y \in Y,$$

equivalently (see [9], [15]) the map

$$(1.3)\qquad (L_0u)(t) = \int_0^t e^{A(t-s)}B_2u(s)ds,$$

can be extended as a continuous map from $L^2(0,\tilde{T};U)$ to $C([0,\tilde{T}];Y)$.

(ii) The pair (C,A) is exponentially detectable, i.e.,there exists $K \in L(Z,Y)$ such that $A+KC$ generates an exponentially stable semigroup $e^{(A+KC)t}$ on Z.

(iii) $D^*\,[C \quad D] = [0 \quad I]$

(iv) For each $y_0 \in Y$, there exists $\tilde{u} \in L^2(\mathbb{R}^+;U)$ such that $\tilde{y}$, the mild solution of (1.1), with $w_i := 0, i \in \mathbb{N}$, satisfies

$$(1.4)\qquad \int_0^\infty (|C\tilde{y}(t)|_Z^2 + |\tilde{u}(t)|_U^2)dt < \infty$$

It is easy to observe that (1.1) can be written in the equivalent form (see [11])

$$(1.5)\qquad \begin{cases} y'(t) = Ay(t) + B_2u(t), & t \in [iT,(i+1)T), i \in \mathbb{N} \\ y(iT+) = y(iT-) + B_1w_i, & i \in \mathbb{N} \\ y(0-) = y_0 \end{cases}$$

The mild solution of (1.1) (or equivalently (1.5)) is

$$(1.6)\qquad y(t) = e^{At}y_0 + \int_0^t e^{A(t-s)}B_2u(s)ds + \sum_{i=0}^{\infty}\chi(t-iT)e^{A(t-iT)}B_1w_i,$$

$$t \in \mathbb{R}^+ \setminus T\mathbb{N}$$

where χ denotes the Heaviside function (i.e.,$\chi(t) = 1$, for $t \geq 0$ and 0, for $t < 0$).

We observe that the mild solution y belongs to $C(\mathbb{R}^+ \setminus T\mathbb{N};Y)$, ($T\mathbb{N} = \{0,T,2T,3T,...\}$) and there exist $y(iT-)$, $y(iT+)$, for $i \in \mathbb{N}$. Denote by $l_2(W)$ the space of sequences $(w_i)_{i\in N}$,which satisfy $\sum_{i=0}^\infty |w_i|_W^2 < \infty$.

Let $\mathcal{F}$ be the set of functions $F(t) = B_2^*Q(t)$, where $Q : \mathbb{R}^+ \to L(Y)$, $Q \in C(Y;C([0,T];Y))$, $Q(t) = Q(t-T[\frac{t}{T}]), \forall t \in \mathbb{R}^+ \setminus T\mathbb{N}$ and the problem

$$\begin{cases} y'(t) = Ay(t) + B_2F(t)y(t), & t \in [0,T] \\ y(0) = y_0, \end{cases}$$

has a mild solution for every $y_0 \in Y$ (i.e., there exists $u \in L^2(0,T;U)$ and $y \in C([0,T];Y)$ such that

$$y(t) = e^{At}y_0 + \int_0^t e^{A(t-s)}B_2u(s)ds, \quad \forall t \in [0,T]$$

and $u(t) = F(t)y(t)$ a.e. in $(0,T)$).

Now, for $F \in \mathcal{F}$ let S_F be the possible multivalued operator defined on $l_2(W)$ by

$$(S_Fw)(t) = \{z(t) = (C + DF(t))\tilde{y}(t);$$

where $\tilde{y}$ is the mild solution of (1.1) for $y_0 := 0$ and $u(t) := F(t)y(t)\}$. Hence

$$(1.7)\qquad \tilde{y}(t) = \int_0^t e^{A(t-s)}B_2F(s)\tilde{y}(s)ds + \sum_{i=0}^{\infty}\chi(t-iT)e^{A(t-iT)}B_1w_i.$$

According to the theory of standard H_∞-problems (see e.g. [13]) we define the H_∞-control problem for system (1.1)-(1.2) as follows: Given $\gamma > 0$, find $F \in \mathcal{F}$ such that for every $w \in l_2(W)$ there exists $z \in S_F(w)$:

$$(1.8)\qquad \|z\|_{L^2(\mathbb{R}^+;Z)} \leq \rho\|w\|_{l_2(W)},$$

where $0 < \rho < \gamma$. In case of single valued S_F, (1.8) can be equivalently written $\|S_F\| \leq \rho$ (here $\|.\|$ is the norm in $L(l_2(W); L^2(\mathbb{R}^+;Z))$).

For related results concerning the continuous H_∞-control problems in the infinite dimensional case we refer to [8], [12], [13]-for bounded control and disturbance operators and to [1], [3], [5], [16], [17]-for unbounded control operators and/or unbounded disturbance operators. In [2] has been treated a continuous H_∞ problem with unbounded observation.

As concern the H_∞-control problems with impulsive controllers and disturbances we mention [7] (for bounded operators and finite dimensional case).

The study of this kind of problems (with unbounded control operator) has a practical motivation including among others: second order hyperbolic equations with Dirichlet boundary control, Euler-Bernoulli equation with Dirichlet/Neumann controls; first-order hyperbolic systems. We refer to the books [9] and [15] for more details.

2. Statement of the main result.

Theorem 2.1. *Suppose $\gamma > 0$ and assumption (i)-(iv) hold. If there exists $F \in \mathcal{F}$ such that (1.8) is satisfied then there exist the operators $P(t) \in L(Y), t \in [0,T]$ such that*

(j) $P(t) = P^*(t) \geq 0, \quad \forall t \in [0,T]$
(jj) $P \in C(Y; C([0,T]; Y))$
(jjj) $P(0)(y + \gamma^{-2}B_1B_1^*P(T)y) = P(T)y, \quad \forall y \in Y$
(jv) $(P(0)B_1w, B_1w) \leq \gamma^2|w|_W^2, \quad \forall w \in W$
(v) *There is a unique optimal pair* $u^*(t,\tau;y_0) \in L^2(\tau,T;U)$ *and* $y^*(t,\tau;y_0) \in C([\tau,T];Y), \tau \in [0,T]$ *of the optimal control problem*

$$\text{Minimize} \quad \int_\tau^T (|Cy(t)|_Z^2 + |u(t)|_U^2)dt + (P(T)y(T), y(T))$$

subject to $u \in L^2(\tau,T;U)$, *where* y *is the mild solution of*

$$\begin{cases} y'(t) = Ay(t) + B_2u(t), & t \in [\tau,T] \\ y(\tau) = y_0 \end{cases}$$

and in addition
(vj) $u^*(t,\tau;y_0) = -B_2^*P(t)y^*(t,\tau;y_0)$ *a.e. in* $[\tau,T]$
(vjj) $(P(t)x_0,y_0) = \int_t^T (Cy^*(s,t;x_0), Cy^*(s,t;y_0))_Z ds + + \int_t^T (u^*(s,t;x_0), u^*(s,t;y_0))_U ds + (P(T)y^*(T,t;x_0), y^*(T,t;y_0)), \forall x_0, y_0 \in Y, \forall t \in [0,T]$

Conversely, if there exists $P(t)$ *satisfying* (j)-(vjj), *then* $F(t) = -B_2^*P(t - T[\frac{t}{T}]), t \in \mathbf{R}^+$, *belongs to* $\mathcal{F}$ *and satisfies* (1.8) *with* $\rho = \gamma$.

Remark. If in addition to $(i) - (iv)$ we have:

(v)* the map $C^*Ce^{At}B_2$ can be extended as a continuous map: $U \to L^1(0,\tilde{T};Y)$, i.e.,

$$\int_0^{\tilde{T}} |C^*Ce^{At}B_2u(t)|dt \leq C_{\tilde{T}}|u|_U, \quad \forall u \in U$$

(vi)* the map $B_2^*e^{A^*t}$ can be extended as a continuous map: $Y \to L^\infty(0,\tilde{T};U)$, i.e.,

$$\sup_{0\leq t\leq T} |B_2^*e^{A^*t}y|_U \leq C_{\tilde{T}}|y|, \quad \forall y \in Y,$$

then we obtain that $P(t)$ also satisfies:

$$(\mathrm{a}_1) \qquad B_2^*P(t) \in L(Y; C([0,T]; Y))$$

$$(\mathrm{a}_2) \qquad \begin{gathered} B_2^*P(t)e^{A(t-\tau)}B_2 \in L(U; L^2(\tau,T;U)) \text{ uniformly in } \tau \text{ and} \\ \sup_{0\leq\tau\leq T} \int_\tau^T |B_2^*P(t)e^{A(t-\tau)}B_2u|_U^2 dt \leq C_T|u|_U^2, \quad \forall u \in U \end{gathered}$$

$$(\mathrm{a}_3) \qquad \begin{aligned} &\frac{d}{dt}(P(t)x_0,y_0) = -(Cx_0,Cy_0)_Z - (P(t)x_0,Ay_0) - (P(t)Ax_0,y_0) + \\ &+ (B_2^*P(t)x_0, B_2^*P(t)y_0)_U, \forall t \in [0,T], \quad \forall x_0,y_0 \in D(A) \end{aligned}$$

3. Proof of the necessary conditions. Firstly we shall assume that the H_∞-problem has a solution $F \in \mathcal{F}$.

Then, on the space $\mathbf{R}^+ \times L^2(\mathbf{R}^+,U) \times l_2(W)$ we define the function

$$(3.1) \qquad \begin{aligned} K(t,u,w) = 2^{-1} &\int_t^\infty (|Cy(t)|_Z^2 + |u(t)|_U^2)dt - \\ &-2^{-1}\gamma^2 \sum_{i=1}^\infty \chi(t - iT)|w_i|_W^2, \end{aligned}$$

where y is the mild solution of the problem

$$(3.2) \qquad \begin{cases} y'(s) = Ay(s) + B_2u(s) + \displaystyle\sum_{i\geq \max\{1,t/T\}} B_1w_i \otimes \delta(iT), & s \in [t,+\infty) \\ y(t) = y_0 \end{cases}$$

Denote $\mathcal{U} = L^2(\mathbf{R}^+;U)$, $\mathcal{U}_t = L^2((t,\infty);U)$, $\mathcal{W} = l_2(W)$, $\mathcal{W}_t = \{(w_i) \in l_2(W); i \in \mathbb{N}, i \geq \max\{1,t/T\}\}$ and consider the max-min problem

$$(PP_t) \qquad \sup_{w\in\mathcal{W}_t} \inf_{u\in\mathcal{U}_t} K(t,u,w) = \phi(t).$$

Clearly, $\phi(t) \geq 0$. We shall prove that problem (PP_t) has a solution $(u_t^*, w_t^*) \in \mathcal{U}_t \times \mathcal{W}_t$. For this, we consider the minimization problem

$$(P_t) \qquad \inf_{u\in\mathcal{U}_t} K(t,u,w),$$

for $w \in \mathcal{W}_t$ fixed.

We shall prove first that (P_0) has a unique optimal control u^*. Indeed, since the H_∞-problem has at least one solution it follows the existence of a $\tilde{u} \in \mathcal{U}$ such that

$$K(0,\tilde{u},w) < \infty.$$

Let us denote by $d = inf\ \{K(0,u,w); u \in \mathcal{U}\}$ and let $\{u_n\}_{n\in\mathbb{N}^*}$ be a sequence such that

$$K(0,u_n,w) < d + \frac{1}{n}, \quad \forall n \in \mathbb{N}^*$$

By the definition of K it follows that $\{u_n\}_{n\in\mathbb{N}^*}$ is bounded in $\mathcal{U}$ and that on a subsequence (also denoted by $\{u_n\}$) we have

$$u_n \to u^*, \text{for } n \to \infty, \text{weakly in } \mathcal{U}.$$

Now, the continuity of the operator L_0 defined by (1.3) implies

$$y_n \to y^*, \text{for } n \to \infty, \text{weakly in } L^2(0,\tilde{T};Y),$$

(y_n is the mild solution of (3.2) corresponding to $u := u_n$) where y^* is the mild solution of (1.1) corresponding to u^*.

Then Fatou's lemma gives

$$\int_0^\infty (|Cy^*(t)|_Z^2 + |u^*(t)|_U^2)dt \leq$$

$$\leq \liminf_{n\to\infty} \int_0^\infty (|Cy_n(t)|_Z^2 + |u_n(t)|_U^2)dt \leq 2d + \gamma^2|w|_W^2,$$

which implies the existence of a solution u^* for problem (P_0). The uniqueness follows from the strict convexity of the functional $u \mapsto K(0,u,w)$.

In the same manner follows (using (iv)) the existence of an optimal control for (P_t).

Theorem 3.1. *The pair (u_t, y_t), which satisfies (3.2) and $u_t \in \mathcal{U}_t$, $Cy_t \in L^2(\mathbb{R}_t^+;Z)$ is optimal for (P_t) if and only if there exists $p_t \in C(\mathbb{R}^+;Y)$ such that*

$$\begin{cases} p_t'(s) = -A^*p_t(s) + C^*Cy_t(s), & s \in (t,\infty) \\ p_t(\infty) = 0 \end{cases} \tag{3.3}$$

(i.e., $p_t(s) = e^{A^*(\tilde{T}-s)}p_t(\tilde{T}) - \int_s^{\tilde{T}} e^{A^*(\tau-s)}C^*Cy_t(\tau)d\tau, \forall \tilde{T} \in \mathbb{R}_t^+, t < s < \tilde{T}$) and

$$B_2^*p_t(s) = u_t(s) \quad a.e. \quad s > t. \tag{3.4}$$

Proof: Consider the minimization problem

$$\inf\{\frac{1}{2}\int_t^n (|Cy(s)|_Z^2 + |u(s)|_U^2)ds;$$

$$y'(s) = Ay(s) + B_2u(s) + \sum_{i\geq\max\{1,t/T\}} B_1w_i \otimes \delta(iT), y(t) = y_0\} = \phi_{n,t}(y_0).$$

Denote by $\phi_t(y_0)$ the optimal value of the cost function in (P_t).

In a standard way it is possible to prove the existence of a unique optimal pair $(u_{n,t}, y_{n,t})$ corresponding to $(P_{n,t})$.

From the obvious inequality $\phi_{n,t}(y_0) \leq \phi_t(y_0)$ it follows the boundedness of the sequence $\{u_{n,t}\}$ in $\mathcal{U}_t$. We infer that on a subsequence (also denoted by $\{u_{n,t}\}$) we have

$$u_{n,t} \to \tilde{u}_t, \text{for } n \to \infty, \text{weakly in } \mathcal{U}_t$$

and

$$Cy_{n,t} \to Cy_t^{\tilde{u}_t}, \text{for } n \to \infty, \text{weakly in } L^2(t,\tilde{T};Z), \quad \forall \tilde{T} \in (t,\infty)$$

(here $y_t^{\tilde{u}_t}$ is the mild solution of (3.2) corresponding to $\tilde{u}_t$).

These convergences imply

$$\liminf_{n\to\infty} \phi_{n,t}(y_0) \geq \phi_t(y_0).$$

Since $\phi_{n,t}(y_0) \leq \phi_t(y_0)$, we may conclude that $\lim_{n\to\infty} \phi_{n,t}(y_0) = \phi_t(y_0)$ and as a consequence :

$$\int_t^n |u_{n,t}(s)|_U^2 ds \to \int_t^\infty |\tilde{u}_t(s)|_U^2 ds, \text{ for } n \to \infty$$

The last convergence and the weak convergence of $\{u_{n,t}\}$ to $\tilde{u}_t$ in $\mathcal{U}_t$ allow us to conclude

$$u_{n,t} \to \tilde{u}_t, \text{for } n \to \infty, \text{strongly in } L^2(\mathbb{R}_t^+;U)$$

$$y_{n,t} \to y_t^{\tilde{u}_t}, \text{for } n \to \infty, \text{strongly in } L^2(t,\tilde{T};Y)$$

$$Cy_{n,t} \to Cy_t^{\tilde{u}_t}, \text{for } n \to \infty, \text{strongly in } L^2(t,\tilde{T};Z.$$

We conclude that (u_t, y_t) ($\tilde{u}_t = u_t$, $y_t^{\tilde{u}_t} = y_t$) is an optimal pair for (P_t). By standard device follows the existence of $p_{n,t} \in C([t,n];Y)$ such that

$$\begin{cases} p_{n,t}'(s) = -A^*p_{n,t}(s) + C^*Cy_{n,t}(s), & s \in [t,n] \\ p_{n,t}(n) = 0 \end{cases} \tag{3.5}$$

(3.6) $$u_{n,t}(s) = B_2^* p_{n,t}(s) \qquad a.e. \quad in \quad (t,n)$$

(see [14], [15]) where (3.5) should be viewed in the "mild" sense i.e.

(3.7)
$$p_{n,t}(s) = e^{A^*(\tilde{T}-s)} p_{n,t}(\tilde{T}) - \int_s^{\tilde{T}} e^{A^*(\tau - s)} C^* C y_{n,t}(\tau) d\tau, t \le s \le \tilde{T} \le n.$$

The function $y_0 \mapsto \phi_{n,t}(y_0)$ is convex and differentiable. Moreover, we have

$$p_{n,t}(s) = -\nabla \phi_{n,s}(y_{n,t}(s)), \qquad \forall s \in [t,n].$$

Indeed, since $(u_{n,t}, y_{n,t})$ is also optimal for $(P_{n,s})$, where $y_0 := y_{n,t}(s)$ $(s \in [t,n])$ we have

$$\phi_{n,s}(y_{n,t}(s)) - \phi_{n,s}(\overline{y}_0) =$$
$$= \frac{1}{2} \int_s^n (|C y_{n,t}(\tau)|_Z^2 - |C \overline{y}(\tau)|_Z^2) d\tau + \frac{1}{2} \int_s^n (|u_{n,t}(\tau)|_U^2 - |u(\tau)|_U^2) d\tau \le$$
$$\le \int_s^n (C y_{n,t}(\tau), C(y_{n,t}(\tau) - \overline{y}(\tau)))_Z + (u_{n,t}(\tau), u_{n,t}(\tau) - u(\tau))_U) d\tau,$$

where

$$\overline{y}'(\tau) = A \overline{y}(\tau) + B_2 u(\tau) + \sum_{\substack{i=1 \\ i \ge \frac{s}{T}}}^{\infty} B_1 w_i \otimes \delta(iT), \qquad \overline{y}(s) = \overline{y}_0$$

and u is the optimal control for $(P_{n,s})$ with $\overline{y}(s) = \overline{y}_0$. By (3.7) we see that

(3.8) $$\phi_{n,s}(y_{n,t}(s)) - \phi_{n,s}(\overline{y}_0) \le -(p_{n,t}(s), y_{n,t}(s) - \overline{y}_0), \quad \forall \overline{y}_0 \in Y$$

as claimed. If in (3.8) we take $\overline{y}_0 = \mu\theta$,with $|\theta| = 1$ we get

(3.9) $\mu |p_{n,t}(s)| \le |y_{n,t}(s)| |p_{n,t}(s)| + \phi_{n,s}(\mu\theta)$, for all $s \in [t,n]$ and $\mu > 0$.

On the other hand we have

$$\phi_{n,s}(\mu\theta) \le \frac{1}{2} \int_s^\infty (|C\tilde{y}|_Z^2 + |B_2^* P_0 \tilde{y}|_U^2) d\tau,$$

where

$$\begin{cases} \tilde{y}'(\tau) = (A - B_2 B_2^* P_0)\tilde{y}(\tau) + \sum_{i=1, i \ge s/T}^{\infty} B_1 w_i \otimes \delta(iT), \tau \ge s \\ \tilde{y}(s) = \mu\theta. \end{cases}$$

Here $P_0 \in L(Y)$ is such that $A_{P_0} = A - B_2 B_2^* P_0$ generates an exponentially stable semigroup and $B_2^* P_0 e^{A_{P_0} t} y_0 \in \mathcal{U}, \forall y_0 \in Y$ (the existence of such a P_0 follows from (iv)-see [10]). After an easy calculation we obtain

(3.10) $$\phi_{n,s}(\mu\theta) \le c(\mu^2 + \sum_{\substack{i=1 \\ i \ge s/T}}^{\infty} |w_i|_W^2),$$

where c is a constant independent of t and μ.

Using (3.9) and (3.10) we may infer that

(3.11) $$|p_{n,t}(s)| \le c(\mu - |y_{n,t}(s)|)^{-1} (\mu^2 + \sum_{\substack{i=1 \\ i \ge s/T}}^{\infty} |w_i|_W^2), \forall s \in [t,n].$$

Since $y_{n,t} \to y_t$, uniformly on bounded subintervals of $\boldsymbol{R}_t^+$ we infer that $\{p_{n,t}\}$ is bounded in $L^\infty(t, \tilde{T}; Y)$ and so on a subsequence (also denoted by $\{p_{n,t}\}$)

$$p_{n,t} \to p_t \qquad \text{weakly star in } L^\infty(t, \tilde{T}; Y).$$

The last convergence and (3.11) imply that

$$|p_t(s)| \le c(\mu - |y_t(s)|)^{-1} (\mu^2 + \sum_{\substack{i=1 \\ i \ge s/T}}^{\infty} |w_i|_W^2), \quad \forall s \ge t.$$

Using the detectability assumption (ii) (which implies $\lim_{s \to \infty} |y_t(s)| = 0$) the above inequality leads to

$$\limsup_{s \to \infty} |p_t(s)| \le (\mu + \mu^{-1} \sum_{\substack{i=1 \\ i \ge s_0/T}}^{\infty} |w_i|_W^2), \quad \forall s_0 \ge t.$$

Choosing now $\mu = (\sum_{\substack{i=1 \\ i \ge s_0/T}}^{\infty} |w_i|_W^2)^{1/2}$, in the last inequality we obtain

$$\limsup_{s \to \infty} |p_t(s)| \le 2c(\sum_{\substack{i=1 \\ i \ge s_0/T}}^{\infty} |w_i|_W^2)^{1/2}, \quad \forall s_0 \in \boldsymbol{R}_t^+.$$

Since $w = (w_i) \in \mathcal{W}_t$, the preceding inequality implies

$$\lim_{s\to\infty} p_t(s) = 0.$$

By (3.7) we have

$$\int_t^{\tilde T} (p_{n,t}(s), \psi(s))ds = \int_t^{\tilde T} (p_{n,t}(\tilde T), e^{A(\tilde T - s)}\psi(s))ds -$$

$$-\int_t^{\tilde T} (\psi(s), \int_s^{\tilde T} e^{A^*(\tau-s)}C^*Cy_{n,t}(\tau)d\tau)ds, \forall \psi \in C^1([t,\tilde T];Y).$$

For $n\to\infty$ we get

$$\int_t^{\tilde T} (p_t(s), \psi(s))ds = (p_{\tilde T}, \int_t^{\tilde T} e^{A(\tilde T - s)}\psi(s)ds) -$$

$$-\int_t^{\tilde T} (\psi(s), \int_s^{\tilde T} e^{A^*(\tau-s)}C^*Cy_t(\tau)d\tau)ds,$$

where $p_{\tilde T} = \text{w-}\lim_{n\to\infty} p_{n,t}(\tilde T)$ (on a subsequence). Hence

$$p_t(s) = e^{A^*(\tilde T - s)}p_{\tilde T} - \int_s^{\tilde T} e^{A^*(\tau-s)}C^*Cy_t(\tau)d\tau, \ a.e.\ s\in(t,\tilde T).$$

We extend p_t by continuity on $[t,\tilde T]$ and we get $p_t \in C(\boldsymbol{R}_t^+;Y)$ and

$$p_t(s) = e^{A^*(\tilde T - s)}p_t(\tilde T) - \int_s^{\tilde T} e^{A^*(\tau-s)}C^*Cy_t(\tau)d\tau, \ \ \forall t\le s\le \tilde T<\infty.$$

i.e., p_t is a mild solution of (3.3).

On the other hand

$$u_{n,t}(s) = B_2^*p_{n,t}(s) = B_2^*e^{A^*(\tilde T - s)}p_{n,t}(\tilde T) -$$

$$(3.12)\qquad -\int_s^{\tilde T} B_2^*e^{A^*(\tau-s)}C^*Cy_{n,t}(\tau)d\tau,$$

for $t\le s\le \tilde T\le n$

Since the map $q\mapsto B_2^*e^{A^*(\tilde T - s)}q$ is continuous from Y to $L^2(0,\tilde T;U)$ and the map $v\mapsto -\int_s^{\tilde T} B_2^*e^{A^*(\tau-s)}v(\tau)d\tau$ is continuous from $L^1(0,\tilde T;Y)$ to $L^2(0,\tilde T;U)$ (see [15]) we may pass to the limit in (3.12) to get

$$u_t(s) = B_2^*p_t(s) \ \ a.e.\ s>t,$$

thus concluding the Theorem 3.1.

Consider $\Gamma_t : \mathcal{W}_t \mapsto \mathcal{U}_t$ defined by $\Gamma_t w = u_t$ $(w=(w_i)_i)$. We shall prove that Γ_t is an affine function and

$$(3.13)\qquad \Gamma_t w = \Gamma_{0t}w + f_{0t}, \forall w\in\mathcal{W}_t$$

where $f_{0t} = arg\ inf\ K(t,u,0)$ and

$$\Gamma_{0t}\in L(\mathcal{W}_t,\mathcal{U}_t).$$

Indeed, Theorem 3.1 allow us to conclude that Γ_t is an affine function. More difficult is to prove that Γ_t is a bounded operator.

Consider $\bar y_{0t}$ the mild solution of (3.2),with $\bar y_{0t}(t) = y_0$, corresponding to $u := f_{0t}, w_i := 0$ and let $\bar v_t$, be the optimal control of (P_t) corresponding to the initial value $y_0 := 0$ and to the input disturbance $w = (w_i)_i$. Denote by $\bar y_t$ the mild solution to (3.2) corresponding to $y_0 := 0, u := \bar v_t$ and w. We get

$$\int_t^\infty (|Cy_t|_Z^2 + |\Gamma_t w|_U^2)dt \le \int_t^\infty (|C(\bar y_0 + \bar y_t)|_Z^2 + |f_{0t}+\bar v_t|_U^2)^2dt \le$$

$$\le 2\int_t^\infty (|C\bar y_{0t}|_Z^2 + |f_{0t}|_U^2)dt + 2\int_t^\infty (|C\bar y_t|_Z^2 + |\bar v_t|_U^2)dt \le C_t + 2\gamma^2\|w\|_{\mathcal{W}_t}^2,$$

$$\forall w\in\mathcal{W}_t,$$

where $C_t>0$ is a constant independent of w (we have used (1.8)). It means that $\Gamma_{0t}\in L(\mathcal{W}_t,\mathcal{U}_t)$. Let $\eta_t : \mathcal{W}_t\to\boldsymbol{R}$ be defined by

$$\eta_t(w) = -K(t,\Gamma_t(w),w).$$

By (3.13) we may reprezent η_t as

$$\eta_t(w) = \|Dw\|_{\mathcal{U}_t}^2 + ((w,f)) + \delta,$$

where $D\in L(\mathcal{W}_t,\mathcal{U}_t)$, $((.,.))$ is the scalar product of $\mathcal{W}_t, f\in\mathcal{W}_t$ and $\delta\in\boldsymbol{R}$. (the proof of this fact is the same as in [3] and uses ingredients from [4] and [6]). Relation (1.8) implies

$$\phi(w)\ge\alpha\|w\|_{\mathcal{W}_t}^2 + \beta, \forall w\in\mathcal{W}_t,$$

where $\alpha > 0$ and $\beta \in \mathbb{R}$ (see for the argument [3]). In a standard way we can prove the existence of a unique $w_t^* \in \mathcal{W}_t$ such that

$$w_t^* = (w_{t,i}^*)_i = arg\ inf\{\eta_t(w); w \in \mathcal{W}_t\}.$$

Theorem 3.2.

$$w_{t,i}^* = -\gamma^{-2}B_1^* p_t(iT), \quad \forall i \in \mathbb{N}^*, i \geq t/T. \tag{3.14}$$

Proof: The inequality

$$\int_t^\infty (|Cy_t^*|_Z^2 + |\Gamma_t w_t^*|_U^2)ds - \gamma^2 |w_t^*|_{\mathcal{W}_t}^2 \geq$$

$$\geq \int_t^\infty (|Cy_t^{w_t^*+\varepsilon w}|_Z^2 + |\Gamma_t(w_t^* + \varepsilon w)|_U^2 ds - \gamma^2 |w_t^* + \varepsilon w|_{\mathcal{W}_t}^2$$

$\forall \varepsilon > 0$ (where y_t^* is the mild solution of (3.2) with $y_t^*(t) = y_0$ and the inputs $u := \Gamma_t w_t^*, w := w_t^*$ and $y_t^{w_t^*+\varepsilon w}$ is the mild solution of (3.2) with $y_t^{w_t^*+\varepsilon w}(t) = y_0$, corresponding to the inputs $u := \Gamma(w_t^* + \varepsilon w)$ and $w := w_t^* + \varepsilon w$) implies

$$\begin{aligned}\int_t^\infty ((Cy_t^*(s), Cx(s))_Z + (\Gamma_{0t} w_t^* + f_{0t}, \Gamma_{0t} w)_U)ds - \\ -\gamma^2 \sum_{\substack{i=1 \\ i \geq t/T}}^\infty (w_{t,i}^*, w_i)_W \leq 0,\end{aligned} \tag{3.15}$$

$\forall w = (w_i)_i \in \mathcal{W}_t$, where x is the mild solution of

$$\begin{cases} x'(s) = Ax(s) + B_2(\Gamma_{0t}w)(s) + \displaystyle\sum_{\substack{i=1 \\ i \geq t/T}}^\infty B_1 w_i \otimes \delta(iT), \ s \in (t, \infty) \\ x(t) = 0 \end{cases}$$

and satisfies $Cx \in L^2(\mathbb{R}_t^+; Z)$.

By (3.3) we deduce that for all $\tilde{T} > t$ we have

$$\begin{aligned}\int_t^{\tilde{T}} (Cy_t^*(s), Cx(s))_Z ds = (p_t(\tilde{T}), x(\tilde{T})) - \\ - \int_t^{\tilde{T}} (B_2(\Gamma_{0t}w)(s), p_t(s))ds + \sum_{\substack{i=1 \\ \tilde{T}/T > i \geq t/T}} (B_1 w_i, p_t(iT))_W.\end{aligned} \tag{3.16}$$

Since (A, C) is exponentially detectable and $Cx \in L^2(\mathbb{R}_t^+; Z)$ we infer that $\lim_{s \to \infty} x(s) = 0$.

Letting $\tilde{T} \to \infty$ in (3.16) it follows

$$\begin{aligned}\int_t^\infty (Cy_t^*(s), Cx(s))_Z ds = -\int_t^\infty ((\Gamma_{0t}w)(s), (\Gamma_{0t}w_t^*)(s) + f_{0t}(s))_U ds - \\ - \sum_{\substack{i=1 \\ i \geq t/T}}^\infty (w_i, B_1^* p_t(iT))_W.\end{aligned}$$

We set

$$P(t)x_0 = -p_t(t), \quad t \in [0, T].$$

The uniqueness of the optimal control (u_t^*, w_t^*) for (PP_t) and the optimality conditions (3.3), (3.4), (3.14) imply that $P(s)y_t^*(s) = -p_t(s)$, $\forall s \geq t$ and

$$\begin{aligned}P(t)y_0 &= -e^{A^*(T-t)}p_t(T) + \int_t^T e^{A^*(\tau - t)}C^*Cy_t^*(\tau)d\tau \\ &= \int_t^T e^{A^*(\tau-t)}C^*Cy_t^*(\tau)d\tau + e^{A^*(T-t)}P(T)y_t^*(T).\end{aligned}$$

Consider now the following minimization problem with finite horizon

$$(\bar{P}_t) \qquad Minimize \quad \int_t^T (|Cy(s)|_Z^2 + |u(s)|_U^2)ds + (P(T)y(T), y(T))$$

subject to $u \in L^2(t, T; U)$, where y is the mild solution of

$$\begin{cases} y'(s) = Ay(s) + B_2 u(s) \quad s \in [t, T] \\ y(t) = y_0 \end{cases} \tag{3.17}$$

Problem $(\bar{P}_t)$ has a unique optimal pair. A standard result says that

$(u^*(s, t; y_0), y^*(s, t; y_0))$ (where $y^*(s, t; y_0)$ is the mild solution of (3.17) corresponding to $u := u^*(s, t; y_0)$) is an optimal pair for $(\bar{P}_t)$ if and only if there exists $q \in C([t, T]; Y)$, a mild solution of

$$\begin{cases} q'(s) = -A^*q(s) + C^*Cy^*(s, t; y_0), \ s \in [t, T] \\ q(T) = -P(T)y^*(T, t; y_0) \end{cases}$$

and in addition

$$u^*(s,t;y_0) = B_2^* q(s), \text{ a.e. } s \in (t,T)$$

(see [10]).

Using the optimality conditions we get that (u_t^*, y_t^*) $((u_t^*, w_t^*, y_t^*)$ is optimal for $(PP_t))$ is an optimal pair for (P_t) and $q(s) = p_t(s)$, $\forall s \in [t,T]$. This implies that $P(t)y_0 = -q(t)$ (it means that the operator $P(t)$ introduced by us is the same with that constructed in the proof of Theorem 3.1 in [15]) and taking into consideration the Theorem 3.1 in [15] we conclude that $P(t) \in L(Y), \forall t \in [0,T]$ and satisfies $(j),(jj),(v),(vj)$ and (vjj).

$(u_T^*, (w_{T,i}^*)_{i\geq 1}, y_T^*)$ is optimal for (PP_T) (where y_T^* is the mild solution of (3.2) with $t := T, u := u_T^*$ and $w_i := w_{T,i}^*, i \geq 1)$ if and only if there exists $p_T \in C(\boldsymbol{R}_T^+; Y)$, a mild solution of

$$\text{(3.18)} \qquad \begin{cases} p_T'(s) = -A^* p_T(s) + C^* C y_T^*(s), \ s \in (T,\infty) \\ p_T(\infty) = 0 \end{cases}$$

and in addition

$$\text{(3.19)} \qquad u_T^*(s) = B_2^* p_T(s) \text{ a.e. } s \in (T,\infty)$$

$$\text{(3.20)} \qquad w_{T,i}^* = -\gamma^{-2} B_1^* p_T(iT), \ i \in \mathbb{N}^*.$$

y_T^* is also a mild solution for the following problem

$$\begin{cases} {y_T^*}'(s) = Ay_T^*(s) + B_2 u_T^*(s) + \sum_{i=2}^{\infty} B_1 w_i^* \otimes \delta(iT), \quad s \in [T, +\infty) \\ y_T^*(T+) = y_0 - \gamma^{-2} B_1 B_1^* p_T(T) = y_0 + \gamma^{-2} B_1 B_1^* P(T) y_0 \end{cases}$$

(we have used the definition of $P(T)$).

As p_T satisfies (3.18),(3.19) and (3.20) (for $i \in \mathbb{N}^* \setminus \{1\}$), it follows that these conditions are necessary and sufficient for the optimality of $(u_T^*(\cdot - T), (w_{T,i-1}^*)_{i\geq 2}, y_T^*(\cdot - T))$ in (PP_0), with initial value $y_0 := y_0 + \gamma^{-2} B_1 B_1^* P(T) y_0$. As a consequence we get

$$p_T(T) = -P(T)y_0 = -P(0)(y_0 + \gamma^{-2} B_1 B_1^* P(T) y_0), \ \forall y_0 \in Y$$

(we have used the definition of $P(0)$). This concludes (jjj).

An easy calculation involving the optimality conditions implies

$$(P(t)x_0, y_0) = \int_t^\infty ((Cx_t^*, Cy_t^*)_Z + (u_t^*, v_t^*)_U) ds -$$

$$-\gamma^2 \sum_{i \geq max\ \{1, t/T\}} (w_{t,i}^*, \bar{w}_{t,i}^*)_W$$

(where $(v_t^*, (\bar{w}_{t,i}^*), x_t^*)$ is optimal for (PP_t), corresponding to $y_0 := x_0$).

For $x_0 = y_0 = B_1\alpha$ and $t := 0$ we obtain

$$(P(0)B_1\alpha, B_1\alpha) = \max_{(w_i)_{i\geq 1} \in \mathcal{W}_t} \ \min_{u \in \mathcal{U}} \int_0^\infty (|Cy|_Z^2 + |u|_U^2) ds - \gamma^2 \sum_{i=1}^{\infty} |w_i|_W^2;$$

y being the mild solution of

$$\begin{cases} y'(s) = Ay(s) + B_2 u(s) + \sum_{i=1}^{\infty} B_1 w_i \otimes \delta(iT), \ s \in \boldsymbol{R}^+ \\ y(0) = B_1 \alpha \end{cases}$$

$= \max_{\substack{(w_i)_{i\geq 0} \in \mathcal{W} \\ w_0 = \alpha}} \ \min_{u \in \mathcal{U}} \int_0^\infty (|Cy|_Z^2 + |u|_U^2) ds - \gamma^2 \sum_{i=0}^{\infty} |w_i|_W^2 + \gamma^2 |\alpha|_W^2$; y being the mild solution of

$$\text{(3.21)} \qquad \begin{cases} y'(s) = Ay(s) + B_2 u(s) + \sum_{i=0}^{\infty} B_1 w_i \otimes \delta(iT), \ s \in \boldsymbol{R}^+ \\ y(0) = 0 \end{cases}$$

which is less or equal than

$$\gamma^2 |\alpha|_W^2 + \max_{(w_i)_{i\geq 0} \in \mathcal{W}} \ \min_{u \in \mathcal{U}} \{ \int_0^\infty (|Cy|_Z^2 + |u|_U^2) ds - \gamma^2 \sum_{i=0}^{\infty} |w_i|_W^2;$$

$$y \text{ being the mild solution of (3.21)} \}$$

(if we translate the argument $t \mapsto t - T$ we can see that the last max-min problem is in fact the (PP_T)-problem with $y_0 := 0) = \gamma^2 |\alpha|_W^2 + (P(T)0, 0) =$ $= \gamma^2 |\alpha|_W^2$.

The last considerations show that (jv) holds.

If in addition $(v)^*$ and $(vi)^*$ hold, then applying Theorem 3.3 in [15] we get that $P(t)$ also satisfies $(a_1),(a_2),(a_3)$.

4. Proof of the sufficiency. Let $w = (w_i)_{i\geq 0} \in l_2(W)$ be fixed and consider the problem (1.1) with the feedback control $u(t) = F(t)y(t)$, where

$F(t) := -B_2^* P(t - [\frac{t}{T}]), \forall t \in R^+$ ($P(t)$ satisfies $(j) - (vjj)$) and the initial condition $y(0) = 0$.

(4.1)

$$\begin{cases} y'(t) = Ay(t) - B_2 B_2^* P(t) y(t) + \sum_{i=0}^{\infty} B_1 w_i \otimes \delta(iT), & t \in R^+ = [0, +\infty) \\ y(0) = 0. \end{cases}$$

This problem can be equivalently written as

$$\text{(4.2)} \qquad \begin{cases} y'(t) = Ay(t) - B_2 B_2^* P(t) y(t), & t \in [iT, (i+1)T), i \in \mathbb{N} \\ y(iT+) = y(iT-) + B_1 w_i \\ y(0-) = y(0) = 0 \end{cases}$$

Using $(v) - (vj)$ we get that there exists at least one mild solution y^w for (4.1) (or (4.2)) and that $(-B_2^* P(t) y^w(t), y^w(t))$ $(w = (w_i)_{i \in \mathbb{N}})$ is the optimal pair for the minimization problem

$$\textit{Minimize} \int_{iT}^{(i+1)T} (|Cy(t)|_Z^2 + |u(t)|_U^2) dt + (P(T) y((i+1)T-), y((i+1)T-)),$$

subject to $u \in L^2((iT, (i+1)T); U)$, where y is the mild solution of

$$\text{(4.3)} \qquad \begin{cases} y'(t) = Ay(t) + B_2 u(t), & t \in [iT, (i+1)T) \\ y(iT+) = y(iT-) + B_1 w_i \\ y(0-) = y(0) = 0 \end{cases}$$

This fact implies (via (vjj))

$$\text{(4.4)} \qquad \begin{aligned} &\int_{iT}^{(i+1)T} (|Cy^w(t)|_Z^2 + |B_2^* P(t) y^w(t)|_U^2) dt = \\ &= (P(0) y^w(iT+), y^w(iT+)) - \\ &- (P(T) y^w((i+1)T-), y^w((i+1)T-)) = \\ &= (P(0)(y^w(iT-) + B_1 w_i), y^w(iT-) + B_1 w_i) - \\ &- (P(T) y^w((i+1)T-), y^w((i+1)T-)) \end{aligned}$$

Using (jjj) and (4.2) we may write

$$\begin{aligned} &(P(0)(y^w(iT-) + B_1 w_i), y^w(iT-) + B_1 w_i) \\ &= (P(0)(y^w(iT-) + \gamma^{-2} B_1 B_1^* P(T) y^w(iT-)), y^w(iT-) + B_1 w_i) \\ &+ (P(0)(B_1 w_i - \gamma^{-2} B_1 B_1^* P(T) y^w(iT-)), y^w(iT-)) + B_1 w_i) \\ &= (P(T) y^w(iT-), y^w(iT-) + B_1 w_i) \\ &- \gamma^{-2} (P(0)(B_1 w_i - \gamma^{-2} B_1 B_1^* P(T) y^w(iT-)), B_1 B_1^* P(T) y^w(iT-)) \\ &+ (P(0)(B_1 w_i - \gamma^{-2} B_1 B_1^* P(T) y^w(iT-)), B_1 w_i) \\ &+ (P(0)(B_1 w_i - \gamma^{-2} B_1 B_1^* P(T) y^w(iT-)), y^w(iT-)) + \\ &+ \gamma^{-2} (P(0)(B_1 w_i - \gamma^{-2} B_1 B_1^* P(T) y^w(iT-)), B_1 B_1^* P(T) y^w(iT-)) \\ &= (P(T) y^w(iT-), y^w(iT-)) + (w_i, B_1^* P(T) y^w(iT-)) \\ &+ (P(0) B_1 (w_i - \gamma^{-2} B_1^* P(T) y^w(iT-)), B_1 (w_i - \gamma^{-2} B_1^* P(T) y^w(iT-))) \\ &+ (B_1 w_i - \gamma^{-2} B_1 B_1^* P(T) y^w(iT-), P(T) y^w(iT-)). \end{aligned}$$

Now taking into account (jv) the last quantity is majorized by

$$\begin{aligned} &P(T) y^w(iT-), y^w(iT-)) + \gamma^2 |w_i - \gamma^{-2} B_1^* P(T) y^w(iT-)|_W^2 \\ &+ 2(w_i, B_1^* P(T) y^w(iT-))_W - \gamma^{-2} (B_1 B_1^* P(T) y^w(iT-), P(T) y^w(iT-)) \\ &= \gamma^2 |w_i|_W^2 + (P(T) y^w(iT-), y^w(iT-)). \end{aligned}$$

This fact and (4.4) imply

$$\int_{iT}^{(i+1)T} (|Cy^w(t)|_Z^2 + |B_2^* P(t) y^w(t)|_U^2) dt \le (P(T) y^w(iT-), y^w(iT-)) -$$

$$-(P(T) y^w((i+1)T-), y^w((i+1)T-)) + \gamma^2 |w_i|^2,$$

from which we get

$$\int_0^{\infty} (|Cy^w(t)|_Z^2 + |B_2^* P(t) y^w(t)|_U^2) dt \le \gamma^2 |w|_W^2, \quad \forall w \in \mathbf{W}.$$

The last relation shows that $F(t)$ belongs to $\mathcal{F}$ and satisfies (1.8) with $\rho = \gamma$.

5. The case of bounded control operator. In the particular case of bounded control operator B_2, we obtain a more refined form for the main Theorem :

Theorem 5.1. *Suppose $\gamma > 0$ and assumption (ii)-(iv) hold. If there exists $F \in \mathcal{F}$ such that (1.8) is satisfied then there exist the operators $P(t) \in L(Y), t \in [0,T]$ such that*

(j) $P(t) = P^*(t) \geq 0, \quad \forall t \in [0,T]$

(jj) $P \in C(Y; C([0,T]; Y))$

(jjj) $P(0)(y + \gamma^{-2} B_1 B_1^* P(T) y) = P(T) y, \quad \forall y \in Y$

(jv) $(P(0) B_1 w, B_1 w) \leq \gamma^2 |w|_W^2, \quad \forall w \in W$

(v) $\frac{d}{dt}(P(t)x_0, y_0) = -(Cx_0, Cy_0)_Z - (P(t)x_0, Ay_0) -$

$-(P(t)Ax_0, y_0) + (B_2^* P(t) x_0, B_2^* P(t) y_0)_U, \ \forall t \in [0,T], \forall x_0, \ y_0 \in D(A)$

Conversely, if there exists $P(t)$ satisfying (j)-(v), then $F(t) = -B_2^* P(t - T[\frac{t}{T}]), t \in R^+$, belongs to $\mathcal{F}$ and satisfies $\|S_F\| \leq \gamma$.

The necessity has been proved in Section 2.

For the proof of the suficiency we have to notice first that if $P(t)$ satisfies $(j) - (v)$ in Theorem 5.1, then $(v), (vj), (vjj)$ from Theorem 2.1 also hold (see [9]). From this point the proof has been done in Section 4.

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