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A note on a system of difference equations

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Abstract We show that the system of difference equations

$$x_{n+1} = \frac{\prod_{j=0}^{k} y_{n-2j}}{\prod_{j=1}^{k} x_{n-(2j-1)} \left(a_n + b_n \prod_{j=0}^{k} y_{n-2j}\right)},$$

$$y_{n+1} = \frac{\prod_{j=0}^{k} x_{n-2j}}{\prod_{j=1}^{k} y_{n-(2j-1)} \left(\alpha_n + \beta_n \prod_{j=0}^{k} x_{n-2j}\right)}, \; n \in \mathbb{N}_0$$

where $k \in \mathbb{N}_0$, the sequences $(a_n)_{n\in\mathbb{N}_0}$, $(b_n)_{n\in\mathbb{N}_0}$, $(\alpha_n)_{n\in\mathbb{N}_0}$, $(\beta_n)_{n\in\mathbb{N}_0}$ are real, and the initial values x_{-2k}, x_{-2k+1},..., x_0, y_{-2k}, y_{-2k+1},..., y_0 are nonzero real numbers, is solved in closed form.

Keywords Difference equations · Systems of difference equations · Closed form solution · Periodicity

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1 Introduction

In the last few decades the area of difference equations has attracted considerable interest of the researchers. Among other topics solving nonlinear difference equations

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reattracted some interest (see, e.g. [1], [2], [4]-[28]), especially after publishing the paper [9] in which Stević explained the formula in paper [2]. Our motivation stems from his papers [1], [9]-[22], as well as papers [5] and [6].

In this paper, we show that the system of difference equations

$$x_{n+1}=\frac{\prod_{j=0}^{k}y_{n-2j}}{\prod_{j=1}^{k}x_{n-(2j-1)}\left(a_n+b_n\prod_{j=0}^{k}y_{n-2j}\right)},$$

$$y_{n+1}=\frac{\prod_{j=0}^{k}x_{n-2j}}{\prod_{j=1}^{k}y_{n-(2j-1)}\left(\alpha_n+\beta_n\prod_{j=0}^{k}x_{n-2j}\right)},\ n\in\mathbb{N}_0 \tag{1.1}$$

where $k\in\mathbb{N}_0$, the sequences $(a_n)_{n\in\mathbb{N}_0}$, $(b_n)_{n\in\mathbb{N}_0}$, $(\alpha_n)_{n\in\mathbb{N}_0}$, $(\beta_n)_{n\in\mathbb{N}_0}$ are real, and the initial values x_{-2k}, x_{-2k+1},..., x_0, y_{-2k}, y_{-2k+1},..., y_0 are nonzero real numbers, is solved in closed form, by using the trick by Stević in [9] (see also [1], [12]-[22]), which transforms a nonlinear difference equation to a linear first-order difference equation.

The following lemma is well known (see, e.g. [3]).

Lemma 1.1 *Consider the linear difference equation*

$$y_{n+1}=a_ny_n+b_n,\ n\in\mathbb{N}_0.$$

Then,

$$y_n=\left(\prod_{i=0}^{n-1}a_i\right)y_0+\sum_{r=0}^{n-1}\left(\prod_{i=r+1}^{n-1}a_i\right)b_r.$$

Moreover if a_n, b_n are constants, that is $a_n=a$ and $b_n=b$, then

$$y_n=a^ny_0+\frac{a^n-1}{a-1}b,\ n\in\mathbb{N}_0,\ a\neq 1,$$
$$y_n=y_0+b\,n,\ n\in\mathbb{N}_0,\ a=1.$$

In the sequel, as usual, assume that $\prod_{j=i}^{k}A_j=1$, $\sum_{j=i}^{k}A_j=0$, $k<i$.

2 Closed form formulas for solution of system (1.1)

In the following, we get a closed form formula for all well defined solutions of the system of difference equations (1.1). By a well defined solution of System (1.1), we mean a solution such that

$$\prod_{j=1}^{k}x_{n-(2j-1)}\left(a_n+b_n\prod_{j=0}^{k}y_{n-2j}\right)\neq 0,\ n\in\mathbb{N}_0,$$

and

$$\prod_{j=1}^{k}y_{n-(2j-1)}\left(\alpha_n+\beta_n\prod_{j=0}^{k}x_{n-2j}\right)\neq 0,\ n\in\mathbb{N}_0.$$

By the change of variables

$$u_n = \frac{1}{\prod_{j=0}^{k} x_{n-2j}},\ v_n = \frac{1}{\prod_{j=0}^{k} y_{n-2j}},\ n \in \mathbb{N}_0 \tag{2.1}$$

the system (1.1) becomes

$$u_{n+1} = a_n v_n + b_n,\ v_{n+1} = \alpha_n u_n + \beta_n,\ n \in \mathbb{N}_0. \tag{2.2}$$

From (2.2) we get

$$u_{n+2} = a_{n+1}\alpha_n u_n + a_{n+1}\beta_n + b_{n+1},\ n \in \mathbb{N}_0, \tag{2.3}$$

$$v_{n+2} = \alpha_{n+1} a_n v_n + \alpha_{n+1} b_n + \beta_{n+1},\ n \in \mathbb{N}_0. \tag{2.4}$$

From (2.3) and (2.4) we have

$$u_{2n+2} = a_{2n+1}\alpha_{2n} u_{2n} + a_{2n+1}\beta_{2n} + b_{2n+1},\ n \in \mathbb{N}_0, \tag{2.5}$$
$$u_{2n+3} = a_{2n+2}\alpha_{2n+1} u_{2n+1} + a_{2n+2}\beta_{2n+1} + b_{2n+2},\ n \in \mathbb{N}_0, \tag{2.6}$$
$$v_{2n+2} = \alpha_{2n+1} a_{2n} v_{2n} + \alpha_{2n+1} b_{2n} + \beta_{2n+1},\ n \in \mathbb{N}_0, \tag{2.7}$$
$$v_{2n+3} = \alpha_{2n+2} a_{2n+1} v_{2n+1} + \alpha_{2n+2} b_{2n+1} + \beta_{2n+2},\ n \in \mathbb{N}_0. \tag{2.8}$$

Let

$$A_n = a_{2n+1}\alpha_{2n},\ B_n = a_{2n+1}\beta_{2n} + b_{2n+1},\ w_n = u_{2n}, \tag{2.9}$$
$$C_n = a_{2n+2}\alpha_{2n+1},\ D_n = a_{2n+2}\beta_{2n+1} + b_{2n+2},\ z_n = u_{2n+1}, \tag{2.10}$$
$$E_n = \alpha_{2n+1} a_{2n},\ F_n = \alpha_{2n+1} b_{2n} + \beta_{2n+1},\ t_n = v_{2n}, \tag{2.11}$$
$$G_n = \alpha_{2n+2} a_{2n+1},\ H_n = \alpha_{2n+2} b_{2n+1} + \beta_{2n+2},\ r_n = v_{2n+1}. \tag{2.12}$$

Then from (2.9)-(2.12), we get the following nonhomogeneous linear first-order difference equations with variable coefficients

$$w_{n+1} = A_n w_n + B_n,\ n \in \mathbb{N}_0, \tag{2.13}$$
$$z_{n+1} = C_n z_n + D_n,\ n \in \mathbb{N}_0, \tag{2.14}$$
$$t_{n+1} = E_n t_n + F_n,\ n \in \mathbb{N}_0, \tag{2.15}$$
$$r_{n+1} = G_n r_n + H_n,\ n \in \mathbb{N}_0. \tag{2.16}$$

From (2.13) and Lemma 1.1, we have

$$w_n = \left[\prod_{j=0}^{n-1} A_j\right] w_0 + \sum_{j=0}^{n-1} \left[\prod_{i=j+1}^{n-1} A_i\right] B_j. \tag{2.17}$$

Then, from (2.9) we obtain

$$u_{2n} = \left[\prod_{j=0}^{n-1} (a_{2j+1}\alpha_{2j})\right] u_0 + \sum_{j=0}^{n-1} \left[\prod_{i=j+1}^{n-1} (a_{2i+1}\alpha_{2i})\right] (a_{2j+1}\beta_{2j} + b_{2j+1}). \tag{2.18}$$

Similarly from (2.14)-(2.16), Lemma 1.1 and (2.10)-(2.12), we obtain

$$\begin{aligned} u_{2n+1} = & \left[\prod_{j=0}^{n-1}\left(a_{2j+2}\alpha_{2j+1}\right)\right]u_1+ \\ & +\sum_{j=0}^{n-1}\left[\prod_{i=j+1}^{n-1}\left(a_{2i+2}\alpha_{2i+1}\right)\right]\left(a_{2j+2}\beta_{2j+1}+b_{2j+2}\right), \end{aligned} \tag{2.19}$$

$$v_{2n} = \left[\prod_{j=0}^{n-1}\left(\alpha_{2j+1}a_{2j}\right)\right]v_0+\sum_{j=0}^{n-1}\left[\prod_{i=j+1}^{n-1}\left(\alpha_{2i+1}a_{2i}\right)\right]\left(\alpha_{2j+1}b_{2j}+\beta_{2j+1}\right), \tag{2.20}$$

$$\begin{aligned} v_{2n+1} = & \left[\prod_{j=0}^{n-1}\left(\alpha_{2j+2}a_{2j+1}\right)\right]v_1+ \\ & +\sum_{j=0}^{n-1}\left[\prod_{i=j+1}^{n-1}\left(\alpha_{2i+2}a_{2i+1}\right)\right]\left(\alpha_{2j+2}b_{2j+1}+\beta_{2j+2}\right). \end{aligned} \tag{2.21}$$

When the coefficients are constants i.e., $a_n = a$, $b_n = b$, $\alpha_n = \alpha$ and $\beta_n = \beta$, formulae (2.18)-(2.21) becomes

$$u_{2n} = (a\alpha)^n u_0 + \frac{(a\alpha)^n - 1}{a\alpha - 1}(a\beta + b), n \in \mathbb{N}_0,\ a\alpha \neq 1, \tag{2.22}$$

$$u_{2n} = u_0 + (a\beta + b)\, n,\ n \in \mathbb{N}_0,\ a\alpha = 1. \tag{2.23}$$

$$u_{2n+1} = (a\alpha)^n u_1 + \frac{(a\alpha)^n - 1}{a\alpha - 1}(a\beta + b), n \in \mathbb{N}_0,\ a\alpha \neq 1, \tag{2.24}$$

$$u_{2n+1} = u_1 + (a\beta + b)\, n,\ n \in \mathbb{N}_0,\ a\alpha = 1, \tag{2.25}$$

$$v_{2n} = (a\alpha)^n v_0 + \frac{(a\alpha)^n - 1}{a\alpha - 1}(\alpha b + \beta), n \in \mathbb{N}_0,\ a\alpha \neq 1, \tag{2.26}$$

$$v_{2n} = v_0 + (\alpha b + \beta)\, n,\ n \in \mathbb{N}_0,\ a\alpha = 1, \tag{2.27}$$

$$v_{2n+1} = (a\alpha)^n v_1 + \frac{(a\alpha)^n - 1}{a\alpha - 1}(\alpha b + \beta), n \in \mathbb{N}_0,\ a\alpha \neq 1, \tag{2.28}$$

$$v_{2n+1} = v_1 + (\alpha b + \beta)\, n,\ n \in \mathbb{N}_0,\ a\alpha = 1. \tag{2.29}$$

Now, from (2.1) we get

$$x_{n+2} = \frac{u_n}{u_{n+2}}x_{n-2k},\ y_{n+2} = \frac{v_n}{v_{n+2}}y_{n-2k},\ n = 0, 1, ..., \tag{2.30}$$

from which it follows, by induction, that for $n = 0, 1, ...,$

$$x_{(2k+2)n+1} = \gamma \prod_{j=1}^{n} \frac{u_{2((k+1)j-1)+1}}{u_{2(k+1)j+1}}, \tag{2.31}$$

$$x_{(2k+2)n+i} = x_{i-(2k+2)} \prod_{j=0}^{n} \frac{u_{(2k+2)j+i-2}}{u_{(2k+2)j+i}},\ i = 2, 3, ..., 2k+2, \tag{2.32}$$

$$y_{(2k+2)n+1} = \lambda \prod_{j=1}^{n} \frac{v_{2((k+1)j-1)+1}}{v_{2(k+1)j+1}}, \tag{2.33}$$

$$y_{(2k+2)n+i} = y_{i-(2k+2)} \prod_{j=0}^{n} \frac{v_{(2k+2)j+i-2}}{v_{(2k+2)j+i}},\ i = 2, 3, ..., 2k+2, \tag{2.34}$$

where

$$\gamma = \frac{\prod_{j=0}^{k} y_{-2j}}{\prod_{j=1}^{k} x_{1-2j} \left(a_0 + b_0 \prod_{j=0}^{k} y_{-2j}\right)},\ \lambda = \frac{\prod_{j=0}^{k} x_{-2j}}{\prod_{j=1}^{k} y_{1-2j} \left(\alpha_0 + \beta_0 \prod_{j=0}^{k} x_{-2j}\right)}.$$

Depending on i is even or odd, we get from (2.32) and (2.34)

$$x_{(2k+2)n+2i} = x_{2i-(2k+2)} \prod_{j=0}^{n} \frac{u_{2((k+1)j+i-1)}}{u_{2((k+1)j+i)}},\ i = 1, 2, ..., k+1,$$

$$y_{(2k+2)n+2i} = y_{2i-(2k+2)} \prod_{j=0}^{n} \frac{v_{2((k+1)j+i-1)}}{v_{2((k+1)j+i)}},\ i = 1, 2, ..., k+1,$$

$$x_{(2k+2)n+2i+1} = x_{2i-(2k+1)} \prod_{j=0}^{n} \frac{u_{2((k+1)j+i-1)+1}}{u_{2((k+1)j+i)+1}},\ i = 1, 2, ..., k,$$

$$y_{(2k+2)n+2i+1} = y_{2i-(2k+1)} \prod_{j=0}^{n} \frac{v_{2((k+1)j+i-1)+1}}{v_{2((k+1)j+i)+1}},\ i = 1, 2, ..., k.$$

In summary, we have prove the following result.

Theorem 2.1 *Let $\{(x_n, y_n)\}_{n\geq -2k}$ be a well defined solution of the system (1.1). Then for $n = 0, 1, ...,$*

$$x_{(2k+2)n+1} = \gamma \prod_{j=1}^{n} \frac{u_{2((k+1)j-1)+1}}{u_{2(k+1)j+1}},$$

$$x_{(2k+2)n+2i} = x_{2i-(2k+2)} \prod_{j=0}^{n} \frac{u_{2((k+1)j+i-1)}}{u_{2((k+1)j+i)}},\ i = 1, 2, ..., k+1,$$

$$x_{(2k+2)n+2i+1} = x_{2i-(2k+1)} \prod_{j=0}^{n} \frac{u_{2((k+1)j+i-1)+1}}{u_{2((k+1)j+i)+1}},\ i = 1, 2, ..., k,$$

$$y_{(2k+2)n+1} = \lambda \prod_{j=1}^{n} \frac{v_{2((k+1)j-1)+1}}{v_{2(k+1)j+1}},$$

$$y_{(2k+2)n+2i} = y_{2i-(2k+2)} \prod_{j=0}^{n} \frac{v_{2((k+1)j+i-1)}}{v_{2((k+1)j+i)}},\ i = 1, 2, ..., k+1,$$

$$y_{(2k+2)n+2i+1} = y_{2i-(2k+1)} \prod_{j=0}^{n} \frac{v_{2((k+1)j+i-1)+1}}{v_{2((k+1)j+i)+1}},\ i = 1, 2, ..., k.$$

where

$$\gamma = \frac{\prod_{j=0}^{k} y_{-2j}}{\prod_{j=1}^{k} x_{1-2j} \left(a_0 + b_0 \prod_{j=0}^{k} y_{-2j}\right)},\ \lambda = \frac{\prod_{j=0}^{k} x_{-2j}}{\prod_{j=1}^{k} y_{1-2j} \left(\alpha_0 + \beta_0 \prod_{j=0}^{k} x_{-2j}\right)}.$$

and for $s = 0, 1,$

$$u_{2n+s} = \left[\prod_{j=0}^{n-1} (a_{2j+1+s}\alpha_{2j+s})\right] u_s + \\ + \sum_{j=0}^{n-1} \left[\prod_{i=j+1}^{n-1} (a_{2i+1+s}\alpha_{2i+s})\right] (a_{2j+1+s}\beta_{2j+s} + b_{2j+1+s}),$$

$$v_{2n+s} = \left[\prod_{j=0}^{n-1} (a_{2j+s}\alpha_{2j+1+s})\right] v_s + \\ + \sum_{j=0}^{n-1} \left[\prod_{i=j+1}^{n-1} (a_{2i+s}\alpha_{2i+1+s})\right] (\alpha_{2j+1+s}b_{2j+s} + \beta_{2j+1+s}),$$

and when the coefficients are constants,

$$\gamma = \frac{\prod_{j=0}^{k} y_{-2j}}{\prod_{j=1}^{k} x_{1-2j} \left(a + b \prod_{j=0}^{k} y_{-2j}\right)},\ \lambda = \frac{\prod_{j=0}^{k} x_{-2j}}{\prod_{j=1}^{k} y_{1-2j} \left(\alpha + \beta \prod_{j=0}^{k} x_{-2j}\right)},$$

and for $s = 0, 1,$

$$u_{2n+s} = (a\alpha)^n u_s + \frac{(a\alpha)^n - 1}{a\alpha - 1}(a\beta + b), \;\; v_{2n+s} = (a\alpha)^n v_s + \frac{(a\alpha)^n - 1}{a\alpha - 1}(\alpha b + \beta),$$

if $a\alpha \neq 1$, *and*

$$u_{2n+s} = u_s + (a\beta + b)n, \;\; v_{2n+s} = v_s + (\alpha b + \beta)n,$$

if $a\alpha = 1$.

Now, we give a result on the periodicity of well defined solutions of the system (1.1) with constant coefficients.

Theorem 2.2 *Assume that* $(a\beta + b)x_0x_{-2}...x_{-2k} = (\alpha b + \beta)y_0y_{-2}...y_{-2k} = 1 - a\alpha$ *and* $(1 - a\alpha)(a\beta + b)(\alpha b + \beta) \neq 0$. *Then all (well defined) solutions of system* (1.1) *are periodic with period* $2k + 2$.

Proof. Assume that

$$x_0x_{-2}...x_{-2k} = \frac{1 - a\alpha}{a\beta + b}, \; y_0y_{-2}...y_{-2k} = \frac{1 - a\alpha}{\alpha b + \beta},$$

then from (2.1) and (2.2), we have

$$u_n = ... = u_0 = \frac{a\beta + b}{1 - a\alpha}, \; v_n = ... = v_0 = \frac{\alpha b + \beta}{1 - a\alpha}, \; n \in \mathbb{N}_0. \tag{2.35}$$

From (2.30) and (2.35), we get

$$x_{n+2} = x_{n-2k}, \;\; y_{n+2} = y_{n-2k}, \; n \in \mathbb{N}_0.$$

That is, the solutions are periodic with period $2k + 2$. □

Remark 2.1 If we take $\alpha_n = a_n$, $\beta_n = b_n, n \in \mathbb{N}_0$, then for the initial values $x_{-i} = y_{-i}$, $i = 0, ..., 2k$, the system (1.1) becomes the equation

$$x_{n+1} = \frac{\prod_{j=0}^{k} x_{n-2j}}{\prod_{j=1}^{k} x_{n-(2j-1)} \left(a_n + b_n \prod_{j=0}^{k} x_{n-2j}\right)}. \tag{2.36}$$

Particular cases of Equation (2.36) with constant coefficients were investigated in [6] ($k = 1$, $a_n = a = 1$, $b_n = b = \pm 1$ or $a_n = a \neq 1$, $b_n = b \in \mathbb{R}$) and [5] ($k = 2$, $a_n = a = \pm 1$ $b_n = b = \pm 1$). It is easy to check that the formulae of the solutions in [6], [5] are direct consequences of our formulae.

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