Analele științifice
ale Universității „Al. I. Cuza" din Iași
Tomul XXX, s. I a, Matematică, 1984 — 4

Romanian-Japanese Colloquium
on Finsler Geometry, Iași,
Romania, 16—25 August, 1984

Greetings

Ladies and Gentlemen,
Dear Friends,

In my position of honorary president of this meeting, it is a very agreeable duty and a great honour for me, to welcome our guests and especially the Japanese mathematicians working in Finsler Geometry, who have covered thousands of kilometers over earth and water, over continents and oceans, to join their Romanian colleagues interested in the same topics. This is a beautiful and a magnificent proof of the universality of our science and of the unity of the Scientific community.

May the spirit of such meetings animate all other human relations.

I think it is our duty, on the occasion of this meeting, to pay a warm and sincere homage to the creator of Finsler Geometry, one of the greatest geometers of the first half of our century, famous not only in geometry, but also in many mathematical fields, to Elie Cartan, whose work is followed even now, in many theories, even if sometimes their origin is forgotten by people who don't like history.

With his genius, starting from Finsler's beautiful 1917 paper on variational calculus, he gave rise to a flourishing geometry.

I am happy that now it is recognized that among the first followers o Cartan — otherwise not only in Finsler Geometry — it was a geometer from the mathematical school of this University; his first paper on Finsler Geometry appeared half a century ago. And since Prof. Matsumoto writes in one of his papers, that this geometer, prof. Mendel Haimovici — my brother — was the father of Romanian Finslerians, it follows logically that I am their uncle; And as an uncle, I wish you all, guests from abroad and all my nephews, a successful colloquium.

I cannot end my short speech, without expressing my deep regret that Prof. A. Kawaguchi is no more with us.

Prof. Adolf Haimovici

Ladies and Gentlemen

The most part of my study consists of Finsler spaces and its generalizations.

Recently, the studies of Finsler geometry have been expanded extremely by the groups of Finsler geometers of Japan and Romania and leaders of the groups are Professor Dr. Makoto Matsumoto and Professor Dr. Radu Miron.

In this time, I am very happy to hear that the Romanian-Japanese Colloquium on Actual problems in Finsler geometry *supported by historical Universities of the ancient cities Iaşi, Braşov and Bucureşti is organised in Romania.*

I hope that the colloquium has many excellent results and contributes to the further development of Finsler geometry.

Thank you

Akitsugu Kawaguchi

Professor Akitsugu Kawaguchi died by an unexpected illness on 30th of July, 1984 at Chigasaki, Japan. Before his demise, he trusted his greetings to Romanian-Japanese Colloquium on Finsler Geometry — where he was one of Honorary Presidents — with Professor Hideo Izumi and me.

Shoji Watanabe

Dear Colleagues,
Distinguished Guests

The theory of Finsler manifolds is one most important and far reaching development of methods and ideas of classical Riemannian geometry. It is fundamental in the modern theory of calculus of variations, analytic mechanics, global analysis and it has of course been extensively worked on from many different points of view. It is a dynamic field of research which owes many important contributions to people present at this conference.

It is a great pleasure for me to welcome on behalf of the Senate of University of Iaşi the specialists from Japan and Romania who have kindly accepted to participate and contribute at this conference on Finsler geometry.

We hope that the results presented at this meeting as well as the exchange of ideas will be of great value for the progress of this field. The personal contacts renewed or newly established at such an occasion are of great importance beyond the immediate scientific interest. Pursuing the road of mutual contacts and human understanding the mathematicians can stengthen the forces securing a peaceful world.

I sincerely wish our distinguished guests from Japan profitable and pleasant days during this conference. I believe that they will use this opportunity to know better our country as well as the achievements of romanian people in spiritual and economic life.

Professor Viorel Barbu
Rector of University „Al. I. Cuza", Iaşi

Ladies and Gentlemen,

BY

Allow me to make a little speech on the inaugural motives of this Symposium. The first motive was my paper on 3-dimensional Finsler spaces dedicated to Prof. Dr. S t a n i s l a w G o l a b on the occasion of his 70th birthday (1973). The second motive was Prof. R. M i r o n's letter to me, dated Sept. 8, 1974. Allow me to quote some lines from the preface of his letter:

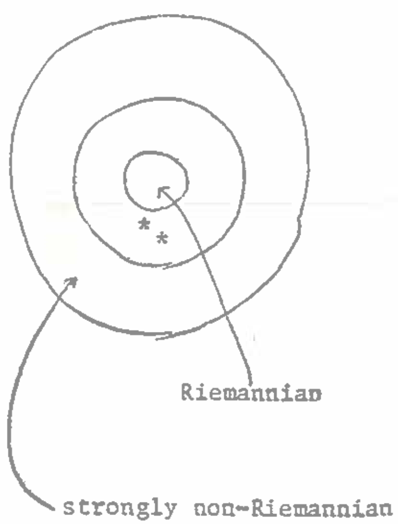
I (Miron) have received some weeks ago your paper A theory of 3-dimensional Finsler spaces in terms of scalars *published in Demonstratio Math. in order to review it for Zentralblatt für Math. The results established by you are very interesting for the case of 3-dimensional Finsler spaces.*

The question is why your method does not work in the general case of Finsler spaces of an arbitrary dimension? I think that the problem could be solved if one determines an invariant orthonormal frame associated with the space.

I should like to collaborate with you in solving this problem and to write together a note for publication.

There was found in this letter an excellent idea to introduce such an orthonormal frame field.

The third motive was the Symposium on Differential Geometry in Debrecen on Aug. 28—Sept. 3 in 1975. At this Symposium I first saw Prof. Miron where he gave a lecture On an invariant theory of the Finsler spaces, *in collaboration with me. On Aug. 31 we had a pleasant excursion to Hortobágy and Eger. At that time Prof. Miron and I asked Prof. F. Brickell the English of the most fitting name of spaces in which our invariant frame field can be defined. Thus, after long consideration and discussion we got the name „strongly non-Riemannian". In 1977 I wrote a paper on such spaces which was published*



in An. şt. Univ. Iaşi. In this paper it was proved that a Finsler space* of dimension >3 with (α, β)-metric is not strongly non-Riemannian.

Thus the time was ripe for holding a joint symposium on Finsler geometry, but the greatest inaugaral motive is Prof. Miron's enthusiasm (élan) about Finsler geometry. He is a passionate leader of studying our geometry in Romania and that he has many staff who were cultivated by glorious geometers. M. Haimovici and so on.

In Japan the seminar on Finsler geometry has been held every year since 1969 and various important papers have been written by members of the seminar. As you know, this field of differential geometry has been not so popular owing to its complicated conditions of metrics and connections. But its geometrical situation has been gradually clarified and we may expect much of applications. I hope you would study Finsler geometry more devotedly by taking this opportunity.

Thank you very much.

M. Matsumoto
松本誠

Ladies and Gentlemen,
Dear Colleagues

By the endeavour of some enthusiastic finslerists, this Romanian-Japanese Colloquium on Finsler geometry is finally under way.

Our guests, famous scientists, professors: Makoto Matsumoto, Masao Hashiguchi, Masaki Fukui, Shun-ichi Hojo, Yoshihiro Ichijyo, Fumio Ikeda, Satoshi Ikeda, Hideo Izumi, Toshio Sakaguchi, Choko Shibata, Hideo Shimada, Shoji Watanabe, Hiroshi Yasuda as well as Mrs Hashiguchi and Mrs Watanabe, travelled a long and tiresome way in order to participate in the first international meeting of large audience devoted to pure Finsler geometry.

We wish them welcome to Romania and great success in the works of the Colloquium. I address my heartful thanks to Professor Masao Hashiguchi, the main supporter of our scientific meeting.

Romanian geometers feel honored by the presence of some members of best school of Finsler geometry in the whole world, the Japanese one. They consider that direct dialogue with the Japanese colleagues in the oldest Romanian University, the „Al. I. Cuza" University followed by the discussions at the Universities of Braşov and Bucureşti, may determine a real progress in a field of the geometry with a brilliant theoretic and applicative perspective.

Much honoured guests from Japan! We know very well your fruitful achievements. They are fundamental in Finsler geometry and its applications. They culminate with the illustrious professor Makoto Matsumoto's monograph Foundations of Finsler Geometry and Special Finsler Spaces. *We highly appreciate them.*

No doubt you are informed that the study of Finsler geometry in Romania has a tradition of half a century. The first paper written by a Romanian is Formules fondamentales dans la théorie des hypersurfaces d'un espace de Finsler *and was published in Comptes Rendus de l'Acad. Sci., Paris, 1934. It represents the beginnings of subspaces of Finsler space theory. We express our grief that the author of this paper, „the father of Romanian finslerists", Academician Mendel Haimovici died 11 years ago. He would have reached today an age allowing him to organise personally this Romanian-Japanese Colloquium. I am sure that he would have done it with great pleasure and competence.*

In the last 50 years, the study of Finsler geometry in Romania followed an ascending trajectory, tangent to some new directions and tendencies, marked by three National Seminars and by the profitable collaboration of the Japanese School.

All these will be common bridges of approaching modern problems interesting the community of the finslerists over the world.

Today, when the mankind looks anxiously to the future, let us hope that Romanian-Japanese Colloquium of Finsler geometry will be a flower of the friendship of our people, in a better and more generous world.

Radu Miron

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ON GENERALIZED MIRON-BERWALD CONNECTIONS

BY

TADASHI AIKOU and MASAO HASHIGUCHI

In the spring last year the second author enjoyed studying in Iaşi at the invitation of Prof. R. M i r o n. Then he was invited to the various conferences and honored to have opportunities to talk about topics of our mutual interest [3]. The significance of axioms in Finsler geometry was especially emphasized by taking account of generalized Berwald spaces [2, 5] and generalized Finsler spaces [9,4].

By Prof. M. M a t s u m o t o [7], generalized Berwald spaces are concerned with Finsler connections of Berwald type with surviving (h) h-torsion $T_j{}^i{}_k$. On the other hand, in his recent paper [8], which is read to this Colloquium, Prof. Matsumoto has treated Finsler connections induced on a hypersurface of a Finsler space and then obtained interesting Finsler connections of Berwald type with surviving (v) hv-torsion $P^i{}_{jk}$.

In the present lecture, continued from [3], we would like to discuss Finsler connections of Berwald type generally (§1) and to consider what kind of Finsler connection should be reasonable as a generalization of the Berwald connection (§2). For fuller particulars refer to our appearing paper [1]. And lastly (§3) we shall notice that the results are applicable to the case of generalized Finsler spaces. The terminology and notations are referred to Matsumoto's monograph [6].

§1. **L-Metrical Finsler Connections**. We are first concerned with a Finsler space. Let L and g_{ij} denote the fundamental function and the fundamental tensor respectively. And we shall express a Finsler connection by its coefficients $(F_j{}^i{}_k, N^i{}_k, C_j{}^i{}_k)$.

In a Finsler space there are known two canonical Finsler connections, that is, the Cartan one, $C\Gamma$, and the Berwald one, $B\Gamma$. They are uniquely determined by the following systems of axioms:

$C\Gamma$ (Matsumoto [6])	$B\Gamma$ (Okada [11])
(C1) $g_{ij|k}=0$,	(B1) $L_{|k}=0$,
(C2) $D^i{}_k\equiv F_0{}^i{}_k-N^i{}_k=0$,	(B2) $D^i{}_k=0$
(C3) $T_j{}^i{}_k\equiv F_j{}^i{}_k-F_k{}^i{}_j=0$,	(B3) $T_j{}^i{}_k=0$,
(C4) $S_j{}^i{}_k\equiv C_j{}^i{}_k-C_k{}^i{}_j=0$,	(B4) $P^i{}_{jk}\equiv \dot\partial_k N^i{}_j-F_k{}^i{}_j=0$,
(C5) $g_{ij}|_k=0$,	(B5) $C_j{}^i{}_k=0$.