(iii) $K\equiv_{W}K'$ *if* $K\leqslant_{W}K'$ *and* $K'\leqslant_{W}K$.

For n-contextual transducers the theorem 3.17 is slightly modified.

4.8. **Theorem.** *Let* W *be a finite subset of* V. *Then* $1\text{-}CT\equiv_{W}n\text{-}CT$ *for any* $n>1$.

Proof. The relation $1\text{-}CT\leqslant_{W}n\text{-}CT$ follows immediately. In order to show that $n\text{-}CT\leqslant_{W}1\text{-}CT$ we consider $T'=(Q', q'_0, P')\in n\text{-}CT$. We initially set $Q=Q'$, $P=\varnothing$. For any rule

$$(q, <c_1, \dots, c_n>)\to(q', <c'_1, \dots, c'_n>)\in P'$$

we add to Q $2n-2$ new states $q_1, \dots, q_{n-1}, q'_1, \dots, q'_{n-1}$ and to P the following set of rules:

$$(q, <c_1>)\to(q_1, <A_1>),$$

$$\dots$$

$$(q_{n-1}, <c_n>)\to(q'_{n-1}, <c'_n>),$$

$$(q'_{n-1}, <A_{n-1}>)\to(q'_{n-2}, <c'_{n-1}>),$$

$$\dots$$

$$(q'_1, <A_1>)\to(q', <c'_1>)$$

where $A_1, \dots, A_{n-1}$ are new symbols that do not belong to W and that are not implicated in the definition of T'. The fact that $T=(Q, q_0, P)\equiv_{W}T'$ is proved in the same manner as in 3.17 #

Since we can consider n-contextual systems as being n-contextual transducers with a single state the following theorem holds true.

4.9. **Theorem.** $CS<CT$.

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15. XI.1987

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ANALELE ŞTIINŢIFICE ALE UNIVERSITĂŢII „AL. I. CUZA" DIN IAŞI
Tomul XXXIII, s. I a. Matematică, 1987, f. 4

ABOUT RECURSIVE DOMAIN EQUATIONS

BY

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Introduction. The paper presents some facts about solving recursive domain equations, which lead us, sometimes, to obtain in a simpler way solutions of such equations.

In the first section we assume that there is a solution in a given category and we try to find a solution in a full subcategory of it.

In the second, a proof of the main theorem of [1] in a more generally case is given. The paper also contains examples of using the facts described above.

1. On Solving Equations in Subcategories. We start by recalling the recursive domain equation problem:

Let $\mathcal{C}$ be a category and let $F:\mathcal{C}\to\mathcal{C}$ be a functor. We try to find an object X of the category $\mathcal{C}$ such that $F(X)$ is isomorphic to X.

We assume that there is an object X of the category $\mathcal{C}$ which is a solution of the problem. Let $\bar{\mathcal{C}}\subseteq\mathcal{C}$ a full subcategory of $\mathcal{C}$. Can we find another solution in $\bar{\mathcal{C}}$ of the problem, by using the solution in $\mathcal{C}$?

Notation. If P is an object of the category $\mathcal{C}$, we define a functor $\mathcal{U}_P:\bar{\mathcal{C}}\to\mathcal{E}ns$ if we set:

1) $\mathcal{U}_P(A)=\mathrm{Hom}_{\mathcal{C}}(P, A)$, for any object A of the category $\bar{\mathcal{C}}$.

2) $\mathcal{U}_P(f)(u)=fu$, for any morphism $f\in\mathrm{Hom}_{\bar{\mathcal{C}}}(A, B)$ $(=\mathrm{Hom}_{\mathcal{C}}(A, B))$ and $u\in\mathrm{Hom}_{\mathcal{C}}(P, A)$.

Definition. *An object* P *of* $\mathcal{C}$ *is said to have a reflection in* $\bar{\mathcal{C}}$ *if the functor* $\mathcal{U}_P$ *is representable (i.e. there is a functorial isomorphism between* $\mathcal{U}_P$ *and* $\mathcal{U}_{\bar{P}}$ *for some object* $\bar{P}$ *of* $\bar{\mathcal{C}}$*).*

Theorem. *An object* P *of* $\mathcal{C}$ *has a reflection in* $\bar{\mathcal{C}}$ *if and only if there is an object* $\bar{P}$ *of* $\bar{\mathcal{C}}$ *and a morphism* $i_P:P\to\bar{P}$ *in* $\mathcal{C}$ *such that for any object* A *of* $\bar{\mathcal{C}}$ *and any morphism* $f:P\to A$ *in* $\mathcal{C}$, *there is a unique morphism* $\bar{f}:\bar{P}\to A$ *such that the following diagram is commutative:*

$$\begin{tikzcd} P \arrow[rr, "i_P"] \arrow[dr, "f"'] & & \bar{P} \arrow[dl, "\bar{f}"] \\ & A & \end{tikzcd}$$

We say that the pair $(\bar{P}, i_P)$ (or briefly $\bar{P}$) reflects P in $\bar{\mathcal{C}}$.

We outline the main ideas of the proof:

„if" We define a functorial isomorphism Φ from $\mathcal{E}_P$ to $\mathcal{E}_{\bar P}$ if we set: $\Phi(A)(f) = f \in \mathrm{Hom}_{\bar{\mathcal{C}}}(\bar P, A)$ for any $f \in \mathrm{Hom}_{\mathcal{C}}(P, A)$

„only if" Let be a functorial isomorphism from $\mathcal{E}_P$ to $\mathcal{E}_{\bar P}$, i_P and $\bar f$ are defined as follows:

$i_{\bar P} = \Phi(\bar P)^{-1}(1_{\bar P})$ and

$\bar f = \Phi(A)(f)$ for any object A of $\bar{\mathcal{C}}$ and any morphism $f : P \to A$ of $\mathcal{C}$.

Examples. 1) Let $\mathcal{cl}$ be the category of commutative domains and $\mathcal{X}$ the full subcategory of fields. Then any object A of $\mathcal{cl}$ is reflected in $\mathcal{X}$ by its field of fractions.

2) Let $\mathcal{T}$ be the category of metric spaces and of uniformly continous maps and $\bar{\mathcal{T}}$ be the full subcategory of complete metric spaces. Then, any metric space A is reflected in $\bar{\mathcal{T}}$ by its completion $\ddot{A}$.

3) Let $\mathcal{T}$ be the category of locally compact topological spaces and $\bar{\mathcal{T}}$ the full subcategory of compact spaces. Then any object A of $\mathcal{T}$ is reflected in $\bar{\mathcal{T}}$ by its Alexandroff compactification $\dot{A}$.

The simple proof of the next result is omitted.

Proposition. (i) *Any two objects of $\mathcal{C}$ which reflect the same object in $\bar{\mathcal{C}}$, are isomorphic.*

(ii) *If two objects of $\mathcal{C}$ are isomorphic and have a reflection in $\bar{\mathcal{C}}$, then they are reflected by isomorphic objects in $\bar{\mathcal{C}}$.*

(iii) *If two objects of $\bar{\mathcal{C}}$ are isomorphic, then any object of $\mathcal{C}$ which reflects one of them in $\bar{\mathcal{C}}$, also reflects the second in $\bar{\mathcal{C}}$.*

Definition. *A functor $F : \mathcal{C} \to \mathcal{C}$ is said to preserve the reflections, if the pair $(F(\bar P), F(i_P))$ reflects the object $F(P)$ of $\mathcal{C}$, for any pair $(\bar P, i_{\bar P})$ which reflects the object P of $\mathcal{C}$.*

Remark. If A is an object of $\bar{\mathcal{C}}$ and F preserves the reflections, then $F(A)$ is an object of $\bar{\mathcal{C}}$, since the pair $(A, 1_A)$ reflects A in $\bar{\mathcal{C}}$.

The proof of the main result of this paragraph is obvious.

Theorem. *If a functor $F : \mathcal{C} \to \mathcal{C}$ preserves the reflections and there is some object X of $\mathcal{C}$ which has a reflection in $\bar{\mathcal{C}}$ such that $F(X)$ is isomorphic to X, then any object of $\mathcal{C}$ which reflects X has the same property.*

Example. Let $\mathcal{T}$ be the category of metric spaces whose metric takes values in $[0, 1]$. A morphism from A to B in $\mathcal{T}$ is a map $f : A \to B$ such that

$$d_B(f(x), f(y)) \leqslant d_A(x, y) \text{ for any } x, y \in A.$$

If A is any set and P an object of $\mathcal{T}$ we define a metric on $A \times P$ by setting:

$$d_{A\times P}((a, x), (b, y)) = \begin{cases} 1, & \text{if } a \neq b \\ 1/2\, d_P(x, y), & \text{if } a = b \end{cases}$$

If P and Q are disjoint metric spaces, we define a metric on $P \cup Q$ by setting:

$$d_{P\cup Q}(x, y) = \begin{cases} 1 & \text{if } x \in P,\ y \in Q \text{ or } x \in Q,\ y \in P, \\ d_P(x, y) & \text{for any } x, y \in P, \\ d_Q(x, y) & \text{for any } x, y \in Q. \end{cases}$$

Let p_0 be an arbitrary element. We define a functor $F : \mathcal{T} \to \mathcal{T}$ as follows: $F(P) = \{p_0\} \cup (A \times P)$, where the union is taken disjoint. Then we obtain the first recursive equation domain equation in [2].

$$P = \{p_0\} \cup (A \times P).$$

We can easily check [1] that : $P = \bigcup_{n=0}^{\infty} F^n(\{p_0\})$ is a solution in $\mathcal{T}$.

It can be checked that F preserves the reflections. Hence the completion $\bar P$ of P is a solution in $\bar{\mathcal{T}}$.

II. **A generalized version of the Smyth-Plotkin Theorem.** We start by recalling some facts [1].

Definition. *Let $\mathcal{C}$ be a category and $F : \mathcal{C} \to \mathcal{C}$ a functor. An F-algebra is a pair (A, α) where A is an object of $\mathcal{C}$ and $\alpha \in \mathrm{Hom}_{\mathcal{C}}(F(A), A)$. If (A, α) and (B, β) are F-algebras then a morphism $f : A \to B$ is said to be an F-algebras morphism if $\beta F(f) = f\alpha$.*

Lemma. *α is an isomorphism if the F-algebra is initial.*

Theorem. a) *Let $\mathcal{C}$ be category with an initial object $\perp$ and $F : \mathcal{C} \to \mathcal{C}$ a functor such that the functor $\Delta = (F^n(\perp), F^n(\alpha_{F(\perp)})) : \omega \to \mathcal{C}$ (α_B is the unique morphism in $\mathcal{C}$ from $\perp$ to B) has an inductive limit (μ, A) and $(F(\mu), F(A))$ is an inductive limit of $F\Delta$ then (A, α) is an initial F-algebra for some $\alpha : F(A) \to A$.*

b) *Let D be an object of $\mathcal{C}$, let $F : \mathcal{C} \to \mathcal{C}$ be a functor and let $\pi : D \to F(D)$ be a morphism in $\mathcal{C}$. If the functor $\Delta = (F^n(D), F^n(\pi)) : \omega \to \mathcal{C}$ has an inductive limit (μ, A) and $(F(\mu), F(A))$ is an inductive limit of $F\Delta$, then $F(A)$ is isomorphic to A.*

We shall prove a generalized version of this theorem.

If $\Delta = (A_n, f_n)_{n\geqslant 0} : \omega \to \mathcal{C}$ is a functor let $\Delta^{(1)} : \omega \to \mathcal{C}$ be the functor $(A_{n+1}, f_{n+1})_{n\geqslant 0}$.

Theorem. *Let $\mathcal{C}$ be a category, let A_0 be an initial object of $\mathcal{C}$, and let $F : \mathcal{C} \to \mathcal{C}$ be a functor. If there is a functor $\Delta = (A_n, f_n)_{n\geqslant 0} : \omega \to \mathcal{C}$ which satisfies the following conditions:*

1) *There is a functorial isomorphism $\Phi : F\Delta \to \Delta^{(1)}$,*

2) $\varinjlim \Delta = (\mu, A)$,

3) $\varinjlim F\Delta = (F(\mu), F(A))$,

then (A, α) is an initial F-algebra for some α.

Proof. We deduce from 1) that for any $n \in \omega$ there is an isomorphism $i_n : F(A_n) \to A_{n+1}$ such that the following diagram is commutative:

$$\begin{tikzcd} F(A_n) \arrow[r, "F(f_n)"] \arrow[d, "i_n"'] & F(A_{n+1}) \arrow[d, "i_{n+1}"] \\ A_{n+1} \arrow[r] & A_{n+2} \end{tikzcd}$$

The sequence of morphisms $\{\mu_{n+1} i_n\}_{n\geqslant 0}$ is an inductive cone from $F\Delta$ to A. Then 3) implies there is an unique morphism $\alpha : F(A) \to A$ such that the following diagram is commutative for any $n \in \omega$:

$$\begin{tikzcd} F(A) \arrow[rr, "\alpha"] & & A \\ & F(A_n) \arrow[ul, "F(\mu_n)"] \arrow[ur, "\mu_{n+1} i_n"'] & \end{tikzcd}$$

We shall prove that (A, α) is an initial F-algebra. Let (B, β) be an arbitrary F-algebra. Let $\nu_0 : A_0 \to B$ be the unique morphism from A_0 to B and $\nu_{n+1} = \beta F(\nu_n)\, i_n^{-1}$.

We show that $\{\nu_n\}_{n\geqslant 0}$ is an inductive cone from Δ to B, by mathematical induction. Condition (2) implies that there is a unique morphism $f : A \to B$ such that $f\mu_n = \nu_n$ for any $n \in \omega$. We check that $f\alpha F(\mu_n) = \beta F(f)\, F(\mu_n)$ for any $n \in \omega$. Since $\varinjlim F\Delta = (F(\mu), F(A))$ it follows that $f\alpha = \beta F(f)$. Hence f is an F-algebras morphism.

If $g : A \to B$ is any F-algebras morphism from (A, α) to (B, β) it is easy to check that $g\mu_n = \nu_n$ for any $n \in \omega$, (using the induction principle). Then (2) implies that $g = f$ and the proof is complete.

Theorem 2. *Let $\mathcal{C}$ be a category, let $F : \mathcal{C} \to \mathcal{C}$ be a functor and let $\Delta = (A_n, f_n)_{n\geqslant 0} : \omega \to \mathcal{C}$ satisfy the following conditions:*

1) *There is a functorial isomorphism* $\Phi : F\Delta \to \Delta^{(1)}$.

2) $\varinjlim \Delta = (\mu, A)$.

3) $\varinjlim F\Delta = (F(\mu), F(A))$;

Then $F(A)$ is isomorphic to A.

Proof. We shall use the same notation as in the proof of the preceding theorem. Let $\mathcal{C}_1$ be the category whose objects are the pairs (d, B), where B is an object of $\mathcal{C}$ and $d : A_0 \to B$ a morphism in $\mathcal{C}$. A morphism in $\mathcal{C}_1$ from (d, B) to (d_1, B_1) is a morphism $f : B \to B_1$ in $\mathcal{C}$ which makes the following diagram commutative:

$$\begin{tikzcd} B \arrow[rr, "f"] & & B_1 \\ & A_0 \arrow[ul, "d"] \arrow[ur, "d_1"'] & \end{tikzcd}$$

Let $F_1 : \mathcal{C}_1 \to \mathcal{C}_1$ be the following functor:

$F_1(d, B) = (F(d)\, i_0^{-1} f_0, F(B))$,

$F_1(f) = f$ for any $f : (d, B) \to (d_1, B_1)$.

Define the functor $\Delta_1 = ((d_n, A_n), f_n)_{n\geqslant 0}$ as follows:

$$d_n = \begin{cases} 1_{A_0} & \text{for } n = 0 \\ f_{n-1} f_{n-2} \dots f_1 f_0 & \text{for } n \geqslant 1. \end{cases}$$

It is easy to check that the definitions of $\mathcal{C}_1$, F_1, Δ_1 are correct and that the sequence of morphisms $\{i_n : F(A_n) \to A_{n+1}\}_{n\geqslant 0}$ is a functorial isomorphism from $F_1\Delta_1$ to $\Delta_1^{(1)}$.

We check that $\varinjlim \Delta_1 = (\mu, (\mu_0, A))$ and that $\varinjlim F_1\Delta_1 = (F(\mu), (F(\mu_0)\, i_0^{-1} f_0, F(A))) = (F_1(\mu), F_1(\mu_0, A))$. $(1_{A_0}, A_0)$ is an initial object in $\mathcal{C}_1$. Theorem 1 implies that (μ_0, A) is isomorphic to $F_1(\mu_0, A) = (F(\mu_0)\, i_0^{-1} f_0, F(A))$ in $\mathcal{C}_1$. We deduce that $F(A)$ is isomorphic to A in $\mathcal{C}$, and the proof is complete.

Example. It is sometimes easier to find a functor $\Delta : \omega \to \mathcal{C}$ which fulfills the conditions listed in theorem 2 and to manipulate this functor, instead of the functor $(F^n(A), F^n(\pi))$ which occurs in [1]. We shall give an example by using another equation of [2], slightly modified for the simplicity of our exposure.

We shall use the same notations as in the example of the first paragraph.

If P is any metric space, let $\mathcal{P}_c(P)$ be the set of its closed subsets. The Hausdorff distance between two closed subsets X, Y of P is:

$$d_{\mathcal{P}_c(P)}(X, Y) = \max\left(\sup_{y\in Y}\inf_{x\in X} d_P(x, y),\ \sup_{x\in X}\inf_{y\in Y} d_P(x, y)\right).$$

Hahn's theorem. *If P is a complete metric space then $\mathcal{P}_c(P)$ is a complete metric space too.*

For any set A, let $\mathcal{P}_c(A \times P)$ denote the set of closed subsets X of $A \times P$ such that if (a, x), $(a, y) \in X$ then $x = y$. The equation to be solved is:

$$P = \{p_0\} \cup \mathcal{P}_c(A \times P) = F(P).$$

Let A^n be the set of strings of elements of A whose length is equal to n.

$$A^{(n)} = \bigcup_{k=1}^{n} A^k,\quad A^+ = \bigcup_{k=1}^{\infty} A^k,\quad A^* = \bigcup_{k=0}^{\infty} A^k,\quad A = A^* \cup A^{\omega},\quad A^{(0)} = \varnothing.$$

Then

$$A^{(0)} \subseteq A^{(1)} \subseteq \dots \subseteq A^{(n)} \subseteq \dots \subseteq A^* \subseteq A.$$

We define a metric on A by setting

$$d(x, y) = \begin{cases} 0 & \text{if } x = y, \\ 1 & \text{if } x = e,\ y \neq e \ (e \text{ the empty string}), \\ 1 & \text{if } x = ax',\ y = by',\ a \neq b, \\ 1/2\, d(x', y') & \text{if } x = ax',\ y = ay'. \end{cases}$$

It is easy to check that d is indeed a metric, that $A^{(0)}, A^{(1)}, \dots, A$ are complete metric spaces and that A is the completion of A^*.

Define the functor $\Delta = (A_n, f_n)_{n\geqslant 0} : \omega \to \mathcal{C}$ by setting:

$$A_n = \{p_0\} \cup \mathcal{P}_c(A^{(n)}) = \{p_0\} \cup \mathcal{P}(A^{(n)}),$$

and $f_n : A_n \to A_{n+1}$ is the inclusion map for each n. Then:

$$F(A_n) = \{p_0\} \cup \mathcal{P}_c((A \times \{p_0\}) \cup (A \times \mathcal{P}(A^{(n)}))).$$

Let $i_n : F(A_n) \to A_{n+1}$ defined by:

$i_n(p_0) = p_0$ and if $M \in \mathcal{P}_c^*((A \times \{p_0\}) \cup (A \times \mathcal{P}(A^{(n)})))$ put $M = M_1 \cup M_2$, where $M_1 = M \cap (A \times \{p_0\})$ and $M_2 = M \cap (A \times \mathcal{P}(A^{(n)}))$. Then $i_n(M) = \{a \mid (a, p_0) \in M_1\} \cup \{ax \mid x \in M',\ (a, M') \in M_2 \text{ for some } M' \subseteq A^{(n)}\}$.

It is easy to check that i_n is an isometry and that the sequence of morphisms $\{i_n\}_{n\geqslant 0}$ is a functorial isomorphism between $F\Delta$ and $\Delta^{(1)}$.

Let $K = \{p_0\} \cup \mathcal{P}_c(A)$ and let $\mu_n : A_n \to K$ be the inclusion map. Then $\varinjlim \Delta = (\mu, K)$ and $\varinjlim F\Delta = (F(\mu), F(K))$ and K is consequently a solution of the given equation.

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Received 15.XI.1987

Analele științifice ale Universității „Al. I. Cuza" din Iași
Tomul XXXIII, s. I a, Matematică, 1987, f.asc. 4

RECENZII

M. BENSON, E. BENSON, R. ILSON: *The BBI Combinatory Dictionary. A guide to word combinations in English*, Amsterdam, John Benjamins B. V., 1986, XXXVI + 286 p., Dfl. 28

Conceput să fie complementar lucrării "The Lexicographic Description of English", "BBI Dictionary of English" este de o excepțională valoare pentru cei ce folosesc limba engleză ca limbă străină, pe care o studiază fără suportul limbii materne. Pentru a face clar de la început conținutul lucrării, autorii explică, în introducere, diferența dintre combinațiile fixe și cele libere, precizînd apoi că doar primele sînt cuprinse în acest dicționar. Combinațiile libere, expresiile fixe, sau alte idiomuri — în care înțelesul întregului nu reflectă sensul fiecărei părți componente — nu sînt, în general, incluse, deși construcțiile fixe mai importante, care nu se încadrează în nici un tip de combinație fixă, pot fi găsite în acest dicționar, sub specificația „misc". Într-o introducere amplă, cele două tipuri de combinații (numite „colocații") — cele gramaticale și cele lexicale — sînt descrise, în ideea de a face dicționarul accesibil și familiar celor ce-l folosesc. O descriere minuțioasă a felului în care sînt ordonate și descrise colocațiile înlesnește găsirea rapidă a cuvintelor căutate. E de remarcat și faptul că autorii dicționarului au acordat atenție diferențelor ce există între engleza britanică și engleza americană (se folosesc prescurtările BE, respectiv AE). Într-un cuvînt, "The BBI Dictionary of English" este un instrument excelent pentru cei care vor să învețe să vorbească și să scrie corect în limba engleză.

Dan Stoica

A. HILBERT: *Mathematik*, Nachschlagebücher für Grundlagenfächer, VEB Fachbuchverlag, Leipzig, 1987, 671 p.

Tratatul de față este un fel de ghid conținînd elemente de bază din toate domeniile matematice și permițînd o informare rapidă, amănunțită și accesibilă relativ la diferitele noțiuni, teoreme, reguli și metode din domeniile respective atît pentru studenții de la diferitele institute tehnice, cît și pentru cei care vor să-și amintească unele cunoștințe matematice necesare în cercetările lor de tehnică, economie sau științele naturii. În acest scop, cartea este împărțită în capitole, paragrafe și subparagrafe, tabla de materii este foarte amănunțită, figurile și exemplele sînt foarte numeroase (524 figuri și 414 exemple), cartea fiind prevăzută și cu o listă foarte bogată de notații cu semnificațiile respective (8 pagini), cît și un indice de materii detaliat. Diferitele ramuri ale matematicii sînt cuprinse în următoarele 13 capitole: 1. Din logica matematică; 2. Din teoria mulțimilor; 3. Numere, domenii de numere; 4. Asupra unui calcul practic cu numere reale și mărimi; 5. Ecuații și inecuații; 6. Funcții reale; 7. Calcul vectorial; 8. Calcul cu matrici; 9. Din teoria probabilităților, statistica descriptivă; 10. Elemente de geometrie; 11. Geometria analitică în plan și spațiu; 12. Calcul infinitezimal (diferențial și integral); 13. Din informatică.

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