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TERM REWRITING SYSTEMS AND P/T-NETS

BY

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Introduction. The aim of this paper is to establish some relationships between P/T-nets and a class of term-rewriting systems. The general framework presented in the first section, follows the line of [1], [3], [4], to which we send for missing concepts. In the second section we recall the notion of term rewriting system with substitution closed balance (SCB for short) which was introduced in [2]. The last section presents the above mentioned relationship.

1.1. Term Rewriting Systems. Let N be the set of natural numbers. $\mathcal{V}$ be a denumerable set of variable symbols (variables, for short), $\mathcal{F}_n$ be a set of functional symbols of arity n and $\mathcal{F}=\cup\{\mathcal{F}_n/n\geq 0\}$. For any $f\in\mathcal{F}$, ar (f) denotes the arity of f. We denote variables by $x, y, z, \dots$; constants (functional symbols of arity 0) by $a, b, c, \dots$, functional symbols of arity at least 1 by $f, g, h, \dots$. The set $M(\mathcal{F}, \mathcal{V})$ of first-order terms constructed from $\mathcal{F}$ and $\mathcal{V}$, is defined by:

(i) $\mathcal{V}\cup\mathcal{F}_0\subseteq M(\mathcal{F}, \mathcal{V})$

(ii) if $f\in\mathcal{F}_n$ and $p_1, \dots, p_n\in M(\mathcal{F}, \mathcal{V})$ then $f(p_1, \dots, p_n)\in M(\mathcal{F}, \mathcal{V})$.

When $\mathcal{F}$ and $\mathcal{V}$ are fixed from the context, we shall usually denote $M(\mathcal{F}, \mathcal{V})$ by $\mathcal{T}$ and $M(\mathcal{F}, \emptyset)$ by $\mathcal{T}_0$. Obviously, $\mathcal{T}_0\subseteq\mathcal{T}$. Also, we denote terms by α, β, p, q, ... Let N^* be the set of sequences of positive integers denoted by π, π_1, π_2, ... ; λ the empty sequence in N^* and let "$\cdot$" be the concatenation operation on sequences (usually omitted). For any term p, we define its set of occurrences as a finite subset $O(p)$ of the set N^*, by:

(i) if $p\in\mathcal{V}\cup\mathcal{F}_0$ then $O(p)=\{\lambda\}$

(ii) if $p=f(p_1, \dots, p_n)$, $n\geq 1$, then $O(p)=\{\lambda\}\cup\{i\pi/1\leq i\leq n,\ \pi\in O(p_i)\}$.

If $\pi\in O(p)$, we inductively define the subterm of p at π, as the term p/π:

(i) $p/\lambda=p$,

(ii) if $p=f(p_1, \dots, p_n)$, $n\geq 1$, and $\pi=i\pi'$, $1\leq i\leq n$, then $p/\pi=p_i/\pi'$.

If $p\in\mathcal{T}$ and $\pi\in O(p)$ then, for any term q, the replacement in p' of q at π is the term $p\,[\pi\leftarrow q]$, defined by:

(i) $p[\lambda\leftarrow q]=q$,

(ii) if $p=f(p_1, \dots, p_n)$, $n\geq 1$, and $\pi=i\pi'$, $1\leq i\leq n$, then $p[i\pi'\leftarrow\lambda]=f(p_1, \dots, p_i\,[\pi'\leftarrow q], \dots, p_n)$.

For any term p, let $\mathcal{V}(p)$ respectively $\mathcal{F}(p)$ denote the set of variables occuring in p, respectively the set of functional symbols occuring in p.

Consider the following functions:

(i) $|\cdot| : \mathcal{T} \to N$ by

$$|p| = \begin{cases} 1 & , \quad \text{if } p \in \mathcal{V} \cup \mathcal{F}_0 \\ 1 + \sum_{i=1}^{n} |p_i|, & \text{if } p = f(p_1, \ldots, p_n) \end{cases}$$

$|p|$ is called the length of the term p;

(ii) $\# : (\mathcal{F} \cup \mathcal{V}) \times \mathcal{T} \to N$ by

$$\#(\alpha, p) = \begin{cases} 0, & \text{if } \alpha,\ p \in \mathcal{V} \cup \mathcal{F}_0 \text{ and } \alpha \neq p \text{ or } \alpha \in \mathcal{F} \setminus \mathcal{F}_0 \text{ and } p \in \mathcal{V}_0 \cup \mathcal{F}_0 \\ 1, & \text{if } \alpha,\ p \in \mathcal{V} \cup \mathcal{F}_0 \text{ and } \alpha = p \\ \sum_{i=1}^{n} \#(\alpha, p_i), & \text{if } \alpha \in \mathcal{F} \setminus \mathcal{F}_0,\ p = f(p_1, \ldots, p_n) \text{ and } \alpha \neq f \\ 1 + \sum_{i=1}^{n} \#(\alpha, p_i), & \text{if } \alpha \in \mathcal{F} \setminus \mathcal{F}_0,\ p = f(p_1, \ldots, p_n) \text{ and } \alpha = f. \end{cases}$$

$\#(\alpha, p)$ is called the number of ocurrences of α in p.

A substitution is an endomorphism σ from $\mathcal{T}$ to itself such that $\sigma(x) = x$ except on a finite set of variables, Dom (σ), which we call the domain of σ. Substitutions can then be represented by the finite set of pairs $\{(x, \sigma(x)) / x \in \text{Dom}\ (\sigma)\}$. The application of a substitution σ to a term $p \in \mathcal{T}$ is written as $\sigma(p)$ or σp. Let Σ be the class of substitutions on $\mathcal{T}$ and ε the identity on $\mathcal{T}$. A term rewriting system (abbreviated TRS) is a finite set of pairs of terms called *rules*, $R = \{(\alpha_i, \beta_i) / 1 \leqslant i \leqslant n,\ n \geqslant 1\}$ such that, for $(\alpha, \beta) \in R$, $\mathcal{V}(\beta) \subseteq \mathcal{V}(\alpha)$. For $1 \leqslant i \leqslant n$, the rule (α_i, β_i) is usually denoted by $\alpha_i \to \beta_i$ or $r_i : \alpha_i \to \beta_i$ or $r_i : \alpha_i$ is its left-hand side (lhs (r_i)) and β_i is its right-hand side (rhs (r_i)). The sum $|r_i| = |\alpha_i| + |\beta_i|$ is called the length of the rule $r_i : \alpha_i \to \beta_i$.

Let $\mathcal{R}$ be the class of TRS's. For two terms p and q, we say that p is rewritten into q in R (in one step), $s \xrightarrow[R]{} t$, if there exist $\pi \in O(p)$, $\alpha_i \to \beta_i \in R$ and $\sigma \in \Sigma$ such that $p/\pi = \sigma(\alpha_i)$ and $q = p[\pi \leftarrow \sigma(\beta_i)]$. Occasionally we keep track of the information by writting $p \xrightarrow[[\pi, r_i, \sigma] R]{} q$. As usual $\xrightarrow[R]{+}$ is the transitive, and $\xrightarrow[R]{*}$ the reflexive and transitive closure of $\xrightarrow[R]{}$.

Let $|p \xrightarrow[R]{*} q| \in N$ denote the smallest n such that $p = s_0 \xrightarrow[R]{} s_1 \xrightarrow[R]{} \ldots \xrightarrow[R]{} s_n = q$ (if it exists).

Let $R \in \mathcal{R}$, p, $q \in \mathcal{T}$ and ρ be an equivalence relation on $\mathcal{T}$. We say that p is rewritten into q in R with respect to ρ, $p \xrightarrow[R, \rho]{} q$, if there exist p', $q' \in \mathcal{T}$ such that $p \rho p'$, $q \rho q'$ and $p' \xrightarrow[R]{} q'$.

1.2. P/T-nets *A fivetuple $N = (S, T, F, K, W)$ is called a place/transition net (P/T-net) iff:*

(a) *(S, T, F) is a net where the S-elements are called places and the T-elements are called transitions;*

(b) *$K : S \to N^+ \cup \{\infty\}$ is a capacity function;*

(c) *$W : F \to N^+$ is a weight function.*

For technical purposes, it is sometimes convenient to replace both F and W by a single function $W' : (S \times T) \cup (T \times S) \to N$ where $(x, y) \notin F$ iff $W'(x, y) = 0$ and $(x, y) \in F$ iff $W'(x, y) = W(x, y)$. In this paper we shall assume P/T-nets to be of the form (S, T, F, K, W) where W is a function from the set $(S \times T) \cup (T \times S)$ into the set of non-negative integers which agrees with F in the sense that $(x, y) \notin F$ iff $W(x, y) = 0$.

Let $N = (S, T, F, K, W)$ be a P/T-net. A function $M : S \to N$ is called a marking of N iff $M(s) \leqslant K(s)$ for all $s \in S$. A transition $t \in T$ is enabled at M iff for all $s \in S$: $W(s, t) \leqslant M(s) \leqslant K(s) - W(t, s)$.

If $t \in T$ is a transition which is enabled at a marking M then t may occur, yielding a new marking M' given by the equation:

$$M'(s) = M(s) - W(s, t) + W(t, s) \quad \text{for all } s \in S.$$

The occurrence of t changes the marking M into the new marking M'; we may denote this fact by $M[t > M'$.

In this paper, we will restrict ourselves to P/T-nets without capacity constraints (that is, we will assume all capacities to be infinite). A P/T-net will be thus denoted by $N = (S, T; F, W)$.

1.3. Multisets. Let A be a nonvoid set. A multiset on A is an unordered collection of elements of A, possibly with repetitions. For instance, (1, 2, 1, 3, 15, 15) is a multiset on N which is the same with the multiset (1, 1, 2, 3, 15, 15).

In the following, χ (maybe with a subscript or a superscript) denotes an arbitrary multiset (on a set A). For every multiset χ and every $a \in A$ let $\#(a, \chi)$ denote the number of occurrences of a in χ. By analogy with sets, the notation $a \in \chi$ means that a is an element of the multiset χ, that is, $\#(a, \chi) > 0$. If a is not an element of χ, we write $a \notin \chi$. Obviously, $a \notin \chi$ iff $\#(a, \chi) = 0$. By $|\chi|$ we denote the cardinal number of χ, that is, $|\chi| = \sum_{a \in \chi} \#(a, \chi)$. Because we shall use only finite multisets, infinity exceptions are not considered. If χ_1 and χ_2 are multisets on A, $\chi_1 \subseteq \chi_2$ denotes inclusion, that is, for every $a \in \chi_1$, $\#(a, \chi_1) \leqslant \#(a, \chi_2)$. Equality of the multisets χ_1 and χ_2, in symbols $\chi_1 = \chi_2$, holds iff $\chi_1 \subseteq \chi_2$ and $\chi_2 \subseteq \chi_1$. If $\chi_1 = \chi_2$ does not hold, we write $\chi_1 \neq \chi_2$. Proper inclusion is denoted by $\chi_1 \subset \chi_2$; by definition $\chi_1 \subset \chi_2$ means that $\chi_1 \subseteq \chi_2$ and $\chi_1 \neq \chi_2$. We extend the operations of union, intersection and difference from sets to finite multisets as follows:

Let χ_1, χ_2 be two multisets on A;

(a) The multiset $\chi_1 \cup \chi_2$ given by:

(i) $a \in \chi_1 \cup \chi_2$ iff $a \in \chi_1$ or $a \in \chi_2$

(ii) $\#(a, \chi_1 \cup \chi_2) = \#(a, \chi_1) + \#(a, \chi_2)$

is called the union of the multisets χ_1 and χ_2;

(b) The multiset $\chi_1 \cap \chi_2$ given by:

(i) $a \in \chi_1 \cap \chi_2$ iff $a \in \chi_1$ and $a \in \chi_2$

(ii) $\#(a, \chi_1 \cap \chi_2) = \min\ \{\#(a, \chi_1),\ \#(a, \chi_2)\}$

is called the intersection of the multisets χ_1 and χ_2;

(c) The multiset $\chi_1 \setminus \chi_2$ given by:

(i) $a \in \chi_1 \setminus \chi_2$ iff $a \in \chi_1$

(ii) $\#(a, \chi_1 - \chi_2) = \#(a, \chi_1) - \#(a, \chi_2)$

is called the difference of the multisets χ_1 and χ_2.

2. Term Rewriting Systems with Substitution Closed Balance. In the following, the concept of term rewriting system with substitution closed balance is essential.

Definition 2.1. *Let $R = \{\alpha_i \to \beta_i / 1 \leqslant i \leqslant n,\ n \geqslant 1\}$ be a TRS.*

(i) *For every $r_i : \alpha_i \to \beta_i \in R$, the number $B(r_i) = |\alpha_i| - |\beta_i|$ is called the balance of the rule r_i;*

(ii) *We say that R is with substitution closed balance iff for every $r_i \in R$ and every $\sigma \in \Sigma$, $B(r_i) = B(\sigma r_i)$, where $B(\sigma r_i) = |\sigma\alpha_i| - |\sigma\beta_i|$.*

In the following, we shall denote by SCB the class of term rewriting systems with substitution closed balance and we shall prove some properties of these types of TRS's.

Proposition 2.2. *Let $R \in \mathcal{R}$. Then, $R \in SCB$ iff for every $r \in R$ and every $x \in V$, $\#(x, \text{lhs}(r)) = \#(x, \text{rhs}(r))$.*

Proof. Let $R \in SCB$, $r \in R$ and $x \in V$. If $\#(x, \text{lhs}(r)) = 0$, then $\#(x, \text{rhs}(r)) = 0$, because $\mathcal{V}(\text{rhs}(r)) \subseteq \mathcal{V}(\text{lhs}(r))$. If $\#(x, \text{lhs}(r)) > 0$, let $k = \#(x, \text{lhs}(r))$. We shall prove that $\#(x, \text{rhs}(r)) = k$. Suppose by contradiction that $\#(x, \text{rhs}(r)) \neq k$ and let $\#(x, \text{rhs}(r)) = l < k$. Let σ be the substitution $\{x \leftarrow p\}$ where $p \in \mathcal{T}$ and $|p| > 1$. Then, $B(\sigma(r)) = |\sigma(\text{lhs}(r))| - |\sigma(\text{rhs})(r))| = (|\text{lhs}(r)| - k|x| + k|p|) - (|\text{rhs}(r)| - l|x| + l|p|) = |\text{lhs}(r)| - |\text{rhs}(r)| + (k-l)(|p|-1) = B(r) + (k-l)(|p|-1)$. Because $k > l$ and $|p| > 1$ we deduce that $B(\sigma(r)) > B(r)$, contradicting the fact that $R \in SCB$. Analogously, if $\#(x, \text{rhs}(r)) = l > k$.

Conversely, let $R \in \mathcal{R}$ be such that for every $r \in R$ and every $x \in V$, $\#(x, \text{lhs}(r)) = \#(x, \text{rhs}(r))$.

Let $r : \alpha \to \beta \in R$ and $\sigma \in \Sigma$. Without loss of generality we can assume that $\text{Dom}(\sigma) = \mathcal{V}(\alpha)$ $(= \mathcal{V}(\beta))$. It is easy to verify by induction on $|\mathcal{V}(\alpha)|$ that $B(r) = B(\sigma(r))$, and hence $R \in SCB$. Q.E.D.

Corollary 2.3. *If $R \in SCB$, then R is regular, that is, for every $r : \alpha \to \beta \in R$, $\mathcal{V}(\alpha) = \mathcal{V}(\beta)$.*

Proposition 2.4. *Let $R \in SCB$ and $p, q \in \mathcal{T}$ be such that $p \xrightarrow[R]{*} q$. Then, for every $x \in V$, $\#(x, p) = \#(x, q)$.*

Proof. Let $R \in SCB$ and $p, q \in \mathcal{T}$ be such that $p \xrightarrow[R]{*} q$. From Corollary 2.3 it follows that R is regular. Let $x \in V$. If $x \notin \mathcal{V}(p)$, then $x \notin \mathcal{V}(q)$, since for every $\alpha \to \beta \in R$, $\mathcal{V}(\beta) \subseteq \mathcal{V}(\alpha)$. If $x \in \mathcal{V}(p)$, suppose that q does not contain the variable x. Then, x has to vanish in $p \xrightarrow[R]{*} q$, contradicting the fact that R is regular. Hence, for every $x \in V$, $\#(x, p) = \#(x, q)$. Q.E.D

Definition 2.5. *Let $R = \{\alpha_i \to \beta_i / 1 \leqslant i \leqslant n,\ n \geqslant 1\} \in \mathcal{R}$ and $f \in \mathcal{F}$. For every $1 \leqslant i \leqslant n$, the number $B_i(f) = \#(f, \alpha_i) - \#(f, \beta_i)$ is called the balance of f in the rule r_i.*

Proposition 2.6. Let $R = \{\alpha_i \to \beta_i / 1 \leqslant i \leqslant n,\ n \geqslant 1\} \in SCB$ and $f \in \mathcal{F}$. Then, for every $\sigma \in \Sigma$ and every $i \in \{1, \ldots, n\}$, $B_i(f) = \#(f, \sigma(\alpha_i)) - \#(f, \sigma(\beta_i))$.

Proof. Let $R = \{\alpha_i \to \beta_i / 1 \leqslant i \leqslant n,\ n \geqslant 1\} \in SCB$, $f \in \mathcal{F}$, $i \in \{1, \ldots, n\}$ and $\sigma \in \Sigma$.

We must consider only substitutions of the form $\sigma = \{x \leftarrow p\}$, where $x \in \mathcal{V}(\alpha_i)$ and $p \in \mathcal{T}$.

Let $\#(x, \alpha_i) = k \geqslant 1$. Since $R \in SCB$, by Proposition 2.2, $(x, \beta_i) = k$. Then, $\#(f, \sigma(\alpha_i)) - \#(f, \sigma(\beta_i)) = (\#(f, \alpha_i)) + k\,\#(f, p) - (\#(f, \beta_i) + k\,\#(f, p)) = \#(f, \alpha_i) - \#(f, \beta_i) = B_i(f)$.

Q.E.D.

3. TRS and P/T-nets. Let $R \in \mathcal{R}$ and $p \in \mathcal{T}$. We shall denote by $\mathcal{F}(R)$ the set $\mathcal{F}(R) = \bigcup_{r \in R} \mathcal{F}(\text{lhs}(r)) \cup \bigcup_{r \in R} \mathcal{F}(\text{rhs}(r))$ and by $\mathcal{F}(R; p)$ the set $\mathcal{F}(R; p) = \mathcal{F}(R) \cup \mathcal{F}(p)$.

Definition 3.1. *Let $R = \{\alpha_i \to \beta_i / 1 \leqslant i \leqslant n,\ n \geqslant 1\} \in SCB$. The P/T-net $N_R = (S_R, T_R; F_R, W_R)$ given by:*

(i) $S_R = \{s_f / f \in \mathcal{F}(R)\}$,

(ii) $T_R = \{t_i / 1 \leqslant i \leqslant n\}$,

(iii) $(s_f, t_i) \in F_R$, iff $\#(f, \alpha_i) \geqslant 1$,

$(t_i, s_f) \in F_R$, iff $\#(f, \beta_i) \geqslant 1$,

(iv) $W_R : F_R \to N$ by:

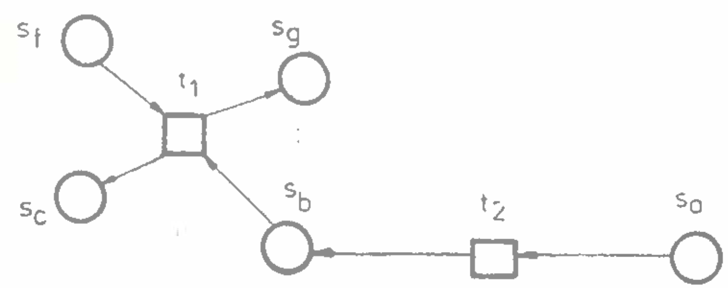
$$W_R(s_f, t_i) = \#(f, \alpha_i)$$

$$W_R(t_i, s_f) = \#(f, \beta_i)$$

is called the P/T-net associated to R (or induced by R).

In the following, for any $R \in SCB$, by $N_R = (S_R, T_R; F_R, W_R)$ we shall denote the P/T-net induced by R.

Example 3.2. Let $R = \{f(x, b) \to g(x, c),\ a \to b\} \in SCB$. The P/T-net induced by R is:



Remark 3.3. There exist $R_1, R_2 \in SCB$, $R_1 \neq R_2$, such that $N_{R_1} = N_{R_2}$. For instance, $N_R = N_{R'}$, where $R = \{f(x, b) \to g(x, c),\ a \to b\}$ and $R' = \{f(b, x) \to g(c, x),\ a \to b\}$.

Definition 3.4. *Let $R \in SCB$, $p \in \mathcal{T}$ and $N_R = (S_R, T_R; F_R, W_R)$ be the P/T-net induced by R. The mapping $M_p : S_R \to N$ given by:*

$$M_p(s_f) = \#(f, p) \text{ for any } s_f \in S_R,$$

is called the marking of N_R induced by p.

Example 3.5. For R from Example 3.2 and $p = f(c, a)$, M_p is the marking:

$$M_p(s_f) = M_p(s_c) = M_p(s_a) = 1,$$

$$M_p(s_g) = M_p(s_b) = 0.$$

Lemma 3.6. *Let $R=\{\alpha_i\to\beta_i/1\leqslant i\leqslant n,\ n\geqslant 1\}\in SCB$, N_R be the P/T-net induced by R and M be a marking of N_R. Then, a transition t_i is enabled at M iff for any $s_f\in{}^{\cdot}t_i$, $\#(f,\alpha_i)\leqslant M(s_f)$.*

Proof. t_i is enabled at M iff for any $s_f\in{}^{\cdot}t_i$, $W(s_f,t_i)\leqslant M(s_f)$. But, $W(s_f,t_i)\leqslant M(s_f)$ iff $\#(f,\alpha_i)\leqslant M(s_f)$ *Q.E.D.*

Lemma 3.7. *Let $R=\{\alpha_i\to\beta_i/1\leqslant i\leqslant n,\ n\geqslant 1\}$ be a TRS and $p,q\in\mathcal{T}$. If $p\xrightarrow[[\pi,r_i,\sigma]R]{}q$ then $\#(f,\alpha_i)\leqslant\#(f,p)$ for any $f\in\mathcal{F}(R;p)$.*

Proof. Let $R=\{\alpha_i\to\beta_i/1\leqslant i\leqslant n,\ n\geqslant 1\}\in\mathcal{R}$, $p,q\in\mathcal{T}$, $\pi\in O(p)$, $\sigma\in\Sigma$ be such that $p\xrightarrow[[\pi,r_i,\sigma]R]{}q$. Let $f\in\mathcal{F}(R;p)$. Because $p\xrightarrow[[\pi,r_i,\sigma]R]{}q$ it follows that $p/\pi=\sigma(\alpha_i)$ and hence $\#(f,\alpha_i)\leqslant\#(f,p)$. Q.E.D.

Theorem 3.8. *Let $R=\{\alpha_i\to\beta_i/1\leqslant i\leqslant n,\ n\geqslant 1\}\in SCB$, $p,q\in\mathcal{T}$ and N_R $N_R=(S_R,T_R;F_R,W_R)$ be the P/T-net induced by R. If $p\xrightarrow[R]{*}q$ then there exists $u\in T_R^*$ such that $M_p[u>M_q$ and $|p\xrightarrow[R]{*}q|=|u|$.*

Proof. By induction on $k=|p\xrightarrow[R]{*}q|$.

Basis: $k=1$. Then, $p\xrightarrow[R]{}q$ and hence there exist $\alpha_i\to\beta_i\in R$, $\pi\in O(p)$, $\sigma\in\Sigma$ such that:

(i) $p/\pi=\sigma(\alpha_i)$,

(ii) $q=p[\pi\leftarrow\sigma(\beta_i)]$.

We prove that $M_p[t_i>M_q$. From Proposition 2.6 it follows that

$$B_i(f)=\#(f,p)-\#(f,q),\ \forall f\in\mathcal{F}(R),$$

that is,

$$(*)\qquad \#(f,\alpha_i)-\#(f,\beta_i)=\#(f,p)-\#(f,q).$$

We prove that t_i is enabled at M_p, that is:

$$(**)\qquad \#(f,\alpha_i)\leqslant M_p(s_f)\quad\text{(by Lemma 3.6).}$$

But from Lemma 3.7, $\#(f,\alpha_i)\leqslant\#(f,p)$ that is, $\#(f,\alpha_i)\leqslant M_p(s_f)$ and hence t_i is enabled at M_p. Then, t_i may occur, yielding a new marking given by the equation:

$$\#(f,q)=\#(f,p)-\#(f,\alpha_i)+\#(f,\beta_i)\ \text{ that is,}$$

$$M_q(s_f)=M_p(s_f)-W(s_f,t_i)+W(t_i,s_f)\ \text{ and hence}$$

$$M_p[t_i>M_q.$$

Moreover, $|p\xrightarrow[R]{}q|=1=|t_i|$.

Induction step: Suppose that for every $p'\in\mathcal{T}$ such that $p\xrightarrow[R]{*}p'$ in at most k steps, we have:

$$(1)\qquad \exists u'\in T_R^*: M_p[p'>M_p,\ \text{and}\ |p\xrightarrow[R]{*}p'|=|u'|.$$

Let $q\in\mathcal{T}$ be such that $p\xrightarrow[R]{*}q$ in $(k+1)$ steps. Then, there exists $p'\in\mathcal{T}$

such that $p\xrightarrow[R]{*}p'\xrightarrow[R]{}q$ and by the inductive hypothesis, (1) hold. Analogously, $M_p[t_i>M_q$ where r_i is the rule used in the R-derivation $p'\xrightarrow[R]{}q$. Then, $M_p[u't_i>M_q$ and $|p\xrightarrow[R]{*}q|=|p\xrightarrow[R]{*}p'|+1=|u'|+1=|u't_i|$. Q.E.D.

The converse of Theorem 3.8 is generally false. But, we shall prove that for some P/T-net $N=(S,T;F,W)$ and any initial marking $M_0\in N^S$, there exists a TRS, R_N, and an equivalence relation on $\mathcal{T}$, ρ, such that for every $M\in[M_0>$ we have: $p\xrightarrow[R_{N,\rho}]{}q$, where p and q are „terms induced by M_0 and M".

We shall start with the remark that every $p\in\mathcal{T}$ induces a multiset on $\mathcal{F}$ denoted χ_p. Indeed, we can consider χ_p given by:

(i) $f\in\chi_p$ iff $f\in\mathcal{F}(p)$,

(ii) $\#(f,\chi_p)=\#(f,p)$.

For instance, considering $p=f(a,g(f(b,x),h(a)))$ the multiset induced is $\chi_p=(f,a,g,f,b,h,a)$. Now, the following question arises:

— when a multiset on $\mathcal{F}$ does induce at least one term? The answer of this question is given in the following

Lemma 3.9. (1) *For every $p\in\mathcal{T}_0$ we have:*

$$|\chi_p|=1+\sum_{f\in\chi_p}\#(f,\chi_p)\,\mathrm{ar}(f)$$

(2) *For every $p\in\mathcal{T}\setminus\mathcal{T}_0$ we have:*

$$|\chi_p|<1+\sum_{p\in\chi_p}\#(f,\chi_p)\,\mathrm{ar}(f).$$

Proof. It follows from the following remarks:

— χ_p does not contain variables;

— the number of elements in χ_p is computed by adding, for any functional symbol in p, the number of its succesors; the first functional symbol in p is also counted. Q.E.D.

Lemma 3.10. *Let χ be a multiset on $\mathcal{F}$. If*

$$(2)\qquad \sum_{f\in\chi}\#(f,\chi)\,\mathrm{ar}(f)\geqslant|\chi|-1$$

then there exists $p\in\mathcal{T}$ such that $\chi_p=\chi$.

Proof. By induction on $k=|\chi|$

Basis: $k=1$. Then, $\chi=(f)$, where $f\in\mathcal{F}$. If $\mathrm{ar}(f)=0$ ((2) holds), then, we choose $p=f$ and hence $\chi_p=\chi$. If $\mathrm{ar}(f)>0$ ((2) holds), let $m=\mathrm{ar}(f)$. Then, we choose $p=f(x_1,\ldots,x_m)$ with $x_i\in\mathcal{V}$, $1\leqslant i\leqslant m$, and $\chi_p=\chi$ is true.

Induction step: Suppose that for every multiset χ' on $\mathcal{F}$ such that:

$$(1')\qquad |\chi'|\leqslant k,\ k>1,$$

$$(2')\qquad \sum_{f\in\chi'}\#(f,\chi')\,\mathrm{ar}(f)\geqslant|\chi'|-1$$

there exists $p'\in T$ such that $\chi_{p'}=\chi'$. Let $\chi=(f_1,\ldots,f_k,f_{k+1})$ be a multiset on $\mathcal{T}$ such that $|\chi|=k+1$ and χ satisfies (2). There are two cases:

Case 1: there exists $j \in \{1, \dots, k, k+1\}$ such that $\mathrm{ar}(f_j)=0$. Then we consider the multiset $\chi'=(f_1, \dots, f_{j-1}, f_{j+1}, \dots, f_{k+1})$ and we prove that χ' satisfies (2'). Indeed, because χ satisfies (2) we have:

$$\sum_{f \in \chi} \#(f, \chi') \, \mathrm{ar}(f) = \sum_{\substack{l=1 \\ l \neq j}}^{k+1} \#(f_l, \chi) \, \mathrm{ar}(f_l) =$$

$$= \sum_{\substack{l=1 \\ l \neq j}}^{k+1} \#(f_l, \chi) \, \mathrm{ar}(f_l) + \#(f_j, \chi) \, \mathrm{ar}(f_j) = \sum_{l=1}^{k+1} \#(f_l, \chi) \, \mathrm{ar}(f_l) \geq |\chi|-1.$$

Because $|\chi|=|\chi'|+1$, we deduce that $\sum_{f \in \chi} \#(f, \chi') \, \mathrm{ar}(f) > |\chi'|-1$. From the inductive hypothesis it follows that there exists $p' \in \mathcal{T}$ such that $\chi_{p'}=\chi'$. Moreover, $p' \in \mathcal{T} \setminus \mathcal{T}_0$ (by Lemma 3.9). Hence, there exists $\pi \in \mathcal{O}(p')$ such that $p'/\pi \in \mathcal{V}$. Then, the term $p=p'[\pi \leftarrow f_j]$ satisfies the lemma.
Case 2: for any $j \in \{1, \dots, k+1\}$, $\mathrm{ar}(f_j) \geq 1$. Consider in this case the smallest $j \in \{1, \dots, k+1\}$ such that $\mathrm{ar}(f_j)=\min\{\mathrm{ar}(f_l)/1 \leq l \leq k+1\}$. Let $\chi'=(f_1, \dots, f_{j-1}, f_{j+1}, \dots, f_{k+1})$ and $m=\mathrm{ar}(f_j)$.
We have:

(3) $$\sum_{f \in \chi} \#(f, \chi) \, \mathrm{ar}(f) \geq m|\chi|$$

and hence $\sum_{f \in \chi} \#(f, \chi') \, \mathrm{ar}(f) = \sum_{l=1}^{k+1} \#(f_l, \chi) \, \mathrm{ar}(f_l) - \mathrm{ar}(f_j) =$

$$= \sum_{l=1}^{k+1} \#(f_l, \chi) \, \mathrm{ar}(f_l) - m \geq m|\chi'|-m = m(|\chi|-1) = m(k+1-1) =$$

$$= mk = m|\chi'| > m|\chi'|-1.$$

From the inductive hypothesis it follows that there exists p' such that $\chi_{p'}=\chi'$.
Consider the term:

$$p = \begin{cases} f_j(p'), & \text{if } m=1 \\ f_j(p', x_2, \dots, x_m), & \text{if } m>1 \end{cases}$$

where $x_2, \dots, x_m \in \mathcal{V}$.
Then, we can choose $\chi_p=\chi$ to end the proof. Q.E.D.

Corollary 3.11. *Let χ be a multiset on $\mathcal{F}$.*
1) *If*

(2.1) $$\sum_{f \in \chi} \#(f, \chi) \, \mathrm{ar}(f) = |\chi|-1$$

then there exists $p_0 \in \mathcal{T}_0$ such that $\chi_{p_0}=\chi$;
2) *If*

(2.2) $$\sum_{f \in \chi} \#(f, \chi) \, \mathrm{ar}(f) > |\chi|-1,$$

then there exists $p \in \mathcal{T} \setminus \mathcal{T}_0$ such that $\chi_p=\chi$.

Proof. (1) From Lemma 3.10 it follows that there exists $p_0 \in \mathcal{T}$ such that $\chi_{p_0}=\chi$. If $p_0 \notin \mathcal{T}_0 (p_0 \in \mathcal{T} \setminus \mathcal{T}_0)$, from Lemma 3.9 it follows that

$$\sum_{f \in \chi_{p_0}} \#(f, \chi_{p_0}) \, \mathrm{ar}(f) > |\chi_{p_0}|-1$$

and because $\chi_{p_0}=\chi$ we deduce that

$$\sum_{f \in \chi} \#(f, \chi) \, \mathrm{ar}(f) > |\chi|-1$$

which is a contradiction. Hence, $p_0 \in \mathcal{T}_0$.
The proof of (2) is analogous to that of (1). Q.E.D.

Lemma 3.12. *Let $p, q \in \mathcal{T}$, $\pi \in \mathcal{O}(p)$, $p'=p[\pi \leftarrow q]$ and $\sigma=\{x_i \leftarrow q_i / 1 \leq i \leq n, n \geq 1\}$, where $x_i \in \mathcal{V}(p)$ and $q_i \in \mathcal{T}$, $1 \leq i \leq x$. Then:*

(1) $$\chi_{p'}=(\chi_p \setminus \chi_{p/\pi}) \cup \chi_q$$

(2) $$\chi_{\sigma(p)}=\chi_p \cup \bigcup_{i=1}^{n} \chi_{q_i}$$

Proof. Directly from the corresponding definitions. Q.E.D.

Lemma 3.13. *Let χ_1, χ_2, be two multisets on $\mathcal{F}$ such that:*
(i) $\sum_{f \in \chi_i} \#(f, \chi_i) \, \mathrm{ar}(f) \geq |\chi_i|-1$, $i=1, 2$

(ii) $\chi_1 \subseteq \chi_2$.
Then, there exist $p_1, p_2 \in \mathcal{T}$, $\sigma \in \Sigma$ and $\pi \in \mathcal{O}(p_2)$ such that:

(1) $$\chi_{p_i}=\chi_i, \quad i=1, 2$$

(2) $$p_2/\pi=\sigma(p_1).$$

Proof. Let χ_1, χ_2 be two multisets on $\mathcal{F}$, such that χ_1 and χ_2 satisfy (i) and (ii).
From (i) and Lemma 3.10, it follows that there exists $p_1 \in \mathcal{T}$, such that $\chi_{p_1}=\chi_1$. There are two cases.

Case 1. $\sum_{f \in \chi_1} \#(f, \chi_1) \, \mathrm{ar}(f) = |\chi_1|-1$. Then, from Corollary 3.11 it follows that $p_1 \in \mathcal{T}_0$ and from (i) and (ii) it follows that:

$$\sum_{f \in \chi_2} \#(f, \chi_2) \, \mathrm{ar}(f) = \sum_{f \in \chi_1} \#(f, \chi_1) \, \mathrm{ar}(f) + \sum_{f \in \chi_2 \setminus \chi_1} \#(f, \chi_2 \setminus \chi_1) \, \mathrm{ar}(f) =$$

$$= |x_1|-1 + \sum_{f \in \chi_2 \setminus \chi_1} \#(f, \chi_2 \setminus \chi_1) \, \mathrm{ar}(f) \geq |\chi_2|-1.$$

Hence, $\sum_{f \in \chi_2 \setminus \chi_1} \#(f, \chi_2 \setminus \chi_1) \, \mathrm{ar}(f) \geq |\chi_2| - |\chi_1| = |\chi_2 \setminus \chi_1| > |\chi_2 \setminus \chi_1|-1$ and from Corollary 3.11 it follows that there exists $p' \in \mathcal{T} \setminus \mathcal{T}_0$ such that $\chi_{p'} = \chi_2 \setminus \chi_1$. Moreover, there exists $\pi' \in \mathcal{O}(p')$ such that $p'/\pi' \in \mathcal{V}$. Consider $p_2=p'[\pi' \rightarrow p_1]$; we prove that $\chi_{p_2}=\chi_2$.
Indeed, from Lemma 3.12, it follows that $\chi_{p_2}=(\chi_{p'} \setminus \chi_{p'/\pi'}) \cup \chi_{p_1}=(\chi_2 \setminus \chi_1 \setminus \emptyset) \cup \chi_1=\chi_2$.
We prove that there exist $\sigma \in \Sigma$ and $\pi \in \mathcal{O}(p_2)$ such that $p_2/\pi=\sigma(p_1)$. Indeed, we choose $\sigma=\varepsilon$ and $\pi=\pi'$ and then we have $p_2/\pi=p_1=\varepsilon(p_1)$.

Case 2. $\sum_{f\in\chi_1} \#(f,\chi_1)\ \mathrm{ar}(f) > |\chi_1| - 1$. Then, $p_1 \in \mathcal{T}\setminus\mathcal{T}_0$ (by Corollary 3.11). Let $k \in \mathbb{N}$ be such that

$$\sum_{f\in\chi_1} \#(f,\chi_1)\ \mathrm{ar}(f) = |\chi_1| - 1 + k.$$

In p_1 there exist k variables, $x_1, \ldots, x_k$. From (i) and (ii) we deduce that :

$$\sum_{f\in\chi_2} \#(f,\chi_2)\ \mathrm{ar}(f) = \sum_{f\in\chi_1} \#(f,\chi_1)\ \mathrm{ar}(f) +$$
$$+ \sum_{f\in\chi_2\setminus\chi_1} \#(f,\chi_2\setminus\chi_1)\ \mathrm{ar}(f) = |\chi_1| - 1 + k +$$
$$+ \sum_{f\in\chi_2\setminus\chi_1} \#(f,\chi_2\setminus\chi_1)\ \mathrm{ar}(f) \geq |\chi_2| - 1.$$

$$\text{Hence, } \sum_{f\in\chi_2\setminus\chi_1} \#(f,\chi_2\setminus\chi_1)\ \mathrm{ar}(f) + k > |\chi_2\setminus\chi_1|.$$

There are two subcases.

(2.1) In $\chi_2\setminus\chi_1$ there exist k constants, $a_1, \ldots, a_k$. Consider the multiset $\overline{\chi_2\setminus\chi_1} = (\chi_2\setminus\chi_1) - (a_1, \ldots, a_k)$. Then, $\sum_{f\in\chi_2\setminus\chi_1} \#(f,\chi_2\setminus\chi_1)\ \mathrm{ar}(f) = \sum_{f\in\overline{\chi_2\setminus\chi_1}} \#(f,\overline{\chi_2\setminus\chi_1})\ \mathrm{ar}(f)$ and $|\chi_2\setminus\chi_1| = |\overline{\chi_2\setminus\chi_1}| + k$. From $\sum_{f\in\overline{\chi_2\setminus\chi_1}} \#(f,\chi_2\setminus\chi_1)\ \mathrm{ar}(f) + k \geq |\overline{\chi_2\setminus\chi_1}| + k$ it follows that $\sum_{f\in\overline{\chi_2\setminus\chi_1}} \#(f,\overline{\chi_2\setminus\chi_1})\ \mathrm{ar}(f) \geq |\overline{\chi_2\setminus\chi_1}| > |\overline{\chi_2\setminus\chi_1}| - 1$ and hence there exists $p' \in \mathcal{T}$ such that $\chi_{p'} = \overline{\chi_2\setminus\chi_1}$. Moreover, p' contains at least a variable. Let $\pi' \in \mathcal{O}(p')$ be such that $p'/\pi' \in \mathcal{V}$. Consider the substitution $\sigma' = \{x_i \leftarrow a_i / 1 \leq i \leq k\}$ and the term $p_2 = p'[\pi' \leftarrow \sigma'(p_1)]$. We prove that $\chi_{p_2} = \chi_2$. Indeed, from Lemma 3.12 it follows that $\chi_{p_2} = (\chi_{p'} \setminus \cup\chi_{p'/\pi'})\chi_{\sigma'(p_1)} = (\chi_{p'}\setminus\varnothing) \cup \chi_{p_1} \cup \bigcup_{i=1}^{k} \chi_{a_i} = \overline{\chi_2\setminus\chi_1} \cup \chi_{p_1} \cup \bigcup_{i=1}^{k} \cup \chi_{a_i} = (\chi_2 \setminus \chi_1) \cup \chi_1 = \chi_2$.

Choosing $\pi = \pi'$ and $\sigma = \sigma'$ we deduce that p_1 and p_2 satisfy the lemma ;

(2.2) In $\chi_2\setminus\chi_1$ there exist m constants, $m < k$. Let $a_1, \ldots, a_m$ be these constants Let $(\chi_2\setminus\chi_1)\setminus(a_1, \ldots, a_m) = (f_1, \ldots, f_l)$.

Consider the substitution $\sigma' = \{x_i \leftarrow a_i / 1 \leq i \leq m\}$ and the term $p_2 = f_1(f_2(\ldots(f_l(\sigma'(p_1), \ldots,), \ldots) \ldots), \ldots)$ where the free positions stand for new variables (distinct each other and distinct from $x_1, x_2, \ldots, x_k$).

Then, p_1, p_2, $\sigma = \sigma'$ and $\pi = 11\ldots1$ (l times) satisfy the lemma.

Q. E. D.

Let $N = (S, T; F, W)$ be a P/T-net and $\varphi : S \to \mathcal{F}$ an injective function. Suppose that the S-elements and respectively the T-elements are ordered :

$$S : s_1 < s_2 < \ldots < s_m,$$

$$T : t_1 < t_2 < \ldots < t_n.$$

Let M be a marking of N. M induces a multiset on $\mathcal{T}$, denoted χ_M, as follows :

(i) $f \in \chi_M$ iff there exists $l \in \{1, \ldots, k\}$ such that $f = \varphi(s_{i_l})$

(ii) $\#(\varphi(s_{i_l}), \chi_M) = M(s_{i_l})$, for any $l \in \{1, \ldots, k\}$, where $\{s_{i_1}, \ldots, s_{i_k}\} = \{s \in S / M(s) \neq \varnothing\}$.

Conversely, a (finite) multiset on $\mathcal{F}$, $\chi = (f_1, \ldots, f_l)$ induces a marking of N, M_χ, by :

$$M_\chi(s) = \#(\varphi(s), \chi), \text{ for any } s \in S.$$

Let $t \in \mathcal{T}$. The transition t induces two multisets on $\mathcal{F}$, denoted $\chi_{\cdot t}$ and $\chi_{t\cdot}$, respectively by :

(i) $f \in \chi_{\cdot t}$ iff there exists $l \in \{1, \ldots, k\}$ such that $f = \varphi(s_{i_l})$

(ii) $\#(\varphi(s_{i_l}), \chi_{\cdot t}) = W(s_{i_l}, t)$, for any $l \in \{1, \ldots, k_1\}$

and

(i') $f \in \chi_{t\cdot}$ iff there exists $l \in \{1, \ldots, k_2\}$ such that $f = \varphi(s_{j_l})$

(ii') $\#(\varphi(s_{j_l}), \chi_{t\cdot}) = W(t, s_{j_l})$, for any $l \in \{1, \ldots, k_2\}$, where ${}^{\cdot}t = \{s_{i_1}, \ldots, s_{i_{k_1}}\}$ and $t^{\cdot} = \{s_{j_1}, \ldots, s_{j_{k_2}}\}$.

Conversely, two finite multisets on $\mathcal{F}$, χ_1 and χ_2, induce a transition t by :

(i) $(s, t) \in F$ iff $\varphi(s) \in \chi_1$ and in this case
$W(s, t) = \#(\varphi(s), \chi_1)$

(ii) $(t, s) \in F$ iff $\varphi(s) \in \chi_2$ and in this case
$W(t, s) = \#(\varphi(s), \chi_2)$.

Lemma 3.14. *Let $N = (S, T; F, W)$ be a P/T-net, $\varphi : S \to \mathcal{F}$ be an injective function, M be a marking of N and $t \in T$. If t is enabled at M, then $\chi_t \subseteq \chi_M$.*

Proof. Because t is enabled at M, for any $s \in S$, $W(s, t) \leq M(s)$. That is, for any $s \in S$, $\#(\varphi(s), \chi_{\cdot t}) \leq \#(\varphi(s), \chi_M)$ which implies $\chi_{\cdot t} \subseteq \chi_M$.

Q.E.D.

Lemma 3.15. *Let $R \in SCB$, $r : \alpha \to \beta \in R$ and $p, q \in \mathcal{T}$ be such that $p \xrightarrow[(\pi, r, \sigma) R]{} q$. Then, $\chi_q = (\chi_p \setminus \chi_\alpha) \cup \chi_\beta$*

Proof. From Lemma 3.12 and from the definition of rewriting. *Q.E.D.*

Let us note that Lemma 3.15 is not true for $R \in \mathcal{R}\setminus SCB$.

Lemma 3.16. *Let $N = (S, T; F, W)$ be a P/T-net, $\varphi : S \to \mathcal{F}$ be an injective function, M be a marking of N and $t \in T$. If $M[t > M_1$, then $\chi_{M_1} = (\chi_M \setminus \chi_{\cdot t}) \cup \chi_{t\cdot}$.*

Proof. Let $f \in \chi_{M_1}$. Then, there exists $s \in S$ such that $\varphi(s) = f$. Because $M[t > M_1$, it follows that for any $s \in S$, $M_1(s) = M(s) - W(s, t) + W(t, s)$, from which we obtain that $\#(\varphi(s), \chi_{M_1}) = \#(\varphi(s), \chi_M) - \#(\varphi(s), \chi_{\cdot t}) + \#(\varphi(s), \chi_{t\cdot})$, that is, $\varphi(s) = f \in (\chi_M \setminus \chi_{\cdot t}) \cup \chi_{\cdot t}$. Hence, $\chi_{M_1} \subseteq (\chi_M \setminus \chi_{\cdot t}) \cup \chi_{t\cdot}$.

The converse follows in the same way. *Q.E.D.*

Let ρ be the following relation on $\mathcal{T}$:

$$p \rho q \text{ iff } \chi_p = \chi_q.$$

It is easy to see that ρ is an equivalence relation.

Theorem 3.17. *Let $N = (S, T; F, W)$ be a P/T-net, $\varphi : S \to \mathcal{F}$ be an injective function and M_0, M be two markings of N such that there exists $u \in T^*$ with $M_0[u > M$. If*

(i) $\sum_{f\in\chi_{\cdot t}} \#(f, \chi_{\cdot t})\ \mathrm{ar}\,(f)\geq|\chi_{\cdot t}|-1$

$\sum_{f\in\chi_{t\cdot}} \#(f, \chi_{t\cdot})\ \mathrm{ar}(f)\geq|\chi_{t\cdot}|-1$, for any $t\in T$

(ii) $\sum_{s\in S} M_0(s)\ \mathrm{ar}\,(\varphi(s))\geq|\{\varphi(s)\mid M_0(s)>0\}|-1$

(iii) $\sum_{s\in\cdot t} W(s, t)\ \mathrm{ar}\,(\varphi(s))-|\{s\in\cdot t\mid \mathrm{ar}\,(\varphi(s))=0\}|=\sum_{s\in t\cdot} W(s, t)\ \mathrm{ar}\,(\varphi(s))-$
$-|\{s\in t\cdot\mid \mathrm{ar}\,(\varphi(s))=0\}|$, for any $t\in T$,

then there exist p_{M_0}, $p_M\in\mathcal{T}$ and $R_N\in SCB$ such that:

(1) $$p_{M_0}\xrightarrow[R_{N,\rho}]{}p_M,$$

(2) $$|p_{M_0}\xrightarrow[R_{N,\rho}]{*}p_M|=|u|.$$

Proof. By induction on $k=|u|$.

Basis. $k=1$. Then, $u=t\in\mathcal{T}$ and $M_0[t>M$. Because t is enabled at M_0, from Lemma 3.14 it follows that $\chi_{\cdot t}\subseteq\chi_{M_0}$ and from Lemma 3.13 it follows that there exist p_{M_0}, $\alpha_t\in\mathcal{T}$, $\pi\in O(p_{M_0})$ and $\sigma\in\Sigma$ such that:

(a) $$\chi_{M_0}=\chi_{p_{M_0}},\ \chi_{\cdot t}=\chi_{\alpha_t}$$

(b) $$p_{M_0}/\pi=\sigma(\alpha_t).$$

From (i) and Lemma 3.10 it follows that there exists $\beta_t\in\mathcal{T}$ such that $\chi_{t\cdot}=\chi_{\beta_t}$ and (iii) implies that α_t and β_t have the same number of variables. Moreover, we can choose β_t such that for every $x\in\mathcal{V}$, $\#(x, \alpha_t)=\#(x, \beta_t)$ (as in the proof of Lemma 3.13).

Consider the term rewriting system $R_N=\{\alpha_t\to\beta_t\}$. By Proposition 2.2, $R_N\in SCB$. From (b) it follows that the rule $\alpha_t\to\beta_t$ can be used in the rewriting of p_{M_0} and hence there exists $q\in\mathcal{T}$ such that $p_{M_0}\xrightarrow[R]{}q$. We show that $\chi_q=\chi_M$. Indeed, from Lemma 3.15, it follows that $\chi_q=(\chi_{p_{M_0}}\setminus\chi_{\cdot t})\cup\chi_{t\cdot}$ and from Lemma 3.16 it follows that $\chi_M=(\chi_{M_0}\setminus\chi_{\cdot t})\cup\chi_{t\cdot}$. Because $\chi_{p_{M_0}}=\chi_{M_0}$, $\chi_{\alpha_t}=\chi_{\cdot t}$ and $\chi_{\beta_t}=\chi_{t\cdot}$ we deduce that $\chi_q=\chi_M$. We can then take $p_M=q$.

Induction step: Suppose that for every $u'\in T^*$, such that $|u'|\leq k$ $(k>1)$ and $M_0[u'>M'$, there exist $R'_N\in SCB$ and $p_{M_0}, p_{M'}\in\mathcal{T}$ such that $p_{M_0}\xrightarrow[R'_{N,\rho}]{*}p_{M'}$ and $|p_{M_0}\xrightarrow[R'_{N,\rho}]{}p_{M'}|=|p'|$.

Let $u\in T$ be such that:

(a') $$|u|=k+1,$$

(b') $$M_0[u>M.$$

Then, $u=u't$ with $u'\in T^*$, $|u'|=k$, $t\in T$ and $M_0[u'>M'[t>M$. From the inductive hypothesis it follows that there exist $R'_N\in SCB$ and p_{M_0}, $p_{M'}\in\mathcal{T}$ such that $p_{M_0}\xrightarrow[R'_{N,\rho}]{*}p_{M'}$ and $|p_{M_0}\xrightarrow[R'_{N,\rho}]{*}p_{M'}|=|u'|$.

From $M'[t>M$ we deduce that there exist $R''_N=\{\alpha_t\to\beta_t\}\in SCB$ and $q_{M'}$, $p_M\in\mathcal{T}$ such that $q_{M'}\xrightarrow[R''_{N,\rho}]{}p_M$. We show that $p_{M'}\rho q_{M'}$, that is $\chi_{p_{M'}}=\chi_{q_{M'}}$. Indeed, $\chi_{p_{M'}}=\chi_{M'}=\chi_{q_{M'}}$ and hence $p_{M'}\rho q_{M'}$. Then, $p_{M_0}\xrightarrow[R_{N,\rho}]{*}p_M$ and $|p_{M_0}\xrightarrow[R_{N,\rho}]{*}p_M|=|u|$ where $R_N=R'_N\cup R''_N\in SCB$. *Q.E.D.*

Conclusions. Although the obtained results are not so general as we expected, we hope that future developments could add new insights to the general concepts of parallelism and nondeterminism as expressed in term rewriting systems and Petri nets.

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