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REVERSIBLE AND STRICT REVERSIBLE P/T-SYSTEMS

BY

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1. Preliminaries. A sixtuple $\Sigma=(S, T; F, K, W, M_0)$ is called a place/transition system (P/T-system) iff :

(i) $(S, T; F)$ is a net where the S-elements are called places and the T-elements are called transitions ;

(ii) $K: S\rightarrow \mathbb{N}^+ \bigcup\{\infty\}$ is a capacity function ;

(iii) $W: F\rightarrow \mathbb{N}^+$ is a weight function ;

(iv) $M_0: S\rightarrow \mathbb{N}$ is an initial marking function which satisfies $M_0(s)\leqslant K(s)$ for all $s\in S$.

In the graphical reprezentation of P/T-systems, the arcs $f\in F$ are labelled by $W(f)$ whenever $W(f)>1$. We can extend the function W to W' : $(S\bigcup T)\times(S\cup T)\rightarrow \mathbb{N}$ by :

(i) $W'(x, y)=0$, if $(x, y) \notin F$

(ii) $W'(x, y)=W(x, y)$, if $(x, y)\in F$.

Since $W'|_F=W$, there can be no disagreement between W' and W on arguments for which both are defined. Thus we shall for convenience use W instead of W' from here on. Hence, $(x, y)\notin F$ or $(x, y)\in F$ iff $W(x, y)=0$ or $W(x, y)>0$, respectively.
As usual, ${}^{\cdot}x$ denotes the set of F-predecessors of $x\in S\bigcup T$ and $x^{\cdot}$ denotes the set of F-successors of $x\in S\bigcup T$.

Let $\Sigma=(S, T; F, K, W, M_0)$ be a P/T-system. A function $M: S\rightarrow \mathbb{N}$ is called a *marking* of Σ iff $M(s)\leqslant K(s)$ for all $s\in S$. A transition $t\in T$ is *enabled* at M (or has *concession* at M) iff for every $s\in S$: $W(s, t)\leqslant M(s)\leqslant K(s)-W(t, s)$. If $t\in T$ is a transition which is enabled at a marking M then t may occur, yielding a new marking M' given by the equation : $M'(s)=M(s)-W(s, t)+W(t, s)$ for all $s\in S$. The occurrence of t changes the marking M into the new marking M' ; we may denote this fact by $M[t>M'$ or by $M\xrightarrow{t}M'$. We denote by $[M_0>$ the smallest set of markings of Σ such that :

(i) $M_0\in[M_0>$;

(ii) if $M_1\in[M_0>$ and $M_1[t>M_2$ for some $t\in T$ then $M_2\in[M_0>$.

Let λ be the neutral element of T^*. We then write: $M[\lambda>M$ $(M\xrightarrow{\lambda}M)$ for any marking $M\in\mathbb{N}^S$ and $M[w>M''$ if $w=ut$ for some $u\in T^*$, $t\in T$ and there exists $M'\in\mathbb{N}^S$ such that $M[u>M'[t>M''$.

A P/T-system $\Sigma=(S, T; F, K, W, M_0)$ is called :

(i) *pure* iff for every $x, y\in S\bigcup T$, if $(x, y)\in F$ then $(y, x)\notin F$;

(ii) *live* iff for every $t\in T$ and every $M\in[M_0>$ there exists $M'\in[M>$ such that t is enabled at M';

(iii) *weakly connected* iff all $x, y\in S\cup T$ are in the relation $(F\cup F^{-1})^*$;

(iv) *strongly connected* iff all $x, y\in S\cup T$ are in the relation F^*.

In order to define a Petri net language, one has to specify four quantities: the underlying Petri net $\Sigma=(S, T; F, K, W)$, the initial marking $M_0\in\mathbb{N}^S$, a usually finite set of final markings $\mathcal{M}\subseteq\mathbb{N}^S$ and a labelling function $l: T\to X$, where X is an alphabet. Given this information $\Gamma=(\Sigma, M_0, \mathcal{M}, l)$ and by extending l to a homomorphism $l: T^*\to X^*$ in the obvious way or omitting l if it is the identity, we define the following type of Petri net language:

$$L(\Gamma)=\{l(w)\mid w\in T^*,\ \exists M'\in\mathcal{M}: M_0[w>M'\}.$$

$L(\Gamma)$ is called the $\mathcal{L}$-type or terminal language of Σ. In this paper we shall consider l as being the identity.

We shall study in this paper the properties of P/T-systems $\Sigma=(S, T; F, K, W, M_0)$ which have four restrictions:

Restriction 1.1. $K: S\to\{\infty\}$. For the sake of simplicity and brevity, we will change the sixtuple notation of Σ into a fivetuple notation; thus, $\Sigma=(S, T; F, W, M_0)$ will henceforth a denote a P/T-system with infinite capacities.

Restriction 1.2. Σ is finite, that is, $S\cup T$ is a finite set.

Restriction 1.3. Σ is weakly connected.

Restriction 1.4. For every $t\in T$ there exists $M\in[M_0>$ such that t is enabled at M.

2. Reversible and Strictly Reversible P/T-systems.

Definition 2.1. *A P/T-system $\Sigma=(S, T; F, W, M_0)$ is called reversible iff for every $M\in[M_0>$ and every $t\in T$ such that t is enabled at M, there exists $w\in T^*$ such that $M'[w>M$, where $M[t>M'$.*

Definition 2.2. *A P/T-system $\Sigma=(S, T; F, W, M_0)$ is called strictly reversible iff for every $M\in[M_0>$ and every $t\in T$ such that t is enabled at M, there exists $t'\in T$ such that $M'[t'>M$, where $M[t>M'$.*

Remark 2.3. A strictly reversible P/T-system is a reversible P/T-system.

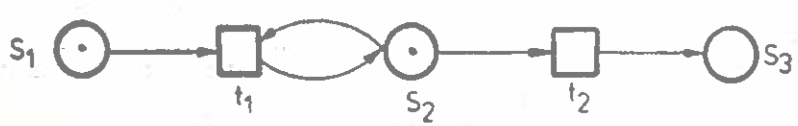
In the following we give some characterizations of reversible and strictly reversible P/T-systems.

Definition 2.4. *Let $\Sigma=(S, T; F, W, M_0)$ be a P/T-system, $t\in T$ and $s\in S$. We say that:*

(i) *t affects s iff $W(s, t)\neq W(t, s)$;*

(ii) *t affects S iff there exists $s_1, \ldots, s_k\in S$ such that t affects s_i, $1\leq i\leq k$.*

Example 2.5. Let Σ be the P/T-system drawn below together with its initial marking M_0:



t_1 does not affect s_2 and s_3 but affects s_1.

Notation 2.6. Let $\Sigma=(S, T; F, W, M_0)$ be a P/T-system, $t\in T$ and $s\in S$. We denote by Af (t) the set Af $(t)=\{s\in S\mid t$ affects $s\}$ and by $T(s)$ the set $T(s)=\{t\in T\mid t$ affects $s\}$. Also, the number $W(t, s)-W(s, t)$ is denoted by $C(s, t)$.

Remark 2.7. Let $\Sigma=(S, T; F, W, M_0)$ be a P/T-system, $t\in T$ and $s\in S$. If t not affect s, then either $(t, s), (s, t)\in F$ and $W(t, s)=W(s, t)$ or it does not exist any arc between t and s.

The following proposition can easily be verified.

Proposition 2.8. *Let $\Sigma=(S, T; F, W, M_0)$ be a P/T-system, $t\in T$ and $s\in S$. The following statements are equivalent:*

(i) *t does not affect s;*

(ii) *for every $M\in[M_0>$ such that t is enabled at M, $M(s)=M'(s)$, where $M[t>M'$;*

(iii) $C(s, t)=0$.

Definition 2.9. *Let $\Sigma=(S, T; F, W, M_0)$ be a P/T-system and $t_1, t_2\in T$. We say that t_1 and t_2 are duals iff:*

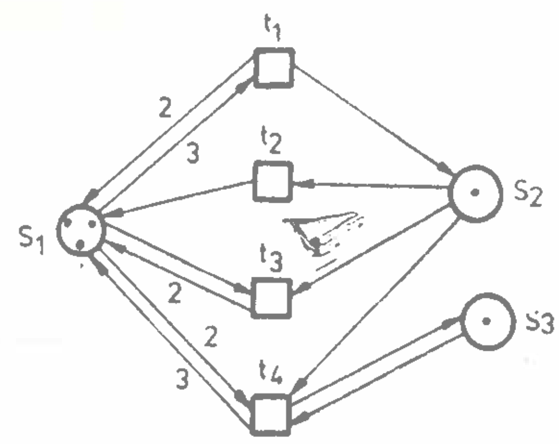
(i) Af $(t_1)=$ Af (t_2);

(ii) *for every* $s\in$ Af (t_1), $C(s, t_1)+C(s, t_2)=0$.

If t_1, t_2 are duals, we also say that t_1 (t_2) is a dual of t_2 (t_1).

A transition may have many duals. The following example illustrate this:

Example 2.10. Let Σ be the P/T-system drawn below together with its initial marking M_0:



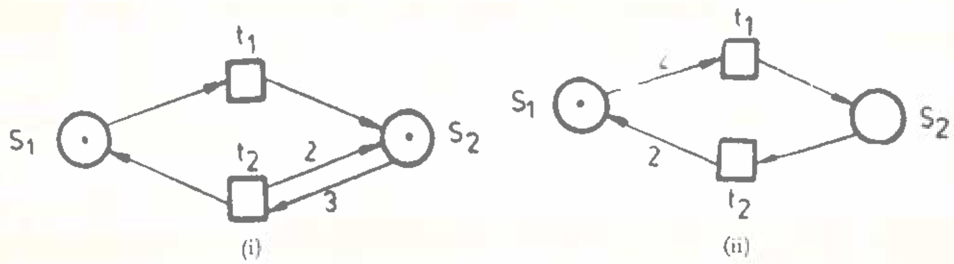
The transitions t_2, t_3, t_4 are duals of the transition t_1.

Notation 2.11. Let $\Sigma=(S, T; F, W, M_0)$ be a P/T-system and $t\in T$. We denote by Dual (t) the set Dual $(t)=\{t'\in T\mid t'\neq t$ and t, t' are duals$\}$.

Remark 2.12. Let $\Sigma=(S, T; F, W, M_0)$ be a P/T-system and $t_1, t_2\in T$.

(i) If t_1 and t_2 are duals and t_1 is enabled at M, $M\in[M_0>$, then it does not follow that t_2 is enabled at M', where $M[t_1>M'$.

For example:



t_1, t_2 are duals and t_1 is enabled at M_0 but t_2 is not enabled at M, where $M_0[t_1>M$.

(ii) If t_1 and t_2 are duals, it does not follow that t_1 or t_2 are enabled at a marking M, $M\in[M_0>$. For example, t_1 and t_2 are duals but they are not enabled at M_0.

Theorem 2.13. *Let $\Sigma=(S, T; F, W, M_0)$ be a P/T-system. If Σ is strictly reversible then for every $t\in T$ such that t affects S, Dual $(t)\neq\varnothing$.*

Proof: Let us suppose that Σ is strictly reversible and let $t\in T$ be such that t affects S. From Restriction 1.4 it follows that there exists $M\in[M_0>$ such that t is enabled at M. Let $M[t>M'$. Because Σ is strictly reversible, there exists $t'\in T$ such that t' is enabled at M' and $M'[t'>M$. We prove that $t'\in$ Dual (t), i.e.:

(i) Af $(t)=$ Af (t')

(ii) for every $s\in$ Af (t), $C(s,t)+C(s,t')=0$.

Suppose by contradiction that Af $(t)\neq$ Af (t'). Then, there are two cases.
Case 1: there exists $s\in S$ such that $s\in$ Af (t) and $s\notin$ Af (t'). From Proposition 2.8 it follows that $C(s,t)\neq0$ and $C(s,t')=0$. From $M(s)=M'(s)+C(s,t')=M(s)+C(s,t)+C(s,t')$ it follows that $C(s,t)=0$ contradicting the fact that $C(s,t)\neq0$;
Case 2: there exists $s\in S$ such that $s\in$ Af (t') and $s\notin$ Af (t). From Proposition 2.8 it follows that $C(s,t)=0$ and $C(s,t)\neq0$. From $M(s)=M'(s)+C(s,t')=M(s)+C(s,t)+C(s,t')=M(s)+C(s,t')$ it follows that $C(s,t')=0$ contradicting the fact that $C(s,t')\neq0$.
Hence, (i) holds.
Let $s\in$ Af (t). Then, $M(s)=M'(s)+C(s,t')=M(s)+C(s,t)+C(s,t')$ and hence $C(s,t)+C(s,t')=0$. We deduce that (ii) holds and therefore Dual $(t)\neq\varnothing$ Q.E.D.

The converse of this result is false. The P/T-system from Remark 2.12 (i) constitute the required counter-example.

Lemma 2.14. *Let $\Sigma=(S, T; F, W, M_0)$ be a pure P/T-system, $M\in[M_0>$ and $t_1, t_2\in T$ such that t_1 and t_2 are duals. If t_1 is enabled at M then t_2 is enabled at M', where $M[t_1>M'$.*

Proof: Let $\Sigma=(S, T; F, W, M_0)$ be a pure P/T-system, $M\in[M_0>$ and $t_1, t_2\in T$ such that t_1 and t_2 are duals. Suppose that t_1 is enabled at M and let $M[t_1>M'$. We shall prove that t_2 is enabled at M', i.e. for every $s\in S$, $W(s,t_2)\leq M'(s)$.

Let $s\in S$. Because t_1 and t_2 are duals, Af $(t_1)=$ Af (t_2). If $s\notin$ Af (t_2) then $W(s,t_2)=0\leq M'(s)$.

If $s\in$ Af (t_2) then there are two cases.
Case 1: $W(s,t_1)\neq0$. In this case, because Σ is pure, $W(t_1,s)=0$. Moreover, $W(s,t_2)=0$ and hence $W(s,t_2)\leq M'(s)$;
Case 2: $W(t_1,s)\neq0$. In this case, because Σ is pure, $W(s,t_1)=0$. Moreover, $W(t_2,s)\neq0$ and $W(s,t_2)=0$. From $C(S,t_1)+C(s,t_2)=0$ (t_1 and t_2 are duals) we deduce that $W(s,t_2)=W(t_1,s)$ and hence $W(s,t_2)\leq M'(s)=M(s)+W(t_1,s)$.

In both cases we deduce that $W(s,t_2)\leq M'(s)$ and hence t_2 is enabled at M'. Q.E.D.

Theorem 2.15. *Let $\Sigma=(S, T; F, W, M_0)$ be a pure P/T-system. Σ is strictly reversible iff for every $t\in T$ such that t affects S, Dual $(t)\neq\varnothing$.*

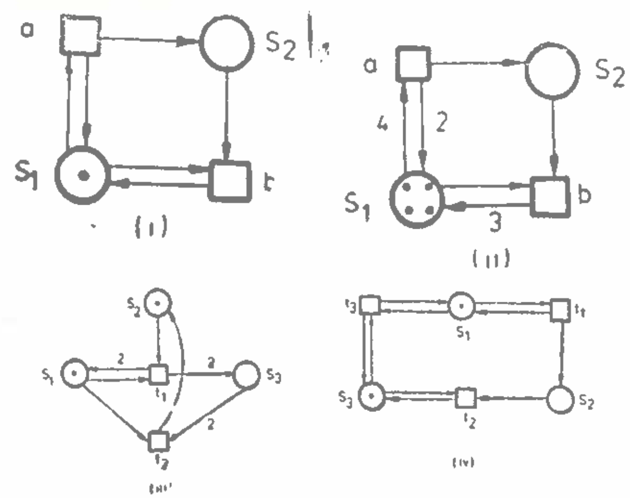
Proof: Let $\Sigma=(S, T; F, W, M_0)$ be a pure P/T-system. It is sufficient to show that if for every $t\in T$ such that t affects S, Dual $(t)\neq\varnothing$, then Σ is strictly reversible.
Let $M\in[M_0>$ and $t\in T$ be such that t is enabled at M. Then, $M[t>M'$ and:

(i) $M'(s)=M(s)$, if $s\notin$ Af (t)

(ii) $M'(s)=M(s)+C(s,t)$, if $s\in$ Af (t).

Because Dual $(t)\neq\varnothing$, there exists $t'\in T$ such that t and t' are duals. From Lemma 2.14 it follows that t' is enabled at M' and let $M'[t'>M''$. We shall prove that $M=M''$. Because Af $(t)=$ Af (t'), for every $s\notin$ Af (t'), $M''(s)=M'(s)=M(s)$. For every $s\in$ Af (t'), $M''(s)=M'(s)+C(s,t')=M(s)+C(s,t)+C(s,t')=M(s)$ since $C(s,t)+C(s,t')=0$. Therefore, $M=M''$ and hence Σ is strict reversible, Q.E.D.

Example 2.16. The following P/T-systems are strictly reversible:



Theorem 2.17. *Let $\Sigma=(S, T; F, W, M_0)$ be a P/T-system. If Σ is reversible, then for every $t\in T$ such that t affects S and every $s\in$ Af (t), there exist $n_1, \ldots, n_{k_s}\in\mathbf{N}$ such that*

$$(1) \qquad \sum_{i=1}^{k_s} n_i C(s, t_i) + C(s, t) = 0,$$

where $T(s) - \{t\} = \{t_1, \ldots, t_{k_s}\}$.

Proof: Let $\Sigma = (S, T; F, W, M_0)$ be a reversible P/T-system and $t \in T$ such that t affects S. For every $s \in \mathrm{Af}\ (t)$, we have $C(s, t) \neq 0$. Because Σ is reversible, for every $M \in [M_0>$ such that t is enabled at M, there exists $u \in T^*$ such that $M'[u>M$, where $M[t>M'$. Hence, there exist $n_1, \ldots, \ldots n_{k_s} \in \mathbb{N}$ such that

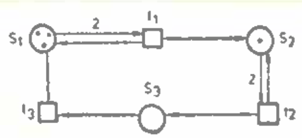
$$M(s) = M'(s) + \sum_{i=1}^{k_s} n_i C(s, t_i), \text{ where } T(s) - \{t\} = \{t_1, \ldots, t_{k_s}\}.$$

Then,

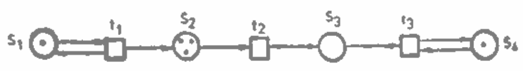
$$M(s) = M(s) + C(s, t) + \sum_{i=1}^{k_s} n_i C(s, t_i).$$

Finally, we deduce that (1) holds. Q.E.D.

Example 2.18. The following P/T-systems are reversible:



(i)



(ii)

3. Some Properties of Reversible and Strictly Reversible P/T**-systems.** The following theorem establishes the relationship between strictly reversible P/T-systems and strongly connected P/T-systems.

Theorem 3.1. *Let* $\Sigma = (S, T; F, W, M_0)$ *be a strictly reversible* P/T-*system. Then,* Σ *is strongly connected iff for any* $t \in T$ *there exist* $s_1, s_2 \in {}^\cdot t$ *such that* $(t, s_1), (s_2, t) \in F$.

Proof: Let $\Sigma = (S, T; F, W, M_0)$ be a strictly reversible and strongly connected P/T-system. Suppose by contradiction that for every $s_1, s_2 \in {}^\cdot t$, $(t, s_1) \notin F$ or $(s_2, t) \notin F$.

There are two cases:

(1) for any $s \in {}^\cdot t$, $(t, s) \in F$ and it does not exist $s' \in {}^\cdot t$ such that $(s', t) \in F$;

(2) for any $s \in {}^\cdot t$, $(s, t) \in F$ and it does not exist $s' \in {}^\cdot t$ such that $(t, s') \in F$. In the first case, for any $x \in S \cup T$, $x \neq t$, $(t, x) \notin F^*$ contradicting the fact that Σ is strongly connected. In the second case, for any $x \in S \cup T$, $x \neq t$, $(x, t) \notin F^*$ contradicting the fact that Σ is strongly connected. Hence, for any $t \in T$ there exists $s_1, s_2 \in {}^\cdot t$ such that $(t, s_1), (s_2, t) \in F$.

Conversely, let $\Sigma = (S, T; F, W, M_0)$ be a strictly reversible P/T-system such that for any $t \in T$ there exists $s_1, s_2 \in {}^\cdot t$ such that $(t, s_1), (s_2, t) \in F$.

For any arbitrary chosen $x, y \in S \cup T$ we have to prove that $(x, y) \in F^*$, i.e. there is a directed F-chain from x to y. Weak connectedness (Restriction 1.3) implies only that there is a $(F \cup F^{-1})$-chain from x to y, i.e. there is a sequence

$$x_0, \ldots, x_m \qquad (m \geq 0,\ x_i \in S \cup T)$$

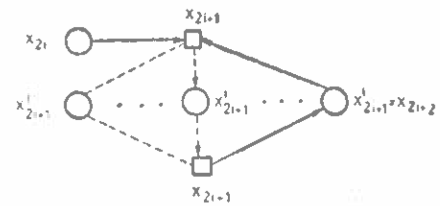
such that $x = x_0$, $x_m = y$ and $(x_i, x_{i+1}) \in (F \cup F^{-1})$ for $0 \leq i \leq m$. Let us now try to construct a F-chain from x to y.

There are four cases.

Case 1: $x, y \in S$. Then, there exists $k \in \mathbb{N}$ such that $m = 2k$ and for every $0 \leq i \leq k-2$, $x_{2i+1} \in T$. We have to prove that for any $0 \leq i \leq k-2$ we can construct a F-chain from x_{2i} to x_{2i+2}. Let $0 \leq i \leq k-2$. Consider the following four cases:

(1.1) $(x_{2i}, x_{2i+1}), (x_{2i+1}, x_{2i+2}) \in F$. Then, $x_{2i}, x_{2i+1}, x_{2i+2}$ is a F-chain from x_{2i} to x_{2i+2};

(1.2) $(x_{2i}, x_{2i+1}) \in F$ but $(x_{2i+1}, x_{2i+2}) \notin F$. Then, $(x_{2i+2}, x_{2i+1}) \in F$ and hence x_{2i+1} affects x_{2i+2}. From Theorem 2.13 it follows that there exists $\bar{x}_{2i+1} \in T$ such that x_{2i+1} and $\bar{x}_{2i+1}$ are duals. Moreover, $(\bar{x}_{2i+1}, x_{2i+2}) \in F$. Let $\mathrm{Af}\ (x_{2i+1}) = \{x^1_{2i+1}, \ldots, x^l_{2i+1}\}$ be, $l \geq 1$. Obvious by, $x_{2i+2} \in \mathrm{Af}\ (x_{2i+1})$ and without loss of generality, we may assume that $x_{2i+2} = x^l_{2i+1}$.



From the hypothesis, there exists $j \in \{1, \ldots, l\}$ such that $(x_{2i+1}, x^j_{2i+1}) \in F_0$ Because Σ is strictly reversible, $(x^j_{2i+1}, \bar{x}_{2i+1}) \in F$. We replace the sequence x_{2i+1}, x_{2i+2} by the sequence $x_{2i}, x_{2i+1}, x^j_{2i+1}, \bar{x}_{2i+1}, x_{2i+2}$ which is a F-chain from x_{2i} to x_{2i+2};

(1.3) $(x_{2i}, x_{2i+1}) \notin F$ but $(x_{2i+1}, \bar{x}_{2i+2}) \in F$. This case is analogous with the case (1.2);

(1.4) $(x_{2i}, x_{2i+1}), (x_{2i+1}, x_{2i+2}) \notin F$. Then, $(x_{2i+1}, x_{2i}), (x_{2i+2}, x_{2i+1}) \in F$ and hence x_{2i+1} affects x_{2i} and x_{2i+1}. From Theorem 2.13 it follows that there exists $\bar{x}_{2i+1} \in T$ such that x_{2i+1} and $\bar{x}_{2i+1}$ are duals. Moreover, $(x_{2i}, \bar{x}_{2i+1})$ $(\bar{x}_{2i+1}, x_{2i+2}) \in F$. We replace the sequence $x_{2i}, x_{2i+1}, x_{2i+2}$ with $x_{2i}, \bar{x}_{2i+1}$ x_{2i+2} which is a F-chain from x_{2i} to x_{2i+2}.

Hence we can construct a F-chain from x to y.

Case 2: $x \in S$ and $y \in T$. Then, $x_{m-1} \in S$ and analogously with Case 1 we can construct a F-chain from x to x_{m-1}. Let $x = x_0, x_{i1}, \ldots, x_{ik} = x_{m-1}$ be a such of F-chain. Consider the following two cases.

(2.1) $(x_{m-1}, x_m) \in F$. Then, $x_0, x_{i1}, \ldots, x_{ik} = x_{m-1}, x_m = y$ is a F-chain from x to y;

(2.2) $(x_{m-1}, x_m) \notin F$. Then, $(x_m, x_{m-1}) \in F$ and hence x_m affects S. From Theorem 2.13 it follows that there exists $\bar{x}_m \in T$ such that x_m and $\bar{x}_m$ are duals. Moreover, $(x_{m-1}, \bar{x}_m) \in F$. From the hypothesis, there exists $s \in S$

such that $(s, x_m) \in F$. Then, $(x_m, s) \in F$ and the sequence $x_0, x_{i_1}, ..., x_{i_k}, x_m, s, x_m$ is a F-chain from x to y.
Hence we can construct a F-chain from x to y.
Case 3: $x \in T$ and $y \in S$. This case is analogous with Case 2.
Case 4: $x, y \in T$. From Case 3 it follows that we can construct a F-chain from x to x_1 and from Case 2 it follows that we can construct a F-chain from x_1 to y. By concatenation of these F-chains, we obtain a F-chain from x to y.
Hence, Σ is strongly connected. Q.E.D.

This result does no longer hold if we consider reversible P/T-systems. The P/T-system from example 2.18(ii) constitute the required counter-example because, it is reversible (but not strictly reversible) and for every $t \in T$ there exist $s_1, s_2 \in {\cdot}t$ such that $(t, s_1), (s_2, t) \in F$. But, it is not strongly connected because $(t_2, s_2) \notin F^*$.

The following theorem establishes the relationship between reversible P/T-systems and live P/T-systems.

Theorem 3.2. *Let $\Sigma=(S, T; F, W, M_0)$ be a P/T-system. If Σ is reversible then Σ is live.*

Proof: Let $\Sigma=(S, T; F, W, M_0)$ be a reversible P/T-system. We have to prove that for every $M \in [M_0>$ and every $t \in T$ there exists $M' \in [M>$ such that t is enabled at M'.
Let $M \in [M_0>$ and $t \in T$. Then, there exists $u \in T^*$ such that $M_0[u>M$. There are two cases.
Case 1: $u=\lambda$. In this case $M=M_0$. From Restriction 1.4 it follows that there exists $M' \in [M_0>$ such that t is enabled at M'.
Case 2: $u \in T^+$. Let $u=t_1 \dots t_n$ be, $t_i \in T$ for all $1 \leqslant i \leqslant n$, $n \geqslant 1$. Then, there exist $M_1, ..., M_n \in [M_0>$ such that $M_0[t_1>M_1[...[t_n>M_n=M$. Because Σ is reversible, there exist $v_1, ..., v_n \in T^*$ such that $M_i[v_i>M_{i-1}$, $1 \leqslant i \leqslant n$. Then $M[v_n...v_1>M_0$. From Restriction 1.4 it follows that there exists $M' \in [M_0>$ such that t is enabled at M'. Hence, there exists $w \in T^*$ such that $M_0[w>M'$. Now, it is sufficient to prove that $M' \in [M>$. But, we have $M[v_n...v_1>M_0[w$ $M_0'[w>M'$ and therefore $M' \in [M>$. We deduce that Σ is live. Q.E.D.

Theorem 3.3. *Let $\Sigma=(S, T; F, W, M_0)$ be a reversible P/T-system and $\Gamma=(\Sigma, M_0, M)$ a structure. If $L(\Gamma) \neq \varnothing$ then for every $w \in L(\Gamma)$ and every decomposition $w=w_1tw_2$, $t \in T$, there exists $u \in T^*$ such that $w_1t(ut)^n w_2 \in L(\Gamma)$ for every $n \geqslant 1$.*

Proof: Let $\Sigma=(S, T; F, W, M_0)$ be a reversible P/T-system, $\Gamma=$ $=(\Sigma, M_0, M)$ a structure such that $L(\Gamma) \neq \varnothing$, $w \in L(\Gamma)$ and the decomposition $w=w_1tw_2$. Because $w \in L(\Gamma)$, there exist $M_1, M_2 \in [M_0>$ and $M_3 \in M$ such that $M_0[w_1>M_1[t>M_2[w_2>M_3$.
Because Σ is reversible, there exists $u \in T^*$ such that $M_2[u>M_1$. Moreover, for every $n \geqslant 1$, $M_2[(ut)^n>M_2$ and hence $w_1t(ut)^n w_2 \in L(\Gamma)$. Q.E.D.

An easy consequence of Theorem 3.3 is:

Corollary 3.4. *Let $\Sigma=(S, T; F, W, M_0)$ be a reversible P/T-system and $\Gamma=(\Sigma, M_0, M)$ a structure. If $L(\Gamma) \neq \varnothing$ then $L(\Gamma)$ is infinite.*

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