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# MATCHING EXTENSIONS FOR PETRI NET LANGUAGES

BY

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**1. Introduction.** In [4], [5], [6] three new operations on languages have been introduced and studied. These operations, called "prolongations" by the author, arise naturally in connection with the behaviour of some real systems. In this paper we study closure properties of classes of Petri net languages under matching extension (prolongation) operations. Unfortunately we have obtained only negative results. A more complex investigation is needed and we shall do this in further work.

We begin with some notions of formal language theory and net theory. For missing concepts and notations, we reffer to [2] and respectively to [3].

Let $V$ be an alphabet, i.e. a finite nonempty set. Then $V^* = V^+ \cup \{\lambda\}$ denotes the free monoid generated by $V$, where $V^+$ is the set of all finite nonempty strings (or words) over $V$ and $\lambda$ is the empty string.

Any subset $L \subseteq V^*$ is a language (over $V$). A family of languages is a nonempty class of languages, $\mathcal{L}$, which is closed under litteral isomorphisms (i.e. changes of alphabets) such that not all elements $L \in \mathcal{L}$ are the empy set.

The union, intersection, product, transpose, left-derivative, right-derivative operations on languages are denoted by $\cup, \cap, \cdot, \sim, \partial^l_a, \partial^r_a$ respectively.

A homomorphism $h : V^* \to U^*$ is called $k$-linear ($k \geq 1$) iff $(\forall u \in V^*)$ $(|u| \leq k\,|h(u)|)$, $|u|$ being the length of the word $u$.

We adopt the standard notation for the classical families of languages of the Chomsky hierarchy : $\mathcal{L}_0$ (recursively enumerable sets), $\mathcal{L}_1$ (context-sensitive sets), $\mathcal{L}_2$ (context-free sets) and $\mathcal{L}_3$ (regular sets).

These families are closed under union, intersection, product, transpose, left-derivatives, right-derivatives and homomorphism, excepting the family $\mathcal{L}_2$ which is not closed under intersection and the family $\mathcal{L}_1$ which is not closed under homomorphism but is closed under $k$-linear homomorphisms.

We are ready to give now the definitions of the matching extensions operations for languages.

Let $L_1, L_2 \subseteq V^*$ be fixed languages. Consider the following properties of a language $L' \subseteq V^*$.

$(P_1(L_1, L_2))$ $L_1 \subseteq L'$ ;

$(P_2(L_1, L_2))$ if $w \in L'$ with $w = w_1 w_2$, $w_1 \in V^*$, $w_2 \in V^+$, and $w_2 a \in L_2$ for some $a \in V$ then $wa \in L'$ ;

$(P_3(L_1, L_2))$ if $w \in L'$ with $w = w_1 w_2$, $w_1 \in V^+$, $w_2 \in V^*$, and $a w_1 \in L_2$ for some $a \in V$ then $aw \in L'$.

For any $L_1, L_2 \subseteq V^*$ we denote by $D(L_1, L_2)$ the smallest language which satisfies $P_1(L_1, L_2)$ and $P_2(L_1, L_2)$, by $S(L_1, L_2)$ the smallest language which satisfies $P_1(L_1, L_2)$ and $P_2(L_1, L_2)$ and by $C(L_1, L_2)$ the smallest language which satisfies $P_1(L_1, L_2)$, $P_2(L_1, L_2)$ and $P_3(L_1, L_2)$. These are respectively the *right-matching-extension*, the *left-matching-extension* and the *left-right-matching-extension operations* (prolongations, in [4], [5], [6].). In [7], [8] it is shown that $\mathcal{L}_0$ and $\mathcal{L}_1$ are closed under $D$, $S$, $C$; $\mathcal{L}_2$ is not closed under $D$, $S$, $C$; $\mathcal{L}_3$ is closed under $D$, $S$ and if $L_1 \in \mathcal{L}_2$, $L_2 \in \mathcal{L}_3$ then $D(L_1, L_2)$, $S(L_1, L_2) \in \mathcal{L}_2$.

The following result is also worth mentioning [8].

**Theorem 1.1.** *Let $V$ be an alphabet and $L_1$, $L_2$ be two languages over $V$. Then:*

(i) $D(L_1, L_2) = \partial_c^l(C(\{c\}L_1, L_2))$;

(ii) $S(L_1, L_2) = \partial_c^r(C(L_1\{c\}, L_2))$;

where $c \notin V$.

It is not known either if $\mathcal{L}_3$ is closed under $C$, nor if $L_1 \in \mathcal{L}_2$ and $L_2 \in \mathcal{L}_3$ imply $C(L_1, L_2) \in \mathcal{L}_2$(or $\mathcal{L}_3$).

Regarding Petri nets, we shall consider $P/T$-nets without capacity constraints; thus, $\Sigma = (S, T; F, W)$ will henceforth denote a $P/T$-net (with infinite capacities). As usual: $S$, $T$, $F$ and $W$ are the set of places, the set of transitions, the flow relation and the weight function, respectively. A marking of $\Sigma = (S, T; F, W)$ is a mapping $M: S \to N$, also represented by a vector $M \in N^{|S|}$, where $|S|$ is the cardinality of $S$ ($N$ is the set of nonnegative integers). $N^S$ denotes the set of all markings of $\Sigma$. The relation "$=$" and "$\leqslant$" and the operations "$+$" and "$-$" are componentwise extended from $N$ to vectors (markings) in $N^k$.

Let $\Sigma = (S, T; F, W)$ be a $P/T$-net, $M$ be a marking of $\Sigma$ and $t$ be a transition. Then $t$ is enabled at $M$, written $M[t>$ iff $M(s) \geqslant W(s, t)$ for all $s \in S$. If $t$ is enabled at $M$ then $t$ may occur, yielding a new marking $M'$ given by:

$$M'(s) = M(s) - W(s, t) + W(t, s) \text{ for all } s \in S.$$

The transformation of $M$ into $M'$ by the occuring of $t$ is denoted by $M[t>M'$. Let $M$ be a marking of the $P/T$ net $\Sigma = (S, T; F, W)$ and $w = t_1 \ldots t_n \in T^*$. Then, $w$ is enabled at $M$, written $M[w>$, iff there exist the markings $M_0, \ldots, M_n$ such that $M = M_0$ and for any $i$, $0 \leqslant i \leqslant n$, $M_i[t_{i+1}> M_{i+1}$. We will once again represent the transformation of $M$ into $M_n$ by $w$ as $M[w> M_n$.

By convention, $M[\lambda> M$ for every $M$ of $\Sigma$. For a making $M$ of $\Sigma$ we denote by $[M>$ the smallest set of markings of such that

(i) $M \in [M>$;

(ii) if $M_1 \in [M>$ and $M_1[t> M_2$ for some $t \in T$, then $M_2 \in [M>$.

In the graphical representation of $P/T$-nets we use circles for places and boxes for transitions.

The arcs $f \in F$ are labelled by $W(f)$ whenever $W(f) > 1$. A marking $M$ is indicated by drawing $M(s)$ black dots (tokens) on each place $s$.

In order to define a Petri net language one has to specify four quantities: the underlying Petri net $\Sigma = (S, T; F, W)$, the initial marking $M_0 \in N^S$,



a (usually finite) set of final markings $\mathcal{M} \subseteq N^S$ and a labelling function $l: T \to X$, where $X$ is an alphabet.

The fourthuple $\Gamma = (\Sigma, M_0, \mathcal{M}, l)$ is sometimes called the labelled $P/T$-net with terminal marking set. For simplicity, we shall call it a structure (as in [3]).

Considering a structure $\Gamma = (\Sigma, M_0, \mathcal{M}, l)$ and extending $l$ to a homomorphism $l: T^* \to X^*$ in the obvious way, or omitting $l$ if it is the identity, we define the following types of Petri net languages and the corresponding language families.

(i) $L(\Gamma) = \{l(w) / w \in T^* \wedge \exists M \in \mathcal{M}: M_0[w> M\}$
is the $L$-type or terminal language of $\Sigma$;

(ii) $G(\Gamma) = \{l(w) / w \in T^* \wedge \exists M' \in \mathcal{M} \wedge \exists M'' \geqslant M' : M_0[w> M''\}$
is the $G$-type or covering language of $\Sigma$;

(iii) $P(\Gamma) = \{l(w) / w \in T^* \wedge M_0[w>\}$ is the $P$-type language of $\Sigma$;

(iv) $T(\Gamma) = \{l(w) / w \in T^* \wedge \exists M \in N^S;\ M_0[w> M \wedge (\forall t \in T:$

$\neg M[t>)\}$ is the $T$-type or deadlock language of $\Sigma$.

The families of $L$-type ($G$-type, $P$-type, $T$-type, respectively) languages are abbreviated by $\underline{L}$ ($\underline{G}$, $\underline{P}$, $\underline{T}$). Among many possibilities for choosing certain labelling functions, the most simple is one where $|T| = |X|$ and $l$ is an isomorphism and may thus be omitted completely by choosing $X \equiv T$. In this situation, the languages of either type are called *free Petri net languages*. Petri net languages obtained by extending the codomain of $l$ to $X \cup \{\lambda\}$ are called *arbitrary Petri net languages* or languages defined by using $\lambda$-transitions.

For any family $X$ of Petri net languages of either type $X \in \{\underline{L}, \underline{G}, \underline{P}, \underline{T}\}$ and $\underline{X}^f$ ($\underline{X}^\lambda$) denote the corresponding class of free (respectively arbitrary) Petri net languages.

**2. Matching Extension Operations on Petri Net Languages.** The matching extension operations have been introduced in [5] to model the behaviour of some economic systems. These are real systems with intrinsic concurrency, for which Petri nets could be their finite descriptions. Matching extension operations applied to Petri net languages could be interpreted in the following natural way: the activity of a $P/T$-net is continued by another $P/T$-net (which "learns" from the first!), yielding a new system. The classes of Petri net languages should thus be closed under these operations. Surprisingly, this fact is not true, so new problems arises: is the definition of a $P/T$-net sufficiently rich to model concurrent behaviour correctly?; are the classes $\underline{L}$, $\underline{P}$, $\underline{T}$, $\underline{G}$ the only interesting families of languages connected to $P/T$-nets?; should the operations $D$, $S$, $C$ be simplified?

The purpose of this paper is only to show that the classes $\underline{X}^f$, $\underline{X} \in \{\underline{L}, \underline{P}, \underline{T}, \underline{G}\}$ are not closed under $D$, $S$, $C$. To start with, we give a technical lemma which will help us to reduce some proofs.

**Lemma 2.1.** *Let $V$ be an alphabet, $a \in V$ and $L$ be a language such that*

*(1) $V$ is the smallest alphabet such that $L \in V^*$*

*(2) there exists $w_1, w_2 \in V^*$ such that for any $n \in N$, $w_1 a^n w_2 \in L$.*

*If* $L \in X^f$, *where* $X \in \{\underline{L}, \underline{G}, \underline{P}, \underline{T}\}$, *then for any structure* $\Gamma=(\Sigma, M_0, \mathcal{M})$ such that $L=X(\Gamma)$ *we have*

(i) $V \subseteq T$;

(ii) *for any* $s \in S$ : $W(s, a) \leq W(a, s)$, *where* $\Sigma=(S, T, F, W)$.

*Proof.* Given $V$, $a \in V$, $X \in \{\underline{L}, \underline{G}, \underline{P}, \underline{T}\}$ and $L \subseteq V^*$ a language such that $L \in X^f$ and $L$ satisfies (1) and (2), let $\Gamma$ be a structure $\Gamma=(\Sigma, M_0, \mathcal{M})$ such that $L=X(\Gamma)$, where $\Sigma=(S, T; F, W)$.

(i) follows directly from the hypotheses.

Let us show that (ii) holds. Suppose by contradiction that there exists $s \in S$ such that $W(s, a) > W(a, s)$. Since $w_1 a w_2 \in X(\Gamma)$ it follows that there exist the markings $M_1, M_2, M_3 \in N^S$ such that

$$M_0[w_1 > M_1[a > M_2[w_2 > M_3.$$

Let $m=M_1(s)$ and $p=W(s, a)-W(a, s)>0$. We have $m \geq p$ since $a$ is enabled at $M_1$. Consider $k=[m/p]+1$. It is easy to see that $a^k$ is not enabled at $M_1$ and hence $w_1 a^k w_2 \notin X(\Gamma)$ contracting the fact that $w_1 a^k w_2 \in L=X(\Gamma)$.

Hence, (ii) holds.

The general method for obtaining negative closure results is to use this lemma to prove that some particular languages are not in the families $X^f$.

**Lemma 2.2.** *The languages* $L_1=\{a, b\}^* \{a\} \cup \{\lambda\}$ *and* $L_2=\{a^n b a^m / n, m \geq 0\} \cup \{a^n b^m / n, m \geq 0\}$ *are not in the families* $\underline{L}^f, \underline{G}^f, \underline{P}^f, \underline{T}^f$.

*Proof.* Let us prove that $L_1 \notin \underline{L}^f$. Suppose by contradiction that $L_1 \in \underline{L}^f$, then, there exists a structure $\Gamma=(\Sigma, M_0, M)$ such that $L_1=L(\Gamma)$. Let $\Sigma=(S, T; F, W)$. From Lemma 2.1 it follows that:

(i) $\{a, b\} \subseteq T$,

(ii) for any $s \in S$ and any $t \in \{a, b\}$ : $W(t, s) \geq W(s, t)$.

Since $ba \in L_1=L(\Gamma)$ we deduce that there exist the markings $M_1 \in N^S$ and $M_2 \in \mathcal{M}$ such that $M_0[b > M_1[a > M_2$. Since $a \in L_1=L(\Gamma)$, $a$ is enabled at $M_0$. Let $M_3 \in N^S$, $M_0[a > M_3$. From (ii), it follows that $M_3 \geq M_0$ and since $b$ is enabled at $M$ it follows that $b$ is enables at $M_3$. Let $M_4 \in N^S$, $M_3[b > M_4$. We have then $M_0[a > M_3[b > M_4$. It is easy to see that $M_4=M_2$ hence $M_4 \in \mathcal{M}$ and $ab \in L(\Gamma)$, but $ab \notin L_1=L(\Gamma)$.

The fact that $L_1$ is not in the families $\underline{G}^f, \underline{P}^f, \underline{T}^f$ can be proved analogously (the only difference is in the way of acceptance of $ab$ and $ba$). The same is true for $L_2$. If $a^2 b^2 \in X(\Gamma)=L_2$ then $ab^2 a \in X(\Gamma)$, but $ab^2 a \notin L_2$.

**Theorem 2.3.** *The families* $\underline{L}^f$, $\underline{G}^f$, $\underline{P}^f$, $\underline{T}^f$ *are not closed under S and D.*

*Proof.* We consider the following languages: $L_1=\{\lambda, a\}$, $L_2=\{\lambda, a, b, aa, ab, ba, bb\}$, $L_3=\{a^n b a^m / n, m>0\} \cup \{a^n / n \geq 0\}$ and $L_4=\{\lambda, b, bb\}$. It is easy to see that $L_1, L_2, L_4$ are in the families $\underline{L}^f, \underline{G}^f, \underline{P}^f, \underline{T}^f$. We show that $L_3 \in X^f$, $X \in \{\underline{L}, \underline{G}, \underline{P}\}$. More precisely, $L_3=X^f(\Gamma)$, where $\Gamma=(\Sigma, M_0, \mathcal{M})$, $\Sigma$ being the $P/T$-net below ($M_0=(2)$, $\mathcal{M}=\{(1), (2)\}$).

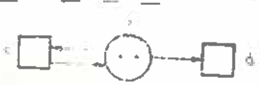

Fig. 1

By changing $\Gamma$ into $\Gamma'=(\Sigma, M_0', \Phi)$, $M_0'=(3)$ we have also $L_3 \in \underline{T}^f$.



Directly from the corresponding definitions we obtain

$$S(L_1, L_2)=\{a, b\}^* \{a\} \cup \{\lambda\}.$$

and

$$D(L_3, L_4)=\{a^n b a^m / n, m \geq 0\} \cup \{a^n b^m / n, m \geq 0\}.$$

But, from Lemma 2.2, these languages are not in $\underline{L}^f, \underline{G}^f, \underline{P}^f, \underline{T}^f$. We proceed in a similar way for $C$. Q.E.D.

**Theorem 2.4.** *The family* $\underline{P}^f$ *is not closed under* $C$.

*Proof.* Let $L'=\{\lambda, a\}$ and $L''=\{\lambda, b, ba, bac, a, ac\}$. The fact $L' \in \underline{P}^f$ is straightforward. To prove that $L'' \in \underline{P}^f$, consider the structure $\Gamma=(\Sigma, M_0, \Phi)$ where $\Sigma$ is the $P/T$-net drawn bellow (together with its initial marking $M_0=(1, 1, 1, 0)$):

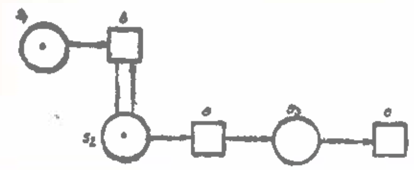

Fig. 2

We have $P(\Gamma)=L''$ and $C(L', L'')=\{\lambda, a, ac, ba, bac\}$. Since $C(L', L'')$ has not the prefix property ($b \notin C(L', L'')$), we deduce that $C(L', L'') \notin \underline{P}^f$ and hence $\underline{P}^f$ is not closed under the $C$ operation. Q.E.D.

**Theorem 2.5.** *The families* $\underline{G}^f$, $\underline{P}^f$, $\underline{T}^f$ *are closed under left-derivatives.*

The proof of this theorem is analogous to that of Lemma 3 [10] in which it is shown that $\underline{L}^f$ is left-derivative-closed, and hence we omit it.

**Theorem 2.6.** *Let* $L \subseteq V^*$ *be an arbitrary language and* $c$ *be a new symbol* ($c \notin V$). *If* $L \in X^f$, $X \in \{\underline{L}, \underline{G}, \underline{T}\}$, *then* $\{c\} L \in X^f$.

*Proof.* Let $X \in \{\underline{L}, \underline{G}, \underline{T}\}$, $L \subseteq V^*$, $L \in X^f$ and $c \notin V$. Then, there exists a structure $\Gamma=(\Sigma, M_0, \mathcal{M})$ such that $L=X(\Gamma)$. Let $\Sigma=(S, T; F, W)$, $V \subseteq T$. We construct the structure $\Gamma'=(\Sigma', M_0', M')$, given by

(1) $\Sigma'=(S', T', ; F', W')$ where

a) $S'=S \cup \{s'\}$, $s' \notin S$;

b) $T'=T \cup \{c\}$ (we can assume that $c \notin T$);

c) $F'=F \cup \{(s', c)\} \cup \{(c, s) / s \in S, M_0(s)>0\}$;

d) $W' : F' \rightarrow N$

$$W'(x, y)=\begin{cases} W(x, y), & \text{if } (x, y) \in F \\ 1, & \text{if } (x, y)=(s', c) \\ M_0(s), & \text{if } (x, y)=(c, s); \end{cases}$$

(2) The initial marking $M_0' : S' \rightarrow N$ is given by

$$M_0'(x)=\begin{cases} 0, & \text{if } x \in S \\ 1, & \text{if } x=s' \text{ and } \underline{X} \in \{\underline{L}, \underline{G}\} \\ 2, & \text{if } x=s' \text{ and } \underline{X}=\underline{T}; \end{cases}$$

(3) For every $M \in \mathcal{M}$ let $M' : S' \rightarrow N$ being defined by

$$M'(x)=\begin{cases} M(x), & \text{if } x \in S \\ 0, & \text{if } x=s'. \end{cases}$$

Let $\mathscr{M}'$ be the set of all these markings (the final markings for $\Gamma'$). Pictorially, $\Gamma'$ can be described as follows

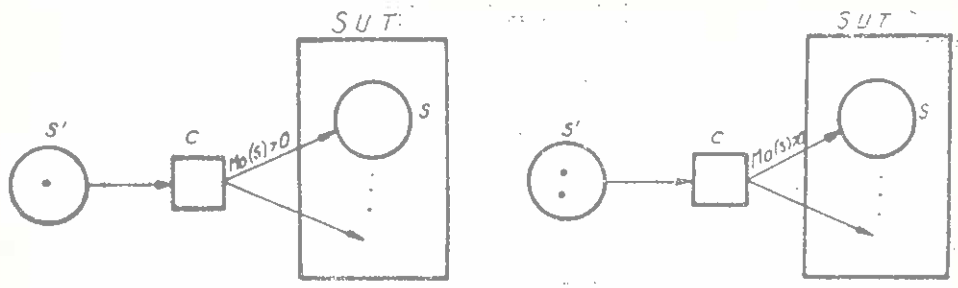


Fig. 3

(i) for $\underline{X} \in \{\underline{L}, \underline{G}\}$ (ii) for $\underline{X} = \underline{T}$

Note that only the transition $c$ is enabled at $M_0$, that $M_0[c\rangle \overline{M}$, where $\overline{M}/S = M_0$ and thus $\Sigma'$ will have the same behaviour as $\Sigma$. Hence, $\{c\}L = X(\Gamma')$.
Q.E.D.

This theorem is not true for $\underline{P}^f$. But a similar result holds for this family.

*Remark* 2.7. Let $L \subseteq V^*$ be an arbitrary language and $c$ be a new symbol ($c \notin V$). If $L \in \underline{P}^f$ then $\{c\}L \cup \{\lambda\} \in P^f$.

**Theorem 2.8.** *The families $\underline{L}^f$, $\underline{G}^f$, $\underline{T}^f$ are not closed under $C$.*

*Proof*. Let $\underline{X} \in \{L, G, T\}$. Suppose that $\underline{X}^f$ is closed under the $C$ operation. From the equality $D(L', L'') = \partial\,(C(\{c\}L', L''))$ (Theorem 2.6) and from Remark 2.7, it follows that $\underline{X}^f$ is closed under the $D$ operation, which is a contradiction (Theorem 2.3). Hence, $\underline{X}^f$ is not closed under the $C$ operation. Q.E.D.

**3. Concluding remarks and open problems.** Free Petri net classes of languages are not closed under the $D$, $S$, $C$ operations. In a further paper we shall investigate more profoundly these operations in order to obtain positive closure results. The question if a change in the definition of a $P/T$-net (or in the form of language acceptance) is really necessary, remains open. Another open problem is to define a "measure" of "closeness" for a class of languages under certain operations.

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# A REFINEMENT OF THE MATCHING EXTENSION OPERATIONS

BY

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**1. Introduction.** We have shown [12] that the classes of Petri net languages are not closed under the matching extension operations, contradicting the intuition. The purpose of this paper is to express these operations in terms of other known operations on languages and in terms of „simpler" operations, to see where the "crash" could stand. One-symbol matching extension operations are introduced and closure properties for families of Petri net languages and for classical families of the Chomsky hierarchy under these operations are studied. Positive results are obtained. For necessary definitions and notations we refer to [12]. We only note that we consider $P/T$-nets with infinite capacities and the families of languages of the Chomsky hierarchy are denoted by $\mathscr{L}_0$, $\mathscr{L}_1$, $\mathscr{L}_2$, $\mathscr{L}_3$.

**2. One-symbol Matching Extension Operations.** Matching extension operations are obtained by recursively "enlarging" a given language by words of another language. What happens if we admit only a one-symbol extension? The answer is that these restricted operations have nice closure properties and the initial operations can be expressed in terms of the new ones. The formal definition of the one-symbol extension operations is given bellow.

Let $L_1$, $L_2$ be fixed languages on an alphabet $V$. We consider the following properties of a language $L' \subseteq V^*$

($P_1'(L_1, L_2)$) $L_1 \subseteq L'$;

($P_2'(L_1, L_2)$) if $w \in L_1$, $w = w_1w_2$ with $w_1, w_2 \in V^*$, $w_2 \neq \lambda$ and $w_2a \in L_2$ (for some $a \in V$), then $wa \in L'$;

($P_3'(L_1, L_2)$) if $w \in L_1$, $w = w_1w_2$ with $w_1, w_2 \in V^*$, $w_1 \neq \lambda$ and $aw_1 \in L_2$ (for some $a \in V$), then $aw \in L'$.

For any $L_1, L_2 \subseteq V^*$ we denote by $D_1(L_1, L_2)$ the smallest language which satisfies $P_1'(L_1, L_2)$ and $P_2'(L_1, L_2)$, by $S_1(L_1, L_2)$ the smallest language which satisfies $P_1'(L_1, L_2)$ and $P_3'(\ _1, L_2)$ and $C_1L(_1, L_2)$ the smallest language which satisfies $P_1'(L_1, L_2)$, $P_2'(L_1, w_2)$ and $P_3'(L_1, L_2)$. These are respectively the *one-symbol right-matching extension*, the *one-symbol left-matching extension* and the *one-symbol left-right-matching extension operations*. To express $D$, $S$, $C$ ([12]) in the terms of $D_1$, $S_1$, $C_1$ let us note that $D$, $S$, $C$ could be obtained by successively applying $D_1$, $S_1$, $C_1$ to the given languages. For any $L_1, L_2 \subseteq V^*$ and $M \in \{D, S, C\}$ we define

(i) $M_1^0(L_1, L_2) = L_1$

(ii) $M_1^n(L_1, L_2) = M_1(M_1^{n-1}(L_1, L_2), L_2)$, for any $n \geq 1$;