Let $\mathcal{M}'$ be the set of all these markings (the final markings for Γ'). Pictorially, Γ' can be described as follows

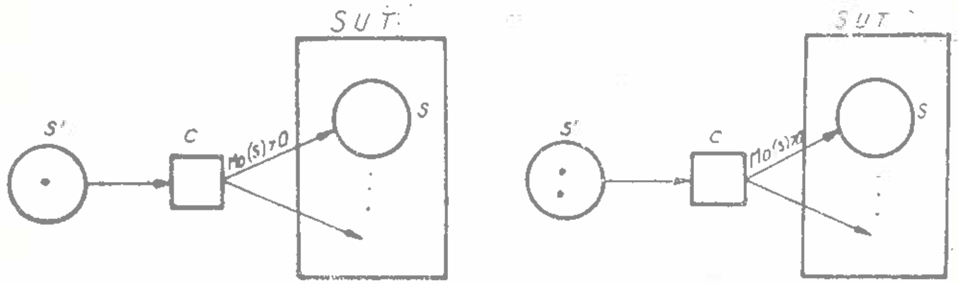

Fig. 3

(i) for $\underline{X}\in\{\underline{L}, \underline{G}\}$ (ii) for $\underline{X}=\underline{T}$

Note that only the transition c is enabled at M_0, that $M_0[c\rangle \overline{M}$, where $\overline{M}/S=M_0$ and thus Σ' will have the same behaviour as Σ. Hence, $\{c\}L=X(\Gamma')$.
Q.E.D.

This theorem is not true for $\underline{P}^f$. But a similar result holds for this family.

Remark 2.7. Let $L\subseteq V^*$ be an arbitrary language and c be a new symbol ($c\notin V$). If $L\in \underline{P}^f$ then $\{c\}L\cup\{\lambda\}\in \underline{P}^f$.

Theorem 2.8. *The families $\underline{L}^f$, $\underline{G}^f$, $\underline{T}^f$ are not closed under C.*

Proof. Let $X\in\{\underline{L}, \underline{G}, \underline{T}\}$. Suppose that X^f is closed under the C operation. From the equality $D(L', L'')=\partial$ $(C(\{c\}L', L''))$ (Theorem 2.6) and from Remark 2.7, it follows that X^f is closed under the D operation, which is a contradiction (Theorem 2.3). Hence, X^f is not closed under the C operation. Q.E.D.

3. Concluding remarks and open problems. Free Petri net classes of languages are not closed under the D, S, C operations. In a further paper we shall investigate more profoundly these operations in order to obtain positive closure results. The question if a change in the definition of a P/T-net (or in the form of language acceptance) is really necessary, remains open. Another open problem is to define a "measure" of "closeness" for a class of languages under certain operations.

REFERENCES

1. Best, E., Fernandez, C. — *Notations and Terminology on Petri Net Theory*, Arbeitspapiere der GMD, 195, Second Edition, 1987.
2. Harison M. A. — *Introduction to Formal Language Theory*, Addison-Wesley, 1978
3. Jantzen, M. — *Language Theory of Petri Nets*, Springer-Verlag LNCS 254, 1986.
4. Păun, Gh. — *A Generative Model of Conversation*, Semiotica, 17, 1, 1976.
5. Păun, Gh. — *Generative Grammars for some Economic Activities (Scheduling and Marshalling Problems)*, Foundations of Control Engineering, Poznan, 2, 1, 1977.
6. Păun, Gh. — *A Formal Linguistic Approach to the Production Process*, Foundations of Control Engineering, Poznan, 1, 3, 1976.
7. Păun, Gh. — *On a Prolongation Operation of Languages*, Bull. Math. de la Soc. de Sci. Math. de R.S.R., 23 (71), 1979.
8. Păun, Gh. — *The Generative Mechanism of Economic Processes*, Ed. Tehnică, Bucureşti 1980. (in Romanian).
9. Reisig, W. — *Place / Transition Systems*, Springer-Verlag, LNCS 254, 1986.
10. Starke, P. H. — *Free Petri Net Languages*, Springer-Verlag LNCS 64, 1978.

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A REFINEMENT OF THE MATCHING EXTENSION OPERATIONS

BY

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1. Introduction. We have shown [12] that the classes of Petri net languages are not closed under the matching extension operations, contradicting the intuition. The purpose of this paper is to express these operations in terms of other known operations on languages and in terms of „simpler" operations, to see where the "crash" could stand. One-symbol matching extension operations are introduced and closure properties for families of Petri net languages and for classical families of the Chomsky hierarchy under these operations are studied. Positive results are obtained. For necessary definitions and notations we refer to [12]. We only note that we consider P/T-nets with infinite capacities and the families of languages of the Chomsky hierarchy are denoted by $\mathcal{L}_0$, $\mathcal{L}_1$, $\mathcal{L}_2$, $\mathcal{L}_3$.

2. One-symbol Matching Extension Operations. Matching extension operations are obtained by recursively "enlarging" a given language by words of another language. What happens if we admit only a one-symbol extension? The answer is that these restricted operations have nice closure properties and the initial operations can be expressed in terms of the new ones. The formal definition of the one-symbol extension operations is given bellow.

Let L_1, L_2 be fixed languages on an alphabet V. We consider the following properties of a language $L'\subseteq V^*$

($P_1'(L_1, L_2)$) $L_1\subseteq L'$;

($P_2'(L_1, L_2)$) if $w\in L_1$, $w=w_1w_2$ with $w_1, w_2\in V^*$, $w_2\neq\lambda$ and $w_2a\in L_2$ (for some $a\in V$), then $wa\in L'$;

($P_3'(L_1, L_2)$) if $w\in L_1$, $w=w_1w_2$ with $w_1, w_2\in V^*$, $w_1\neq\lambda$ and $aw_1\in L_2$ (for some $a\in V$), then $aw\in L'$.

For any $L_1, L_2\subseteq V^*$ we denote by $D_1(L_1, L_2)$ the smallest language which satisfies $P_1'(L_1, L_2)$ and $P_2'(L_1, L_2)$, by $S_1(L_1, L_2)$ the smallest language which satisfies $P_1'(L_1, L_2)$ and $P_3'(\ _1, L_2)$ and $C_1L(_1, L_2)$ the smallest language which satisfies $P_1'(L_1, L_2)$, $P_2'(L_1, w_2)$ and $P_3'(L_1, L_2)$. These are respectively the *one-symbol right-matching extension*, the *one-symbol left-matching extension* and the *one-symbol left-right-matching extension operations*. To express D, S, C([12]) in the terms of D_1, S_1, C_1 let us note that D, S, C could be obtained by successively applying D_1, S_1, C_1 to the given languages. For any $L_1, L_2\subseteq V^*$ and $M\in\{D, S, C\}$ we define

(i) $M_1^0(L_1, L_2)=L_1$

(ii) $M_1^n(L_1, L_2)=M_1(M_1^{n-1}(L_1, L_2), L_2)$, for any $n\geq 1$;

(iii) $M_1^*(L_1, L_2) = \bigcup_{n\geq 0} M_1^n(L_1, L_2)$.

The following facts are immediate from the corresponding definitions.

Remark 2.1. For any $L_1, L_2 \subseteq V^*$ and $M \in \{D, S, C\}$ we have

(i) $M_1^*(L_1, L_2) = \bigcup_{n\geq 1} M_1^n(L_1, L_2)$

(ii) $M_1^n(L_1, L_2) \subseteq M_1^{n+1}(L_1, L_2)$, for any $n \geq 0$;

(iii) if there exists $k \geq 0$ such that $M_1^k(L_1, L_2) = M_1^{k+1}(L_1, L_2)$ then $M_1^*(L_1, L_2) = M_1^k(L_1, L_2)$.

Remark 2.2. If a family of languages is closed under M_1, where $M \in \{D, S, C\}$, then it is closed under M_1^n, for any $n \geq 1$.

Some properties of D_1, S_1, C_1 and the exact relationship between D_1, S_1, C_1 and D, S, C, are emphasized in the following results.

Lemma 2.3. *For any $L_1, L_2 \subseteq V^*$ we have*

(i) $\widetilde{S_1(L_1, L_2)} = D_1(\tilde{w}_1, \tilde{L}_2)$;

(ii) $\widetilde{D_1(L_1, L_2)} = S_1(\tilde{L}_1, \tilde{L}_2)$.

Proof. (i) Let $w \in \widetilde{S_1(L_1, L_2)}$. Then $\tilde{w} \in S_1(L_1, L_2)$. There are two cases

Case 1. $\tilde{w} \in L_1$. Then, $w \in \tilde{L}_1$ and hence $w \in D_1(\tilde{L}_1, \tilde{L}_2)$;
Case 2. there exist $u \in L_1$, $a \in V$ such that $u = u_1u_2$, $u_1 \in V^+$, $u_2 \in V^*$, $au_1 \in L_2$ and $\tilde{w} = au$. Then $\tilde{u} = \tilde{u}_2\tilde{u}_1 \in \tilde{L}_1$, $\tilde{u}_1 \in V^+$, $\tilde{u}_2 \in V^*$, $\tilde{u}_1 a \in \tilde{L}_2$ and hence $\tilde{u}a \in D_1(\tilde{L}_1, \tilde{L}_2)$. Because $w = \tilde{u}a$ we deduce that $w \in D_1(\tilde{L}_1, \tilde{L}_2)$ and hence $\widetilde{S_1(L_1, L_2)} \subseteq D_1(\tilde{L}_1, \tilde{L}_2)$.
The converse inclusion follows in a similar way.

(ii) We consider $\tilde{L}_1$, $\tilde{L}_2$ and from (i) it follows that $\widetilde{S_1(\tilde{L}_1, \tilde{L}_1)} = D_1(L_1, L_2)$, hence $S_1(\tilde{L}_1, \tilde{L}_2) = \widetilde{D_1(L_1, L_2)}$. Q.E.D.

Corollary 2.4. *Any family of languages which is closed under transpose operation is closed under D_1 iff it is closed under S_1.*

Lemma 2.5. *For any $L_1, L_2 \subseteq V^*$ we have $C_1(L_1, L_2) = D_1(L_1, L_2) \cup S_1(L_1, L_2)$.*

Proof. Obviously, $D_1(L_1, L_2) \cup S_1(L_1, L_2) \subseteq C_1(L_1, L_2)$. Let us prove that $C_1(L_1, L_2) \subseteq D_1(L_1, L_2) \cup S_1(L_1, L_2)$. Let $w \in C_1(L_1, L_2)$. There are three cases to consider:
Case 1. $w \in L_1$. Then, $w \in D_1(L_1, L_2) \cup S_1(L_1, L_2)$;
Case 2. there exist $u \in L_1$, $a \in V$ such that $u = u_1u_2$, $u_1 \in V^*$, $u_2 \in V^+$, $u_2a \in L_2$ and $w = ua$. Then, $w \in D_1(L_1, L_2) \subseteq D_1(L_1, L_2) \cup S_1(L_1, L_2)$;
Case 3. there exist $u \in L_1$, $a \in V$ such that $u = u_2$, $u_1 \in V^+$, $u_2 \in V^*$, $au_1 \in L_2$ and $w = au$. Then, $w \in S_1(L_1, L_2) \subseteq D_1(L_1, L_2) \cup S_1(L_1, L_2)$. Q.E.D.
From Lemma 2.5 and Corollary 2.4 it follows

Corollary 2.6. *Any family of languages closed under union, transpose and S_1 (or D_1) it is closed under C_1.*

Theorem 2.7. *For any $L_1, L_2 \subseteq V^*$ and any $M \in \{D, S, C\}$ we have*

$$M(L_1, L_2) = M_1^*(L_1, L_2).$$

Proof. Let us prove that $D(L_1, L_2) = D_1^*(L_1, L_2)$. For S and C the proof follows a similar way. Let $w \in D(L_1, L_2)$. then, There exist $w_0, \ldots, w_n \in V^*$, $a_1, \ldots, a_n \in V$ such that

(i) $w_0 \in L_1$ and $w_n = w$;

(ii) for any i, $0 \leq i \leq n$, $w_i = w_i^1w_i^2$, $w_i^1 \in V^*$, $w_i^2 \in V^+$, $w_i^2a_{i+1} \in L_2$ and $w_{i+1} = w_ia_{i+1}$.

We prove by induction on k, $0 \leq k \leq n$, that $w \in D_1^k(L_1, L_2)$.
Basis : $k = 0$. Then, $w_0 \in L_1$ (from (i)) and hence $w_0 \in D_1^0(L_1, L_2) = L_1$.
Induction step. Suppose that for any i, $0 \leq i \leq k$, $0 \leq k \leq n$ we have: $w_i \in D_1^i(L_1, L_2)$. Let us prove that $w_{k+1} \in D_1^{k+1}(L_1, L_2)$. From (ii) it follows that $w_k = w_k^1w_k^2$, $w_k^1 \in V^*$, $w_k^2 \in V^+$, $w_k^2a_{k+1} \in L_2$ and $w_{k+1} = w_ka_{k+1}$. From the inductive hypothesis and from the fact that $D_2^{k+1}(L_1, L_2) = (D_1(D_1^k(L_1, L_2), L_2)$ it follows that $w_{k+1} \in D_1^{k+1}(L_1, L_2)$.

Now, we deduce that $D_1^n(L_1, L_2) \subseteq D_1^*(L_1, L_2)$ and hence $D(L_1, L_2) \subseteq D_1^*(L_1, L_2)$. Analogously, we can prove that $D_1^*(L_1, L_2) \subseteq D(L_1, L_2)$ and hence $D(L_1, L_2) = D_1^*(L_1, L_2)$ Q.E.D.

In this way C, S, D are iterations of C_1, S_1, D_1. We return now to problem of the expressing D_1, S_1, C_1 in terms of other known operations on languages.

Let V be an alphabet. Then, by c_V we denote a new symbol, i.e. $c_V \notin V$ and by h_V we denote the homomorphism

$$h_V : (V \cup c_V)^* \to V^*$$

defined by

$$h_V(a) = \begin{cases} a, & \text{if } a \in V \\ \lambda, & \text{if } a = c_V \end{cases}$$

(the extension on words is the classical one).

Theorem 2.8. *Let $L_1, L_2 \subseteq V^*$. Then*

(i) $S_1(L_1, L_2) = L_1 \cup h_V(L_1' \cap L_2')$, *where* $L_1' = V\{c_V\}L_1$ *and* $L_2' = (\bigcup_{a\in V} \{ac_V\}\partial_a^l(L_2))V^*$;

(ii) $D_1(L_1, L_2) = L_1 \cup h_V(L_1'' \cap L_2'')$, *where* $L_1'' = L_1\{c_V\}V$ *and* $L_2'' = V^*(\bigcup_{a\in V} \partial_a^r(L_2)\{c_Va\})$.

Proof. i) Let $w \in S_1(L_1, L_2)$. We must again consider two cases
Case 1. $w \in L_1$. Then ,$w \in L_1 \cup h_V(L_1' \cap L_2')$.
Case 2. there exist $u \in L_1$, $a \in V$ such that $u = u_1u_2$, $u_1 \in V^*$, $u_2 \in V^*$, $au_1 \in L_2$ and $w = au$. Then $ac_Vu \in L_1'$, $ac_Vu_1u_2 = ac_Vu \in L_2'$ and hence $ac_Vu \in L_1' \cap L_2'$ from which it follows that $au \in h_V(L_1' \cap L_2')$. We deduce that $L \in L_1 \cup h_V(L_1' \cap L_2')$.
From Case 1 and Case 2, it follows that $S_1(L_1, L_2) \subseteq L_1 \cup h_V(L_1' \cap L_2')$. Let now $w \in L_1 \cup h_V(L_1' \cap L_2')$. If $w \in L_1$ then $w \in S_1(L_1, L_2)$. If $w \in h_V(L_1' \cap L_2')$ then there exists $w' \in L_1' \cap L_2'$ such that $w = h_V(w')$. We deduce that $w' = ac_Vu_1u_2$ with $u_1u_2 \in L_1$ and $au_1 \in L_2$. Moreover, $w = au_1u_2$ and hence $w \in S_1(L_1, L_2)$ and $L_1 \cup h_V(L_1' \cap L_2') \subseteq S_1(L_1, L_2)$.

ii) can be proved analogously.

Corollary 2.9. *The families $\mathcal{L}_3$, $\mathcal{L}_1$, $\mathcal{L}_0$, $\underline{L}^\lambda$, $\underline{P}^\lambda$ are closed under M_1^n, for any $M_1 \in \{D_1, S_1, C_1\}$ and any $n \geq 1$.*

Proof. From Remark 2.2 it follows that it sufficies to prove that the families $\mathcal{L}_3$, $\mathcal{L}_1$, $\mathcal{L}_0$, $\underline{L}^\lambda$, $\underline{P}^\lambda$ are closed under M_1 for any $M_1 \in \{D_1, S_1, C_1\}$. But this follows from Theorem 2.8, Lemma 2.5 and from the fact that these families are closed under union, intersection, concatenation, left-derivatives, right-derivatives and 2-linear homomorphism ([2], [3]). The $D_1(S_1, C_1)$ operation could also be expressed in another way.

Theorem 2.10. *Let* $L_1, L_2 \subseteq V^*$. *Then* $D_1(L_1, L_2) = (L\bar{L} \cap L_1V) \cup L_1$, *where* $L = L_1/(L_2/V - \{\lambda\})$ *and* $\bar{L} = (L_1 . V) \setminus L \cap L_2$ *("/" and "\" denote respectively the right and left quotient of languages).*

Proof. Let $w \in D_1(L_1, L_2)$. If $w \in L_1$ then $w \in (L.\bar{L} \cap L_1 . V) \cup L_1$. Suppose $w \notin L_1$ and $w = va$, $v \in L_1$, $a \in V$, $v = w_1w_2$, $w_2 \neq \lambda$. $w_2a \in L_2$. Then, $w = va$. $v \in L_1$. $a \in V$ and hence $w \in L_1 . V$. Much more, $w = va = w_1w_2a$, $w_2a \in L_2$ and hence $w = w_1 . w_3$ where $w_3 = w_2a$. Let us show that $w_1 \in L$ and $w_3 \in \bar{L}$. We must show that there But $w_3 \in L_2$ from the definition of $D_1(L_1, L_2)$. It suffices to exists $u \in L_1 V$ such that $u = \bar{u}w_3$. $\bar{u} \in L$. But $u = w$ and $\bar{u} = w_1$. But show that $w_1 \in L$. Because $w_2a \in L_2$, $w_2 \neq \lambda$, it follows that $w_2 \in L_2/V - \{\lambda\}$. $w_1w_2 = v \in L_1$ and $w_2 \in L_2/V - \{\lambda\}$ and hence $w_1 \in L_1/(L_2/V - \{\lambda\}) = L$.

Thus, for $w_1 \in L$, $w_3 \in \bar{L}$ we have $w_1w_3 = w \in L . \bar{L}$. From this fact and since $w \in L_1 V$ we deduce that $w \in L\bar{L} \cap L_1 V$.

Conversely, let $w \in (L . \bar{L} \cap L_1 . V) \cup L_1$. If $w \in L_1$, then $w \in D_1(L_1, L_2)$. If $w \notin L_1$ then $w \in L\bar{L} \cap L_1 V$. We have : $w = w_1w_3$, $w_1 \in L$, $w_3 \in \bar{L}$ and $w = va$, $v \in L_1$, $a \in V$. Thus, $w_1w_3 = va$, $w_3 = w_2a$, $w_1w_2 = v$. From $w_3 \in \bar{L}$ it follows that $w_3 \in L_2$ and $w_3 \in L_1V/L$. From $w \in L\bar{L} \cap L_1 V$ it follows that there exist $v \in L_1$ and $a \in V$, $w = va$, $v = w_1w_2$ such that $w_2a = w_3 \in L_2$. We must show that $w \in D_1(L_1, L_2)$ and the only thing to be proved is that $w_2 \neq \lambda$. If $w_2 = \lambda$ then $w_3 = a$, $v = w_1 \in L$. Hence $v \in L_1$ and $v \in L$ which is not possible because $L_2/V - \{\lambda\}$ does not contain λ. Q.E.D.

Theorem 2.11. *The family* $\mathcal{L}_2$ *is not closed under* D_1, S_1, C_1.

Proof. Consider the languages

$$L_1 = \{ba^nba^nba^pba^q / n, p, q \geq 0\},$$

$$L_2 = \{ba^nba^nba^mb / n, m \geq 0\},$$

$$R = \{ba^nba^mba^pba^qb / n, m, p, q \geq 0\}.$$

We note that $L_1, L_2 \in \mathcal{L}_2$ and $R \in \mathcal{L}_3$. Much more, we have :

$$D_1(L_1, L_2) = L_1 \cup \{ba^nba^nba^nba^mb / n, m \geq 0\}$$

and

$$D_1(L_1, L_2) \cap R = \{ba^nba^nba^nba^mb / n, m \geq 0\}$$

from which it follows that $\mathcal{L}_2$ is not closed under D_1 since the language $\{ba^nba^nba^nba^mb / n, m \geq 0\}$ is not in $\mathcal{L}_2$ (and $\mathcal{L}_2$ is closed under intersection with regular sets).

From Corollary 2.4 and from the fact that $\mathcal{L}_2$ is closed under transpose it follows that L_2 is not closed under S_1. To show that $\mathcal{L}_2$ is not closed under C_1 we note that :

$$D_1(L_1, L_2) = h_V(C_1(\{c_V\}L_1, L_2))$$

where $L_1, L_2 \subseteq V^*$.

Now, if $\mathcal{L}_2$ would be closed under C_1 (since L_2 is closed under concatenation and homomorphism), then $\mathcal{L}_2$ would be closed under D_1 which is a contradiction. Q.E.D.

The closure of $\mathcal{L}_3$ under C remains an open problem.

The matching extension of a language $L_1 \in \mathcal{L}_2$ with a language $L_2 \in \mathcal{L}_3$ is another open problem. But it is easy to prove that if $L_1 \in \mathcal{L}_2$, $L_2 \in \mathcal{L}_3$ then $C_1^n(L_1, L_2) \in \mathcal{L}_2$, $\forall n \in N$.

Theorem 2.12. *Let* $L_1 \in \mathcal{L}_2$, $L_2 \in \mathcal{L}_3$. *Then* $D_1^n(L_1, L_2)$, $S_1^n(L_1, L_2)$, $C_1^n(L_1, L_2) \in \mathcal{L}_2$ *for any* $n \geq 0$.

Proof. $\mathcal{L}_3$ is closed under left and right derivatives, under concatenation and finite union. $\mathcal{L}_2$ is closed under concatenation, intersection with regular sets, literal homomorphisms and finite union. From Lemma 2.5 and Theorem 2.8, we see that these are the only operations involved in obtaining S_1, D_1 and C_1.

We proceed now to the study of the closure properties of some classes of Petri net languages under D_1, S_1, C_1.

Theorem 2.13. *The families* $\underline{L}$ *and* $\underline{P}$ *are closed under* D_1^n *for any* $n \geq 1$.

Proof. From Remark 2.2, it follows that it suffices to show that the families $\underline{L}$ and P are closed under D_1.

Let $L_1, L_2 \in \underline{X}$, where $X \in \{L, P\}$. Then, there exist the structure $\Gamma_i = (\Sigma_i, M_0^i, \mathcal{M}_i, l_i)$ such that $L_i = X(\Gamma_i)$, $i = 1, 2$. Let $\Sigma_i = (S_i, T_i; F_i, W_i)$. Without loss of generality we may assume that $S_1 \cap S_2 = 0$ and $T_1 \cap T_2 = 0$.

Some notations and technical considerations are in order before effectively starting the proof.

Let H_c be the set of pairs of transitions with the same label, i.e.

$$H_c = \{(t_1, t_2) \mid (t_1, t_2) \in T_1 \times T_2 \wedge l(t_1) = l(t_2)\}.$$

We consider two disjoint renamings of H_c, denoted T_c and T_c' and we suppose that T_c and T_c' are also disjoint from T_1 and T_2 (e.g. $T_c = \{t'/t \in H_c\}$, $T_c' = \{t''/t \in H_c\}$). These renamings generate two bijections

$$\varphi : T_c \to H_c, \ \psi : T_c' \to H_c$$

$$(\varphi(t') = t, \ \psi(t'') = t)$$

and the functions :

$$\varphi_i : T_c \to T_i, \ \psi_i : T_c' \to T_i, \ i = 1, 2$$

where $\varphi_i(t') = t_i$ iff $\varphi(t') = (t_1, t_2)$ and $\psi_i(t'') = t_i$ iff $\psi(t'') = (t_1, t_2)$.

Let now consider the structure $\Gamma = (\Sigma, M_0, \mathcal{M}, l)$ as being defined by

(i) $S = S_1 \cup S_2 \cup \{s_1, s_2\}$, $s_1, s_2 \notin S_1 \cup S_2$;

(ii) $T = T_1 \cup T_2 \cup T_c \cup T_c'$;

(iii) $F_3 = \{(s, t)/s \in S_1 \wedge t \in T_c \wedge (s, \varphi_1(t)) \in F_1\}$,

$F_4 = \{(t, s)/s \in S_1 \wedge t \in T_c \wedge (\varphi_1(t), s) \in F_1\}$,

$F_5 = \{(s, t)/s \in S_2 \wedge t \in T_c \wedge (s, \varphi_2(t)) \in F_2\}$,

$F_6 = \{(t, s)/s \in S_2 \wedge t \in T_c \wedge (\varphi_2(t), s) \in F_2\}$,

$F_7 = \{(s, t)/s \in S_1 \wedge t \in T_c' \wedge (s, \varphi_1(t)) \in F_1\}$,

$F_8 = \{(t, s)/s \in S_1 \wedge \in T_c' \wedge (\varphi_1(t), s) \in F_1\}$,

$F_9 = \{(s, t)/s \in S_2 \wedge t \in T_c' \wedge (s, \varphi_2(t)) \in F_2\}$.,

$F_{10}=\{(t, s)/s \in S_2 \wedge t \in T_c'' \wedge (\varphi_2(t), s) \in F_2\}$,

$F_{11}=\{(s_1, t)/t \in T_1 T_c''\}$,

$F_{12}=\{(t, s_1)/t \in T_1\}$,

$F_{13}=\{(s_2, t)/t \in T_2 \cup T_c\}$,

$F_{14}=\{(t, s_2)/t \in T_c \cup T_c''\}$.

Then, $F=\bigcup_{i=1}^{14} F_i$;

(iv) $W : F \to N$:

$$W(x, y)=\begin{cases} W_1(x, y) & \text{if } (x, y) \in F_1, \\ W_2(x, y) & \text{if } (x, y) \in F_2, \\ 1 & \text{if } (x, y) \in F_{11} \cup F_{12} \cup F_{13} \cup F_{14}, \\ W_1(x, \varphi_1(y)) & \text{if } (x, y) \in F_3, \\ W_1(\varphi_1(x), y) & \text{if } (x, y) \in F_4, \\ W_1(x, \psi_1(y)) & \text{if } (x, y) \in F_7, \\ W_1(\psi_1(x), y) & \text{if } (x, y) \in F_8, \\ W_2(x, \varphi_2(y)) & \text{if } (x, y) \in F_5, \\ W_2(\varphi_2(x), y) & \text{if } (x, y) \in F_6, \\ W_2(x, \psi_2(y)) & \text{if } (x, y) \in F_9, \\ W_2(\psi_2(x), y) & \text{if } (x, y) \in F_{10}; \end{cases}$$

(b) $M_0 : S \to N$:

$$M_0(x)=\begin{cases} M_0^1(x)^+ & \text{if } x \in S_1, \\ M_0^2(x)^+ & \text{if } x \in S_2, \\ 1 & \text{if } x=s_1 \\ 0 & \text{if } x=s_2; \end{cases}$$

(c) If $X=P$ then $\mathscr{M}=\emptyset$; otherwise $\mathscr{M}$ is constructed as follows: For any marking $M_1 \in \mathscr{M}_1$ let $M \in N^S$ be a new marking such that $M/S_1=M_1, M/S_2=M_0^2, M(s_1)=$ $=1$ and $M(s_2)=0$. Let $\mathscr{M}'$ be the set of these new markings. For any marking $M_1 \in \mathscr{M}_1$ and $M_2 \in \mathscr{M}_2$ let $M \in N^S$ be a new marking such that $M/S_1=M_1$, $M/S_2=M_2$ and $M(s_1)=M(s_2)=1$. Let $\mathscr{M}''$ be the set of these new markings. Then, $\mathscr{M}=\mathscr{M}' \cup \mathscr{M}''$;

(d) $l : T \to X_1 \cup X_2$:

$$l(x)=\begin{cases} l_1(x) & \text{if } x \in T_1, \\ l_2(x) & \text{if } x \in T_2, \\ l_1(\varphi_1(x)) & \text{if } x \in T_c, \\ l_1(\psi_1(x)) & \text{if } x \in T_c'', \end{cases}$$

where $l_i : T_i \to X_i$, $i=1, 2$.

The intuition behind this formale definition can be expressed by the following picture.

The doted arcs represent the actual connections between elements of the corresponding sets and the others the fact that the s_i are connected with every element of the corresponding set. Let us show that $L(\Gamma)=D_1(w_1, w_2)$.

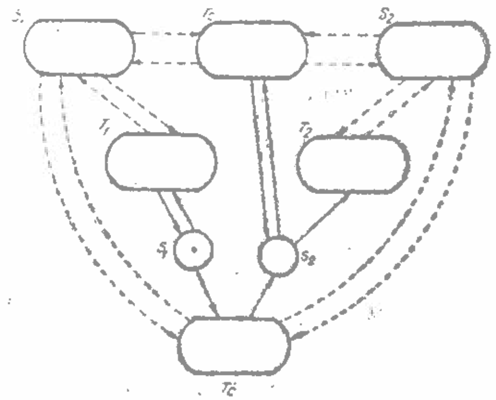

Fig. 1

$L(\Gamma) \subseteq D_1(L_1, L_2)$: Let $w \in L(\Gamma)$. Then there exist $u \in T^*$ and $M \in \mathscr{M}$ such that $M_0[u\rangle M$ and $l(u)=w$. We must distinguish two cases

a) $M \in \mathscr{M}'$. Some transitions from T_1 and any transition from T_c'' are enabled at the initial marking M_0. The application of a T_c''-transsition leads to a new marking in which the token from s_1 is moved into s_2 and transitions from T_c'' are nolonger applicable. Because we have $M(s_1)=1$ and $M(s_2)=0$ we deduce that $u \in T_1^*$ (the same is true if the first transition is T_1). From the fact that $M/S_1 \in \mathscr{M}_1$ we have $w=l(u)=l_1(u) \in L_1$ and hence $w \in D_1(L_1, L_2)$.

b) $M \in \mathscr{M}''$. In this case only the following sequences of transitions are from T_1; a unique transition from T_c''; transition from T_c; a unique transition from T_2. Formally we have $u=u_1 t' u_2 t''$, $u_1 \in T_1^*$, $t' \in T_c''$, $u_2 \in T_c^*$, $t' \in T_2$, from which we obtain that $w=l(u)=l(u_1)l(t)l(u_2)l(t')$.

Let $u_2=t_1^2,\ldots, t_n^2$ and then

$l(u_1)=l_1(u_1)$, $l(t)=l_1(\psi_1(t))$ $l(u_2)=l_1(\varphi_1(t_1^2))\ldots l_1(\varphi_1(t_n^2))$, $l(t')=l_2(t')$.

But $\forall\, i=\overline{1, n}$ we have $l_1(\psi_1(t))=l_2(\psi_2(t))$ and $l_1(\varphi_1(t_i^2))=l_2(\varphi_2(t_i^2))$. Much more, $l_1(u_1)l_1(\psi_1(t))$. $l_1(\varphi_1(t_1^2))\ldots l_1(\varphi_1(t_n^2)) \in L_1$ and $l_2(\psi_2(t))l_2(\varphi_2(t_1^2))\ldots l_2(\varphi_2(t_n^2))l_2(t') \in L_2$. Denoting $l_1(u_1)$ by w_1, $l_1(\psi_1(t))l_1(\varphi_1(t_1^2))\ldots l_1(\varphi_1(t_n^2))$ (which equals $l_2(\psi_2(t))l_2(\varphi_2(t_1^2))\ldots$ $\ldots l_2(\varphi_2(t_n^2))$) by w_2 and $l_2(t')$ by a, we deduce that $w=w_1 w_2 a$, $w_2 \neq \lambda$, $w_1 w_2 \in w_1$, $L_2 a \in w_2$ and hence $w \in D_1(L_1, L_2)$.

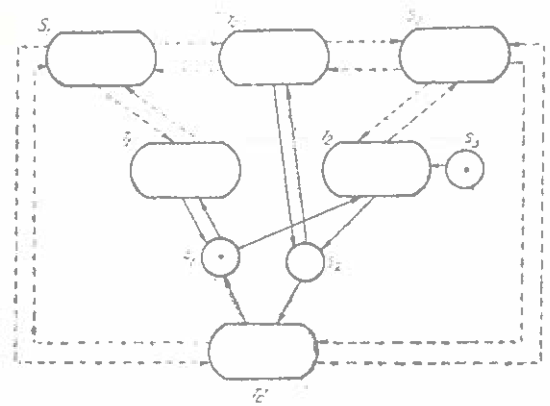

Fig. 2

The converse inclusion can be proved in a similar way.

The previous proof can be modified without major difficulties to obtain $P(\Gamma)=D_1(L_1, L_2)$.

Theorem 2.14. *L is closed under S_1^n, $\forall n \geq 1$.*

Proof. If suffices to show that L is closed under S_1. This proof parallels the prece-

eding proof and we give only the intuitive representation of a structure Γ for which $L(\Gamma)=S_1(L_1, L_2)$, $L_1, L_2 \in L$.
The initial marking is given by

$$M_0(s)=\begin{cases} M_0^1(s) & \text{if } s\in S_1 \\ M_0^2(s) & \text{if } s\in S_2 \\ 1 & \text{if } s\in\{s_1, s_3\} \\ 0 & \text{if } s=s_2. \end{cases}$$

To obtain the set $\mathcal{M}$ of final markings for Γ we define the following sets of markings

$$\mathcal{M}=\{M\in N^S/(\exists M_1\in\mathcal{M}_1,\ M/S_1=M_1)\wedge M/S_2=M_0^2\wedge M(s_1)=M(s_3)= =0\wedge M(s_2)=0\}$$

$$\mathcal{M}''=\{M\in N^S/(\exists M_1\in\mathcal{M}_1)\,(\exists M_2\in\mathcal{M}_2)\,(M/S_1=M_1\wedge M/S_2=M_2\wedge M(s_2)= =M(s_3)=0\wedge M(s_1)=1).$$

Then, $\mathcal{M}=\mathcal{M}'\cup\mathcal{M}''$ and Γ is completely defined.

Corollary 2.14. *The family $\underline{L}$ is closed under C_1^n operation, for any $n\geqslant 1$.*

Proof. The statement follows from Remark 2.2, Lemma 2.5 and the fact that $\underline{L}$ is closed under union [4].

The results obtained in [12] and in this paper can be summarized in the following diagrams.

	$\mathcal{L}_3$	$\mathcal{L}_1$	$\mathcal{L}_0$	$\underline{L}$	$\underline{P}$	$\underline{L}^\lambda$	$\underline{P}^\lambda$	$\mathcal{L}_2\times\mathcal{L}_3$
D_1^n	yes	yes	yes	yes	yes	yes	yes	yes
S_1^n	yes	yes	yes	yes	?	yes	yes	yes
C_1^n	yes	yes	yes	yes	?	yes	yes	yes

where $n\geqslant 1$,

	$\mathcal{L}_3$
D_1	no
S_1	no
C_1	no

	$\underline{L}^f$	$\underline{G}^f$	$\underline{P}^f$	$\underline{T}^f$
D	no	no	no	no
S	no	no	no	no
C	no	no	no	no

3. Concluding Remarks and Open Problems. The main open proble ms are derived by inspecting the above diagrams. Moreover, the results obta ind suggest that a general study of a "measure of closedness" for classes of lang uages is in order. We shall try to do that in future papers.

REFERENCES

1. Best, E., Fernandez, C. — *Notations and Terminology on Petri Net Theory*, Arbeitspapiere der GMD 195, Second Edition, 1987.
2. Hopcroft, J. E., Ullman J. D. — *Formal Languages and Their Relation to Automata*, Addison-Wesley, Publ. Comp., 1969.
3. Harrison M. A. — *Introduction to Formal Language Theory*, Addison-Wesley, 1978.
4. Jantzen, M. — *Language Theory of Petri Nets*, Springer-Verlag, LNCS 254, 1986.
5. Păun, Gh. — *A Generative Model of Conversation*, Semiotica, 17, 1, 1976.
6. Păun, Gh. — *Generative Grammars for some Economic Activities (Scheduling and Marshalling Problems)*, Foundations of Control Engineering, Poznan, 2, 1, 1977.
7. Păun, Gh. — *A Formal Linguistic Approach to the Production Process*, Foundations of Control Engineering, Poznan, 1, 3, 1976.
8. Păun, Gh. — *On a prolongation operation of languages*, Bull. Math. de la Soc. de Sci. Math. de RSR, 23 (71), 3, 1979.
9. Păun, Gh. — *The Generative Mechanisms of Economic Processes*, Ed. Tehnică, 1980 (in Romanian).
10. Reising, W. — *Place / Transition Systems*, Springer-Verlag LNCS 254, 1986.
11. Starke, P. H. — Free Petri Nets Languages, Springer-Verlag, LNCS 63, 1988.
12. Ţiplea, F. L., Jucan, T., Masalagiu, C. — *Matching Extensions for Petri Net Languages* (this volume).

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