$$\bar{D}_X Y - \varphi(v\bar{T}(hX, \varphi^{-1}(hY)) - v\bar{T}(hX, vY) - h\bar{T}(vX, hY) - \varphi^{-1}(h\bar{T}(vX, \varphi(vY)) = \tag{2.6}$$
$$= D_X Y - \varphi(vT(hX, \varphi^{-1}(hY)) - vT(hX, vY) - hT(vX, hY) -$$
$$- \varphi^{-1}(hT(vX, \varphi(vY)). \quad \forall X, Y \in \mathcal{X}(TM).$$

Let D be a fixed normal connection and $\bar{D}$ an arbitrary normal connection. From (2.6) we obtain

Theorem 2.4. *The general solution of system* (2.2) (2.5) *is given by*

$$\overline{\tau}(X, Y) = \varphi(v\underset{1}{\mathcal{B}}(hX, \varphi^{-1}(hY)) + v\underset{1}{\mathcal{B}}(hX, vY) + h\underset{2}{\mathcal{B}}(vX, hY) + \tag{2.7}$$
$$+ \varphi^{-1}(h\underset{2}{\mathcal{B}}(vX, \varphi(vY)) \quad \forall X, Y \in \mathcal{X}(TM),$$

where $\underset{1}{\mathcal{B}}$ *and* $\underset{2}{\mathcal{B}}$ *are arbitrary d-tensor fields.*

Also, we observe that if, $\underset{1}{\mathcal{U}}$ and $\underset{2}{\mathcal{U}}$ are defined by

$$\underset{1}{\mathcal{U}}(X, Y) = v\underset{1}{\mathcal{B}}(hX, vY)\,; \quad \underset{2}{\mathcal{U}}(X, Y) = h\underset{2}{\mathcal{B}}(vY, hX) \quad \forall X, Y \in \mathcal{X}(TM), \tag{2.8}$$

we obtain

$$\overline{\tau}(X, Y) = \varphi(v\underset{1}{\mathcal{U}}(X, \varphi^{-1}(hY)) + \underset{1}{\mathcal{U}}(X, Y) + \underset{2}{\mathcal{U}}(Y, X) + \varphi^{-1}(\underset{2}{\mathcal{U}}(\varphi(vY), X). \tag{2.9}$$

We have

Theorem 2.5. *The group* $G(\varphi)$ *of the general transformations which preserve the class of normal connections, on* TM, *is given by the transformations* (2.1), *where* $\overline{\tau}$ *is given by* (2.7) *or* (2.9).

Theorem 2.6. *The group* $G(\varphi)$ *admit the invariants* $\underset{1}{\mathcal{T}}$ *and* $\underset{2}{\mathcal{T}}$.

Since a $\overset{P}{D}$ connection on TM, is the unique normal connection with the intrinsec torsion T given by (1.12) we obtain.

Theorem 2.7. *The group* $G(\varphi)$ *is generated by* $\overset{P}{D}$.

We observe that locally, in adapted basis $\{\delta/\delta x^i, \partial/\partial y^i\}$ the invariant $\tilde{\mathcal{T}} = \mathcal{T}$ is trivial, since $\mathcal{T} = \tilde{N}$ and we have $\tilde{N} = 0$.

In another note we shall give a generalisation of this to almost seminormal connection on TM.

REFERENCES

1. Miron, R. — *Vector bundles. Finsler geometry*, The Proc. of the Nat. Sem. on Finsler Spaces, Braşov, 1982.
2. Miron, R. and Anastasiei, M. — *Fibrate vectoriale. Spaţii Lagrange. Aplicaţii in teoria relativităţii*. Ed. Acad. R. S. R. Bucureşti, 1987.
3. Stavre, P. — *The d-linear connections*. The Proc. of Nat. Sem. on Finsler and Lagrange Spaces. Braşov, February, 1988.
4. Stavre, P. and Virlan I. — *Some special classes of d-linear connections*. The Proc. of Nat. Conf. on Geometry and Topology. Suceava, România, 1988.

Received 5.V.1989

Department of Mathematics,
University of Craiova
1100 Craiova, România

ANALELE ŞTIINŢIFICE ALE UNIVERSITĂŢII „AL. I. CUZA"
Tomul XXXVI, s. I a. Matematică, 1990, f. 4.

A NOTE ON THE PROJECTIVE CURVATURE TENSOR OF A SEMISYMMETRIC RECURRENT METRIC CONNECTION ON A RIEMANNIAN MANIFOLD

BY

S. MUKHOPADHYAY and B. BARUA

0. Introduction. Let M be an n-dimensional Riemannian manifold with Riemannian metric g and L.C. connection ∇.

An affine connection $\bar{\nabla}$ on a Riemannian manifold is called a recurrent metric connection [1] if there exists a differentiable oneform μ on M such that

$$(\bar{\nabla}_X g)(Y, Z) = \mu(X) g(Y, Z) \tag{0.1}$$

holds for all differentiable vectorfields $X, Y, Z, \ldots$ on M. μ is called the one-form of recurrence. If, further, the torsion tensor $T(X, Y) = \bar{\nabla}_X Y - \bar{\nabla}_Y X - [X, Y]$ of the connection $\bar{\nabla}$ is of the form

$$T(X, Y) = \pi(Y)X - \pi(X)Y \tag{0.2}$$

where π is a differentiable one-form on M, then $\bar{\nabla}$ is called a semisymmetric recurrent metric connection [2] on M.

Ray and Barua [2] have shown that such a connection is of the form

$$\bar{\nabla}_X Y = \nabla_X Y + \alpha(Y)X - g(X, Y)A - \frac{1}{2}\mu(X)Y \tag{0.3}$$

where α is a differentiable one-form given by

$$\alpha(X) = \pi(X) - \frac{1}{2}\mu(X) \tag{0.4}$$

and A is a differentiable vectorfield satisfying

$$g(A, X) = \alpha(X) \tag{0.5}$$

In the present note we have considered M to be a Reimannian manifold admitting a semisymmetric recurrent metric connection.

The aim of the present note is to see under what conditions the projective curvature tensors of the two connections are equal.

1. Projective curvature tensors. Let $\bar{R}(X, Y)Z$ and $\bar{S}(Y, Z)$ denote the curvature tensor and the Ricci tensor of the connection $\bar{\nabla}$. From (0.3) we can see that if $R(X, Y)Z$ is the Riemann curvature tensor then

$$\bar{R}(X, Y)Z = R(X, Y)Z - \lambda(Y, Z)X + \lambda(X, Z)Y - g(Y, Z)LX + g(X, Z)LY - d\mu(Y, Z)X, \tag{1.1}$$

where

$$\lambda(Y, Z) = (\nabla_Y \alpha)(Z) - \alpha(Y)\alpha(Z) - \frac{1}{2}\alpha(A)g(Y, Z) \tag{1.2}$$

and

$$LY = \nabla_Y A - \alpha(Y)A + \frac{1}{2}\alpha(A)Y. \tag{1.3}$$

Also if $S(Y, Z)$ is the Ricci tensor of the connection ∇ then from (1.1)

$$\bar{S}(Y, Z) = S(Y, Z) - (n-2)\lambda(Y, Z) - \operatorname{tr}\lambda \, . \, g(Y, Z) + d\mu(Y, Z). \tag{1.4}$$

From (1.4) we can see that the Ricci tensor $\bar{S}(Y, Z)$ of the connection $\bar{\nabla}$ is not symmetric.

The generalized projective curvature tensor for the connection ∇ is given by [3]

$$\begin{aligned}\bar{W}(X, Y)Z = \bar{R}(X, Y)Z &+ \frac{1}{n+1}\{\bar{S}(X, Y) - \bar{S}(Y, X)\}Z + \\ &+ \frac{1}{n^2-1}[\{n\bar{S}(X, Z) + \bar{S}(Z, X)\}Y - \{n\bar{S}(Y, Z) + \bar{S}(Z, Y)\}X].\end{aligned} \tag{1.5}$$

Substituting for $\bar{R}(X, Y)Z$ and $S(X, Y)$ from (1.1) and (1.4) we find

$$\begin{aligned}\bar{W}(X, Y)Z = W(X, Y)Z &- \frac{2n-1}{n^2-1}\{\lambda(Y, Z)X - \lambda(X, Z)Y\} - \\ &- \frac{n-2}{n^2-1}\{\lambda(Z, X)Y - \lambda(Z, Y)X\} - \{g(Y, Z)LX - g(X, Z)LY\} - \\ &- \frac{\operatorname{tr}\lambda}{n-1}\{g(X, Z)Y - g(Y, Z)X\} - \frac{n-1}{n+1}d\mu(X, Y)Z + \\ &+ \frac{1}{n+1}\{d\mu(X, Z)Y - d\mu(Y, Z)X\} - \frac{2(n-2)}{n+1}d\alpha(X, Y)Z.\end{aligned} \tag{1.6}$$

If

$$\bar{W}(X, Y)Z = W(X, Y)Z \tag{1.7}$$

we get from (1.6)

$$\begin{aligned}&(2n-1)\{\lambda(Y, Z)X - \lambda(X, Z)Y\} + (n-2)\{\lambda(Z, X)Y - \lambda(Z, Y)X\} + \\ &+ (n^2-1)\{g(Y, Z)LX - g(X, Z)LY\} + (n+1)\operatorname{tr}\lambda\{g(X, Z)Y - g(y, Z)X\} + \\ &+ 2(n-1)(n-2)d\alpha(X, Y)Z + (n-1)^2 d\mu(X, Y)Z - \\ &- (n-1)\{d\mu(X, Z)Y - d\mu(Y, Z)X\} = 0.\end{aligned} \tag{1.8}$$

Taking scalar product of both sides of (1.8) with Z, we find

$$2d\alpha(X, Y) + d\mu(X, Y) = 0$$

and substituting for α from (0.4) we get

$$d\pi(X, Y) = 0 \tag{1.9}$$

for all vector fields X, Y on M.

Again, let μ be given by

$$\mu = 2\pi \text{ and } d\pi = 0. \tag{1.10}$$

Then from (0.4) we find $\alpha = 0$ and $LX = 0$ for all X. Hence from (1.1)

$$\bar{R}(X, Y)Z = R(X, Y)Z, \quad \bar{S}(Y, Z) = S(Y, Z)$$

and consequently

$$\bar{W}(X, Y)Z = W(X, Y)Z.$$

We can now state tge following

Theorem 1. *A necessary condition for the generalized projective curvature tensor of a semisymmetric recurrent metric connection be equal to Weyl's projective curvature tensor is that the one-form π is closed, that is, $d\pi = 0$.*

Theorem 2. *Sufficient conditions for the equality of the generalized projective curvature tensors of a semisymmetric recurrent metric connection and the L.C. connection on a Riemannian manifold are that the relations (1.10) hold good.*

REFERENCES

1. Smaranda, D. and Andonie, O. C. — Ann. Fac. Sci. Univ. Nat. Zaire (Kinshasa) Sect. Math. Phys, 2 (1976), 265—270.
2. Ray, A. K. and Barua, B. — *Semi symmetric recurrent metric connection on a Riemannian manifold* (Communicated).
3. Mishra, R. S. — *Structures on differentiable manifold and their applications.* Chandrama Prakashan, Allahabad. 1984.

Received 11.VIII.1989

Department of Pure Mathematics
Calcutt University
Calcutta, India